

# Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

Dr. Werner Rodejohann

Sheet 5

19.11.14

## **Exercise 11:** Electroweak symmetry breaking by a Higgs triplet [8 Points]

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

form a triplet representation of the SU(2) algebra  $[t_a, t_b] = i\epsilon_{abc}t_c$ .

b) Consider the complex triplet Higgs field  $\Phi(x) = (\phi_1(x), \phi_2(x), \phi_3(x))^T$  coupled covariantly to an SU(2)<sub>L</sub> gauge group with

$$D_\mu \Phi = \partial_\mu \Phi - ig t_a W_{a\mu}(x) \Phi,$$

where  $W_{a\mu}$  denote the gauge bosons of the SU(2)<sub>L</sub> group. We introduce a potential  $V(\Phi)$  such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (vev), which can be written  $\Phi_0 = (0, 0, v)^T$ . Derive the gauge boson mass eigenstates of the broken SU(2)<sub>L</sub> symmetry.

c) In the next step we extend the gauge group of the theory to SU(2)<sub>L</sub> × U(1)<sub>Y</sub> so that the covariant derivative now reads

$$D_\mu \Phi = \partial_\mu \Phi - ig t_a W_{a\mu}(x) \Phi - ig' \mathbb{1} B_\mu(x) \Phi,$$

where  $B_\mu$  denotes the gauge boson of the U(1)<sub>Y</sub> group. Using the same vev as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the  $\rho = m_W^2/(m_Z^2 \cos \theta_W)$  parameter compared to the Standard Model (SM)?

d) How does the particle content of part b) and c) differ from the SM (*hint*: count the degrees of freedom)? How can this in principle be observed in an experiment?

## **Exercise 12:** W-polarization [4 Points]

For a W-boson moving along the z-axis, the states of helicity  $\lambda = 0, \pm 1$  can be written as

$$\epsilon_{\lambda=0}^\mu = \frac{1}{m_W} (|k|, E \sin \theta, 0, E \cos \theta)$$
$$\epsilon_{\lambda=\pm 1}^\mu = \frac{1}{\sqrt{2}} (0, \mp \cos \theta, -i, \pm \sin \theta)$$

with the momentum

$$k^\mu = (E, |k| \vec{n}) \quad \text{with} \quad \vec{n} = (\sin \theta, 0, \cos \theta).$$

Check if the completeness relation

$$\sum_{\lambda} \epsilon_{\mu}^* \epsilon_{\nu} = -g_{\mu\nu} + \frac{k_{\mu} k_{\nu}}{m_W^2}$$

is fulfilled.

**Exercise 13:** Running couplings [8 Points]

- a) In QED the coupling  $\alpha = e^2/4\pi$  is a running coupling. For low energies one obtains  $\alpha(\mu = m_e) = 1/137$ . The running of the QED coupling with only one lepton is given by

$$\alpha_{\text{QED}}(\mu) = \frac{\alpha(m_e)}{1 - \frac{2\alpha(m_e)}{3\pi} \ln(\frac{\mu}{m_e})} . \quad (1)$$

At which scale does  $\alpha_{\text{QED}}(\mu)$  become infinite? (If further charged leptons and quarks are considered, the coupling increases even faster.)

- b) In QCD, with  $\alpha_s = g_s^2/4\pi$ , the running coupling

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{(33-2f)\alpha_s(\mu_0)}{6\pi} \ln(\frac{\mu}{\mu_0})} \quad (2)$$

is obtained, where  $f$  denotes the respective number of quark flavours with mass  $2m_q \leq \mu$  in the considered energy range.

The experimental boundary conditions are  $\alpha_s(\mu_0 = M_Z = 91 \text{ GeV}) = 0.12$ ,  $m_t = 175 \text{ GeV}$ ,  $m_b = 4.8 \text{ GeV}$ ,  $m_c = 1.4 \text{ GeV}$  and  $m_u \simeq m_d \simeq m_s \simeq 0$ .

- i) Determine the pole  $\mu = \Lambda_{\text{QCD}}$  of eq. (2).
  - ii) For which  $\mu$  does the coupling  $\alpha_s(\mu)$  become very small (asymptotic freedom), and where does perturbation theory break?
  - iii) Determine  $\alpha_s(\mu)$  in the different energy ranges ( $2m_q \leq \mu \leq 2m_t$  etc.) as well as the values at the thresholds in between.
  - iv)  $\alpha_s(\mu)$  must be continuous. From the thresholds calculated in the last part, however, we saw that this is not true, yet. To make  $\alpha_s(\mu)$  continuous, promote  $\mu_0$  to a function  $\mu_0(f)$  with  $\mu_0(f = 5) = 91 \text{ GeV}$ . Find a relation between  $\mu_0(f)$  and  $\mu_0(f - 1)$  assuming that  $\alpha_s(\mu_0(f)) = 0.12$  for all  $f$ . With this relation determine the values of  $\mu_0(4)$  and  $\mu_0(6)$ .
- c) Draw  $\alpha_s^{-1}(\mu)$  and  $\alpha_{\text{QED}}^{-1}(\mu)$  as functions of  $\ln\mu$ . What could the intersection of the curves indicate?

**Tutor:**

Pascal Humbert, email: [pascal.humbert@mpi-hd.mpg.de](mailto:pascal.humbert@mpi-hd.mpg.de)

Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

**Hand-in and discussion of sheet:**

during tutorial on Thursday, 27.11.14, 9.15 am, kHs, Philosophenweg 12