

# Exercises to “Standard Model of Particle Physics II”

Winter 2015/16

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Sheet 5

November 13, 2015

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## Exercise 11: Stückelberg Mechanism [10 Points]

For a gauged abelian symmetry  $U(1)'$  (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method contains a real scalar field  $\sigma$  together with the  $Z'$ -boson associated to  $U(1)'$ .

Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} + \frac{1}{2}(M_{Z'}Z'_\mu + \partial_\mu\sigma)(M_{Z'}Z'^\mu + \partial^\mu\sigma) + i\bar{\psi}\gamma^\mu(\partial_\mu - ig'Y'Z'_\mu)\psi - m\bar{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with  $U(1)'$  charge  $Y'$ ) and gauge boson are given by

$$\psi \rightarrow e^{-ig'Y'\theta(x)}\psi, \quad Z'_\mu \rightarrow Z'_\mu - \partial_\mu\theta(x).$$

Calculate the gauge transformation of the real scalar  $\sigma$  that makes the Lagrangian invariant and show the invariance of the other terms. Can you fix a gauge to eliminate  $\sigma$  from the theory? Count degrees of freedom in both gauges.

## Exercise 12: Optical Theorem [10 Points]

The scattering of particles  $a + b \rightarrow c + d$  is described by the amplitude

$$f(\theta) = \frac{1}{2ki} \sum_l (2l+1)(\eta_l e^{2i\delta_l} - 1)P_l(\cos\theta),$$

where  $P_l$  are the Legendre-polynomials,  $\theta$  is the scattering angle,  $k$  is the wavenumber in the incident direction and  $\delta_l$  and  $\eta_l$  are both real functions.  $\delta_l$  denotes the phase difference and  $\eta_l$  was introduced to describe inelastic scattering. We have  $\eta_l = 1$  for elastic and  $\eta_l < 1$  for inelastic scattering.

The optical theorem states that the cross section in a forward scattering process is given by

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im}[f(0)].$$

a) Show with the help of the optical theorem that

$$\sigma_{\text{tot}} = \frac{2\pi}{k^2} \sum_l (2l+1)(1 - \eta_l \cos(2\delta_l)).$$

b) The differential cross section for elastic scattering is given by

$$\frac{d\sigma_{\text{el}}}{d\Omega} = |f(\theta)|^2.$$

From this, derive the following expression for the elastic scattering cross section

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l+1)|\eta_l e^{2i\delta_l} - 1|^2.$$

c) From a) and b) it follows that

$$\sigma_{\text{inel}} = \frac{\pi}{k^2} \sum_l (2l+1)(1 - \eta_l^2).$$

Show with this equation that for the reaction  $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$  we obtain the relation

$$\sigma(\nu_\mu + e^- \rightarrow \mu^- + \nu_e) \leq \frac{2\pi}{E_{\text{cm}}^2}, \quad (1)$$

where  $E_{\text{cm}}$  denotes the center-of-mass energy. Note that this is an  $l = 0$  scattering process and that a spin factor  $(2s + 1)$  should be taken into account.

d) In Fermi theory the cross section is given by

$$\sigma = \frac{G_{\text{F}}^2 s}{\pi}, \quad (2)$$

where  $G_{\text{F}}$  is Fermi's constant and  $\sqrt{s}$  denotes the invariant mass.

Use Eqs. (1) and (2) to find the energy at which Fermi theory breaks down.

**Exercise 13: Mass matrices and mixing angles [10 Points]**

A general (Dirac) mass term for fermions is given by

$$\mathcal{L}_M = \bar{\psi}_{i,L} M_{ij} \psi_{j,R} + \text{h.c.}$$

where  $M$  is hermitian and given by a  $n \times n$  Yukawa coupling matrix  $Y$  times the Higgs vev.

- a) Show that one can choose a bi-unitary transformation  $UMV^\dagger$  to diagonalize  $M$ , such that no diagonal element  $UMV^\dagger = D := \text{diag}(m_1, m_2, \dots, m_n)$  is negative. The matrices  $U$  and  $V$  are unitary.
- b) Show that for a real mass matrix  $M$  one can choose orthogonal diagonalization matrices.
- c) As an example for calculable mixing angles consider a simple  $2 \times 2$  mass matrix of the form

$$M = \begin{bmatrix} 0 & a \\ a^* & b \end{bmatrix}.$$

The unitary matrix that diagonalizes  $M$  can be described by a single parameter: a *mixing angle*  $\theta$ . Show that the following relation between mixing angle and masses holds:

$$\tan \theta = \sqrt{\frac{m_1}{m_2}}.$$

Compare this with the Cabibbo angle and the down and strange quark masses.

- d) A completely different situation holds for the symmetric mass matrix

$$M = \begin{bmatrix} a & b & b \\ b & \frac{1}{2}(a+b+d) & \frac{1}{2}(a+b-d) \\ b & \frac{1}{2}(a+b-d) & \frac{1}{2}(a+b+d) \end{bmatrix}.$$

Give the mixing matrix for this mass matrix (*Hint*: try first a 23-rotation).

**Tutor:**

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Lecture webpage: [http://www.mpi-hd.mpg.de/manitop/StandardModel2/index\\_WS15.html](http://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html)

**Hand-in and discussion of sheet:**

Tuesday, 17.11.15, 16.15 am, INF 501 / CIP R. 103.