Exercises to "Standard Model of Particle Physics II"

Winter 2018/19

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 4 November 7, 2018

Problem 8: *Electroweak symmetry breaking by a Higgs triplet* **[10 Points]**

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}, \qquad t_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a, t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\Phi(x) = (\phi_1(x),\, \phi_2(x),\, \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU}(2)_{\mathrm{L}}$ gauge group with

$$D_\mu\Phi = \partial_\mu\Phi - igt_aW^a_\mu(x)\Phi,$$

where W^a_μ denote the gauge bosons of the $\mathrm{SU}(2)_{\mathrm{L}}$ group. We introduce a potential $V(\Phi)$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (VEV), which can be written $\Phi_0 = (0,\, 0,\, v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU}(2)_{\mathrm{L}}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ so that the covariant derivative now reads

$$D_\mu\Phi = \partial_\mu\Phi - igt_aW^a_\mu(x)\Phi - ig'\mathbb{1}B_\mu(x)\Phi,$$

where B_μ denotes the gauge boson of the $\mathrm{U}(1)_{\mathrm{Y}}$ group. Using the same VEV as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2\cos^2\theta_{\mathrm{W}})$ parameter compared to the SM?

d) How does the particle content of part b) and c) differ from the SM? How can these theories be distinguished in an experiment?

Problem 9: *Scalar electrodynamics* [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = 2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = -2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
November 14, 2018, 16:00, Philosophenweg 12, R106