Exercises to "Standard Model of Particle Physics II"

Winter 2016/17

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Problem 11: *Running couplings* [**10 Points**]

a) In QED the coupling $\alpha = e^2/4\pi$ is a running coupling. For low energies one obtains $\alpha(\mu = m_e) = 1/137$. The running of the QED coupling with only one lepton is given by

$$\alpha_{\text{QED}}(\mu) = \frac{\alpha(m_e)}{1 - \frac{2\alpha(m_e)}{3\pi}\ln(\frac{\mu}{m_e})} \ . \tag{1}$$

At which scale does $\alpha_{\text{QED}}(\mu)$ become infinite? (If further charged leptons and quarks are considered, the coupling increases even faster.)

b) In QCD, with $\alpha_s = g_3^2/4\pi$, the running coupling

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{(33-2f)\alpha_s(\mu_0)}{6\pi}\ln(\frac{\mu}{\mu_0})} \tag{2}$$

is obtained, where f denotes the respective number of quark flavours with mass $2m_q \leq \mu$ in the considered energy range.

The experimental boundary conditions are $\alpha_s(\mu_0 = m_Z = 91 \text{ GeV}) = 0.12$, $m_t = 175$ GeV, $m_b = 4.8$ GeV, $m_c = 1.4$ GeV and $m_u \simeq m_d \simeq m_s \simeq 0$.

i) Determine the pole $\mu = \Lambda_{\text{QCD}}$ of eq. (2).

ii) For which μ does the coupling $\alpha_s(\mu)$ become very small (asymptotic freedom), and where does perturbation theory break down?

iii) Determine the value of $\alpha_s(\mu)$ in the different energy ranges ($2m_q \leq \mu \leq 2m_t$ etc.) at the thresholds.

iv) $\alpha_s(\mu)$ should be continuous. However, from the threshold values calculated in the last part we saw that this is not yet true. To make $\alpha_s(\mu)$ continuous, promote μ_0 to a function $\mu_0(f)$ with $\mu_0(f = 5) = 91$ GeV. Find a relation between $\mu_0(f)$ and $\mu_0(f-1)$ assuming that $\alpha_s(\mu_0(f)) = 0.12$ for all f. With this relation determine the values of $\mu_0(4)$ and $\mu_0(6)$.

c) Draw $\alpha_s^{-1}(\mu)$ and $\alpha_{\text{QED}}^{-1}(\mu)$ as functions of $\ln(\mu)$. What could the intersection of the curves indicate?

Problem 12: *W-polarisation* [**5 Points**]

For a massive vector boson with four-momentum $k^\mu = (E,\, |k|\vec{n})$ propagation along the direction $\vec{n} = (\sin\theta,\, 0,\, \cos\theta)$, the polarisation vectors corresponding to the helicities $\lambda = 0, \pm 1$ can be written as

$$\epsilon^\mu_{\lambda=0} = m_W^{-1}\left(|k|,\, E\sin\theta,\, 0,\, E\cos\theta\right),$$
$$\epsilon^\mu_{\lambda=\pm 1} = \frac{1}{\sqrt{2}}\left(0,\, \mp\cos\theta,\, -i,\, \pm\sin\theta\right).$$

Check that the completeness relation holds, i.e. verify that

$$\sum_\lambda \epsilon^{\mu\,*}_\lambda \epsilon^\nu_\lambda = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}.$$

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Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106