Exercises to "Standard Model of Particle Physics II"

Winter 2016/17

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann | Sheet 4 | 9.11.16

Problem 7: *Electroweak symmetry breaking by a Higgs triplet* [**10 Points**]

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a, t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\mathbf{\Phi}(x) = (\phi_1(x),\, \phi_2(x),\, \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU}(2)_{\mathrm{L}}$ gauge group with

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - igt_a W_\mu^a(x)\mathbf{\Phi},$$

where W_μ^a denote the gauge bosons of the $\mathrm{SU}(2)_{\mathrm{L}}$ group. We introduce a potential $V(\mathbf{\Phi})$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (VEV), which can be written $\mathbf{\Phi}_0 = (0,\, 0,\, v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU}(2)_{\mathrm{L}}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ so that the covariant derivative now reads

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - igt_a W_\mu^a(x)\mathbf{\Phi} - ig'\mathbb{1}B_\mu(x)\mathbf{\Phi},$$

where B_μ denotes the gauge boson of the $\mathrm{U}(1)_{\mathrm{Y}}$ group. Using the same VEV as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2 \cos^2\theta_{\mathrm{W}})$ parameter compared to the SM?

d) How does the particle content of part b) and c) differ from the SM? How can these theories be distinguished in an experiment?

Problem 8: *Scalar electrodynamics* [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = 2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = -2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.

Tutor:
Moritz Platscher
e-mail: moritz.platscher@mpi-hd.mpg.de

Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106