

Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

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Sheet 4

12.11.14

Exercise 8: Noether’s theorem revised [5 Points]

Consider a Lagrange density of the general form $\mathcal{L} = \mathcal{L}_0(\psi, \partial_\mu\psi)$. From Noether’s theorem we know that if the Lagrange density is left invariant by a symmetry transformation this leads to the conserved current

$$J^\mu = \frac{\partial \mathcal{L}_0}{\partial(\partial_\mu\psi)} \delta\psi.$$

Now imagine that we add a term \mathcal{L}_1 to the Lagrange density (so that $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$) that does not respect the symmetry.

Show that in this case the current is not conserved any more, i.e.

$$\partial_\mu J^\mu \neq 0.$$

Exercise 9: Higgs sector and gauge bosons [5 Points]

Consider the kinetic term of the Standard Model Higgs doublet,

$$\mathcal{L} = (D_\mu\phi)^\dagger (D^\mu\phi),$$

with $D_\mu\phi = \partial_\mu\phi + igT^a W_\mu^a\phi + ig'\frac{Y}{2}B_\mu\phi$. In the covariant derivative, $T^a = \sigma^a/2$ are the generators of $SU(2)_L$ and Y denotes hypercharge. With a suitable isospin rotation the Higgs doublet takes on the form

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}.$$

Derive the Feynman rules for the hW^+W^- and the hhW^+W^- couplings from the kinetic term of the Higgs.

Exercise 10: Young tableaux [10 Points]

Familiarize yourself with the method of Young tableaux (source: Review of Particle Physics, K. Hagiwara et al., Phys. Rev. D 66 (2002) 010001; URL: <http://pdg.lbl.gov/2013/reviews/rpp2012-rev-young-diagrams.pdf>).

Verify, using Young tableaux, the following decompositions of products of representations into irreducible representations.

- SU(3):
- a) $3 \otimes 3 = \bar{3} \oplus 6$
 - b) $3 \otimes 3 \otimes \bar{3} = 3 \oplus 3 \oplus \bar{6} \oplus 15$
 - c) $8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \bar{15}$
 - d) $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$
- SU(6):
- e) $6 \otimes \bar{6} = 1 \oplus 35$
 - f) $6 \otimes 6 \otimes 6 = 20 \oplus 56 \oplus 70 \oplus 70$

Tutor:

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Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

Hand-in and discussion of sheet:

during tutorial on Thursday, 20.11.14, 9.15 am, kHs, Philosophenweg 12