Exercises to "Standard Model of Particle Physics II"

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Problem 9: *Optical Theorem* **[10 Points]**

The scattering of particles $a + b \to c + d$ is described by the amplitude

$$f(\theta) = \frac{1}{2ki} \sum_l (2l+1)(\eta_l \mathrm{e}^{2i\delta_l} - 1) P_l(\cos\theta),$$

where P_l are the Legendre-polynomials, θ is the scattering angle, k is the wavenumber in the incident direction and δ_l and η_l are both real functions. δ_l denotes the phase difference and η_l was introduced to describe inelastic scattering. We have $\eta_l = 1$ for elastic and $\eta_l < 1$ for inelastic scattering.
The optical theorem states that the cross section in a forward scattering process is given by

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \mathrm{Im}\left[f(0)\right].$$

a) Show with the help of the optical theorem that

$$\sigma_{\text{tot}} = \frac{2\pi}{k^2} \sum_l (2l+1)(1 - \eta_l \cos(2\delta_l)).$$

b) The differential cross section for elastic scattering is given by

$$\frac{d\sigma_{\text{el}}}{d\Omega} = |f(\theta)|^2.$$

From this, derive the following expression for the elastic scattering cross section

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l+1)|\eta_l \mathrm{e}^{2i\delta_l} - 1|^2.$$

c) From a) and b) it follows that

$$\sigma_{\text{inel}} = \frac{\pi}{k^2} \sum_l (2l+1)(1 - \eta_l^2).$$

Show with this equation that for the reaction $\nu_\mu + e^- \to \mu^- + \nu_e$ we obtain the relation

$$\sigma(\nu_\mu + e^- \to \mu^- + \nu_e) \leq \frac{2\pi}{E_{\text{cm}}^2}, \tag{1}$$

where E_{cm} denotes the center-of-mass energy. Note that this is an $l = 0$ scattering process and that a spin factor $(2s+1)$ should be taken into account.

d) In Fermi theory the cross section is given by

$$\sigma = \frac{G_{\text{F}}^2 s}{\pi}, \tag{2}$$

where G_{F} is Fermi's constant and $\sqrt{s}$ denotes the invariant mass.

Use Eqs. (1) and (2) to find the energy at which Fermi theory breaks down.

Problem 10: *Stückelberg Mechanism* [**10 Points**]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method contains a real scalar field σ together with the Z'-boson associated to $U(1)'$.
Consider the Lagrangian

$$\mathscr{L} = -\frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} + \frac{1}{2}(M_{Z'}Z'_\mu + \partial_\mu\sigma)(M_{Z'}Z'^\mu + \partial^\mu\sigma) + i\overline{\psi}\gamma^\mu(\partial_\mu - ig'Y'Z'_\mu)\psi - m\overline{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge Y') and gauge boson are given by

$$\psi \to e^{-ig'Y'\theta(x)}\psi, \qquad Z'_\mu \to Z'_\mu - \partial_\mu\theta(x).$$

Calculate the gauge transformation of the real scalar σ that makes the Lagrangian invariant and show the invariance of the other terms. Can you fix a gauge to eliminate σ from the theory? Count degrees of freedom in both gauges.

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Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106