Exercises to "Standard Model of Particle Physics II"

Winter 2014/15

Dr. Werner Rodejohann Sheet 5 19.11.14

Exercise 11: Electroweak symmetry breaking by a Higgs triplet [**8 Points**]

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a,\ t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\mathbf{\Phi}(x) = (\phi_1(x),\ \phi_2(x),\ \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU(2)_L}$ gauge group with

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - \mathrm{i} g t_a W_{a\mu}(x)\mathbf{\Phi},$$

where $W_{a\mu}$ denote the gauge bosons of the $\mathrm{SU(2)_L}$ group. We introduce a potential $V(\mathbf{\Phi})$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (vev), which can be written $\mathbf{\Phi}_0 = (0,\ 0,\ v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU(2)_L}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ so that the covariant derivative now reads

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - \mathrm{i} g t_a W_{a\mu}(x)\mathbf{\Phi} - \mathrm{i} g' \mathbb{1} B_\mu(x)\mathbf{\Phi},$$

where B_μ denotes the gauge boson of the $\mathrm{U(1)_Y}$ group. Using the same vev as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2 \cos\theta_{\mathrm{W}})$ parameter compared to the Standard Model (SM)?

d) How does the particle content of part b) and c) differ from the SM (*hint:* count the degrees of freedom)? How can this in principle be observed in an experiment?

Exercise 12: W-polarization [**4 Points**]

For a W-boson moving along the z-axis, the states of helicity $\lambda = 0,\ \pm 1$ can be written as

$$\epsilon^\mu_{\lambda=0} = \frac{1}{m_{\mathrm{W}}}(|k|,\ E\sin\theta,\ 0,\ E\cos\theta)$$
$$\epsilon^\mu_{\lambda=\pm1} = \frac{1}{\sqrt{2}}(0,\ \mp\cos\theta,\ -\mathrm{i},\ \pm\sin\theta)$$

with the momentum

$$k^\mu = (E,\ |k|\vec{n}) \quad \text{with} \quad \vec{n} = (\sin\theta,\ 0,\ \cos\theta).$$

Check if the completeness relation

$$\sum_\lambda \epsilon^*_\mu \epsilon_\nu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_{\mathrm{W}}^2}$$

is fulfilled.

Exercise 13: Running couplings [**8 Points**]

a) In QED the coupling $\alpha = e^2/4\pi$ is a running coupling. For low energies one obtains $\alpha(\mu = m_e) = 1/137$. The running of the QED coupling with only one lepton is given by

$$\alpha_{\mathrm{QED}}(\mu) = \frac{\alpha(m_e)}{1 - \frac{2\alpha(m_e)}{3\pi}\ln(\frac{\mu}{m_e})} \,. \tag{1}$$

At which scale does $\alpha_{\mathrm{QED}}(\mu)$ become infinite? (If further charged leptons and quarks are considered, the coupling increases even faster.)

b) In QCD, with $\alpha_{\mathrm{s}} = g_3^2/4\pi$, the running coupling

$$\alpha_{\mathrm{s}}(\mu) = \frac{\alpha_{\mathrm{s}}(\mu_0)}{1 + \frac{(33-2f)\alpha_{\mathrm{s}}(\mu_0)}{6\pi}\ln(\frac{\mu}{\mu_0})} \tag{2}$$

is obtained, where f denotes the respective number of quark flavours whit mass $2m_q \leq \mu$ in the considered energy range.

The experimental boundary conditions are $\alpha_{\mathrm{s}}(\mu_0 = M_{\mathrm{Z}} = 91\text{ GeV}) = 0.12$, $m_t = 175$ GeV, $m_b = 4.8$ GeV, $m_c = 1.4$ GeV and $m_u \simeq m_d \simeq m_s \simeq 0$.

i) Determine the pole $\mu = \Lambda_{\mathrm{QCD}}$ of eq. (2).

ii) For which μ does the coupling $\alpha_{\mathrm{s}}(\mu)$ become very small (asymptotic freedom), and where does perturbation theory break?

iii) Determine $\alpha_{\mathrm{s}}(\mu)$ in the different energy ranges ($2m_q \leq \mu \leq 2m_t$ etc.) as well as the values at the thresholds in between.

iv) $\alpha_{\mathrm{s}}(\mu)$ must be continuous. From the thresholds calculated in the last part, however, we saw that this is not true, yet. To make $\alpha_{\mathrm{s}}(\mu)$ continuous, promote μ_0 to a function $\mu_0(f)$ with $\mu_0(f = 5) = 91$ GeV. Find a relation between $\mu_0(f)$ and $\mu_0(f-1)$ assuming that $\alpha_s(\mu_0(f)) = 0.12$ for all f. With this relation determine the values of $\mu_0(4)$ and $\mu_0(6)$.

c) Draw $\alpha_{\mathrm{s}}^{-1}(\mu)$ and $\alpha_{\mathrm{QED}}^{-1}(\mu)$ as functions of $\ln\mu$. What could the intersection of the curves indicate?

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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html`

Hand-in and discussion of sheet:

during tutorial on Thursday, 27.11.14, 9.15 am, kHs, Philosophenweg 12