

Exercises to “Standard Model of Particle Physics II”

Winter 2019/20

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Lecture webpage: <https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html>

Hand-in of solutions:

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15:45, Philosophenweg 12, kHS

Discussion of solutions:

November 27, 2019

Problem 10: *Optical Theorem* [10 Points]

In the context of partial wave expansion, the scattering amplitude at an angle θ for the process $a + b \rightarrow c + d$ is given by

$$f(\theta) = \frac{1}{2ki} \sum_l (2l + 1)(\eta_l e^{2i\delta_l} - 1) P_l(\cos \theta),$$

where P_l are the Legendre-polynomials, θ is the scattering angle, k is the wavenumber in the incident direction and δ_l and η_l are both real functions. δ_l denotes the phase difference and η_l was introduced to describe inelastic scattering. We have $\eta_l = 1$ for elastic and $\eta_l < 1$ for inelastic scattering.

The optical theorem states that the cross section in a forward scattering process is given by

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} [f(0)].$$

a) Show with the help of the optical theorem that

$$\sigma_{\text{tot}} = \frac{2\pi}{k^2} \sum_l (2l + 1)(1 - \eta_l \cos(2\delta_l)).$$

b) The differential cross section for elastic scattering is given by

$$\frac{d\sigma_{\text{el}}}{d\Omega} = |f(\theta)|^2.$$

From this, derive the following expression for the elastic scattering cross section

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l + 1) |\eta_l e^{2i\delta_l} - 1|^2.$$

c) From a) and b) it follows that

$$\sigma_{\text{inel}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - \eta_l^2).$$

Show with this equation that for the reaction $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ we obtain the relation

$$\sigma(\nu_\mu + e^- \rightarrow \mu^- + \nu_e) \leq \frac{2\pi}{E_{\text{cm}}^2}, \quad (1)$$

where E_{cm} denotes the center-of-mass energy (k should be considered in the center-of-mass system). Note that this is an $l = 0$ scattering process and that a spin factor $1/(2s + 1)$ should be taken into account.

d) In Fermi theory the cross section is given by

$$\sigma = \frac{G_F^2 s}{\pi}, \quad (2)$$

where G_F is Fermi's constant and \sqrt{s} denotes the invariant mass.

Use Eqs. (1) and (2) to find the energy at which Fermi theory breaks down.

Problem 11: Stückelberg Mechanism [10 Points]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method contains a real scalar field σ together with the Z' -boson associated to $U(1)'$.

Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} + \frac{1}{2}(M_{Z'}Z'_\mu + \partial_\mu\sigma)(M_{Z'}Z'^\mu + \partial^\mu\sigma) + i\bar{\psi}\gamma^\mu(\partial_\mu - ig'Y'Z'_\mu)\psi - m\bar{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge Y') and gauge boson are given by

$$\psi \rightarrow e^{-ig'Y'\theta(x)}\psi, \quad Z'_\mu \rightarrow Z'_\mu - \partial_\mu\theta(x).$$

Calculate the gauge transformation of the real scalar σ that makes the Lagrangian invariant and show the invariance of the other terms. Can you fix a gauge to eliminate σ from the theory? Count degrees of freedom in both gauges.