Exercises to "Standard Model of Particle Physics II"

Winter 2020/21

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Sheet 06 - December 9, 2020

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
December 16, 2020 - via e-mail, **before 14:00**

Discussion of solutions:
December 16, 2020 - on zoom

Problem 11: *Optical Theorem* [**10 Points**]

In the context of partial wave expansion, the scattering amplitude at an angle θ for the process $a + b \to c + d$ is given by

$$f(\theta) = \frac{1}{2ki} \sum_l (2l+1)(\eta_l \mathrm{e}^{2i\delta_l} - 1) P_l(\cos\theta),$$

where P_l are the Legendre-polynomials, θ is the scattering angle, k is the wavenumber in the incident direction and δ_l and η_l are both real functions. δ_l denotes the phase difference and η_l was introduced to describe inelastic scattering. We have $\eta_l = 1$ for elastic and $\eta_l < 1$ for inelastic scattering.
The optical theorem states that the cross section in a forward scattering process is given by

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \mathrm{Im}\left[f(0)\right].$$

a) Show with the help of the optical theorem that

$$\sigma_{\text{tot}} = \frac{2\pi}{k^2} \sum_l (2l+1)(1 - \eta_l \cos(2\delta_l)).$$

b) The differential cross section for elastic scattering is given by

$$\frac{d\sigma_{\text{el}}}{d\Omega} = |f(\theta)|^2.$$

From this, derive the following expression for the elastic scattering cross section

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l+1)|\eta_l \mathrm{e}^{2i\delta_l} - 1|^2.$$

c) From a) and b) it follows that

$$\sigma_{\text{inel}} = \frac{\pi}{k^2} \sum_l (2l+1)(1 - \eta_l^2).$$

Show with this equation that for the reaction $\nu_\mu + e^- \to \mu^- + \nu_e$ we obtain the relation

$$\sigma(\nu_\mu + e^- \to \mu^- + \nu_e) \leq \frac{2\pi}{E_{\text{cm}}^2}, \tag{1}$$

where E_{cm} denotes the center-of-mass energy (k should be considered in the center-of-mass system). Note that this is an $l = 0$ scattering process and that a spin factor $1/(2s+1)$ should be taken into account.

d) In Fermi theory the cross section is given by

$$\sigma = \frac{G_{\mathrm{F}}^2 s}{\pi}, \tag{2}$$

where G_{F} is Fermi's constant and $\sqrt{s}$ denotes the invariant mass.

Use Eqs. (1) and (2) to find the energy at which Fermi theory breaks down.

Problem 12: *Custodial Symmetry and the ρ-Parameter* [**10 Points**]

The 1-loop correction to the ρ-parameter is given by

$$\Delta\rho = \frac{3G_F}{8\pi^2 2\sqrt{2}} \left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log\frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 \sin^2\theta_W \log\frac{m_h^2}{m_Z^2} \right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

a) Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.

b) As discussed in the lecture, the Higgs potential is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$. Define

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and } \Phi = \left(\phi, \tilde{\phi}\right),$$

with ϕ the Higgs doublet, $\tilde{\phi} = i\sigma_2\phi^*$, and σ_2 the second Pauli matrix. Show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ symmetry only if $g_t = g_b$.

c) Show by analyzing the kinetic term of the Higgs boson that it is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry only if the hypercharge gauge coupling is zero. Start by writing down the covariant derivative on Φ (taking into account the fact that ϕ and $\tilde{\phi}$ have hypercharges 1 and -1 respectively).