Exercises to "Standard Model of Particle Physics II"

Winter 2019/20

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: November 13, 2019 15:45, Philosophenweg 12, kHS **Discussion of solutions:** November 20, 2019

Problem 8: *Electroweak symmetry breaking by a Higgs triplet* **[10 Points]**

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a, t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\Phi(x) = (\phi_1(x),\, \phi_2(x),\, \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU(2)_L}$ gauge group with

$$D_\mu \Phi = \partial_\mu \Phi - igt_a W^a_\mu(x)\Phi,$$

where W^a_μ denote the gauge bosons of the $\mathrm{SU(2)_L}$ group. We introduce a potential $V(\Phi)$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (VEV), which can be written $\Phi_0 = (0,\, 0,\, v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU(2)_L}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ so that the covariant derivative now reads

$$D_\mu \Phi = \partial_\mu \Phi - igt_a W^a_\mu(x)\Phi - ig'\mathbb{1}\, B_\mu(x)\Phi,$$

where B_μ denotes the gauge boson of the $\mathrm{U(1)_Y}$ group. Using the same VEV as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2 \cos^2\theta_{\mathrm{W}})$ parameter compared to the SM?

d) How does the particle content of part b) and c) differ from the Standard Model? How can these theories be distinguished in an experiment?

Problem 9: *Scalar electrodynamics* [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = -2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = 2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.