Exercises to “Standard Model of Particle Physics II”

Winter 2020/21

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann

Sheet 04 - November 25, 2020

Tutor: Cristina Benso **e-mail:** benso@mpi-hd.mpg.de
Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:	**Discussion of solutions:**
December 2, 2020 - via e-mail, **before 14:00**	December 2, 2020 - on zoom

---

Problem 6: *The parity asymmetry of weak interactions* [**20 Points**]

The first measurement of the P2 experiment aims for a high precision determination of the weak mixing angle $\sin^2(\theta_W)$ at a four-momentum transfer of $Q^2 = 4.5 \times 10^{-3}\text{GeV}^2$.
This requires a measurement of the parity violating cross section asymmetry in the elastic electron-proton scattering of a polarized electron beam impinging on a target, allowing for the extraction of the weak charge of the proton $Q_W(p)$ (the analog of the electric charge which determines the strength of the neutral-current weak interaction) which is directly connected to the weak mixing angle $\sin^2(\theta_W)$. Contributions from diagrams at level higher than the tree level can be neglected at this small beam energy ($E_{\text{beam}} = 155$ MeV).

As already said, the process considered is the elastic scattering of polarized electrons (meaning purely LH or purely RH) on unpolarized protons, $e_{L/R}(p_1) + p(p_2) \to e_{L/R}(p_3) + p(p_4)$.

The parity violating asymmetry of interest for the experiment is defined as

$$A^{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

where σ_L (σ_L) is the cross section of LH (RH) electrons.

a) Which Feynman diagrams give contribution to the process?
Write down the Feynman diagrams contributing for each of the two polarizations of interest, and the relative Feynman rules for both electromagnetic and weak interaction. [**2 Points**]

b) At the energies at which the experiment operates, the masses of which particles can be neglected and which are instead to be taken into account? [**1 Point**]

c) Remember that the vertex of interaction of electrons with the Z boson is expressed by

$$-i\frac{g}{2\cos(\theta_W)}\gamma^\mu(g_V^e - g_A^e\gamma_5)$$

while instead the one describing the interaction of the protons with the Z boson is expressed by

$$-i\frac{g}{2\cos(\theta_W)}\gamma^\mu\frac{1}{2}(Q_W(p) - 2g_A^p\gamma_5)$$

where $Q_W(p) = 1 - 4\sin^2(\theta_W)$. How can you justify this latter expression of $Q_W(p)$ at an elementary level? [**2 Points**]
Hint: pay attention to the factors of 2!

d) Write down the amplitude of the process corresponding to each Feynman diagram (*do not forget to polarize the electrons!*). **[2 Points]**

e) Which terms of the amplitude squared contribute to the asymmetry between σ_L and σ_R? **[1 Point]**

f) Give the expression of the parity violating cross section asymmetry as a function of $2 \times Q_W(p)$ and of the Fermi constant G_F for very low elastic momentum transfer $Q^2 \to 0$. **[12 Points]**
Hint: It is enough to calculate the asymmetry between the amplitudes squared

$$A^{PV} = \frac{\overline{|\mathcal{M}_L|^2} - \overline{|\mathcal{M}_R|^2}}{\overline{|\mathcal{M}_L|^2} + \overline{|\mathcal{M}_R|^2}}$$

since the phase space is the same for the two polarizations and, thus, simplifies.