Exercises to "Standard Model of Particle Physics II"

Winter 2014/15

Dr. Werner Rodejohann | Sheet 4 | 12.11.14

Exercise 8: Noether's theorem revised **[5 Points]**

Consider a Lagrange density of the general form $\mathscr{L} = \mathscr{L}_0(\psi,\ \partial_\mu\psi)$. From Noether's theorem we know that if the Lagrange density is left invariant by a symmetry transformation this leads to the conserved current

$$J^\mu = \frac{\partial \mathscr{L}_0}{\partial(\partial_\mu \psi)}\delta\psi.$$

Now imagine that we add a term $\mathscr{L}_1$ to the Lagrange density (so that $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1$) that does not respect the symmetry.
Show that in this case the current is not conserved any more, i.e.

$$\partial_\mu J^\mu \neq 0.$$

Exercise 9: Higgs sector and gauge bosons **[5 Points]**

Consider the kinetic term of the Standard Model Higgs doublet,

$$\mathscr{L} = (D_\mu\phi)^\dagger (D^\mu \phi),$$

with $D_\mu\phi = \partial_\mu\phi + igT^aW^a_\mu\phi + ig'\frac{Y}{2}B_\mu\phi$. In the covariant derivative, $T^a = \sigma^a/2$ are the generators of $\mathrm{SU(2)_L}$ and Y denotes hypercharge. With a suitable isospin rotation the Higgs doublet takes on the form

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}.$$

Derive the Feynman rules for the hW^+W^- and the hhW^+W^- couplings from the kinetic term of the Higgs.

Exercise 10: Young tableaux **[10 Points]**

Familiarize yourself with the method of Young tableaux (source: Review of Particle Physics, K. Hagiwara et al., Phys. Rev. D 66 (2002) 010001; URL: http://pdg.lbl.gov/2013/reviews/rpp2012-rev-young-diagrams.pdf).
Verify, using Young tableaux, the following decompositions of products of representations into irreducible representations.

SU(3): a) $3 \otimes 3 = \overline{3} \oplus 6$

b) $3 \otimes 3 \otimes \overline{3} = 3 \oplus 3 \oplus \overline{6} \oplus 15$

c) $8 \otimes \overline{3} = \overline{3} \oplus 6 \oplus \overline{15}$

d) $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$

SU(6): e) $6 \otimes \overline{6} = 1 \oplus 35$

f) $6 \otimes 6 \otimes 6 = 20 \oplus 56 \oplus 70 \oplus 70$

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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html`

Hand-in and discussion of sheet:
during tutorial on Thursday, 20.11.14, 9.15 am, kHs, Philosophenweg 12