Exercises to "Standard Model of Particle Physics II"

Winter 2015/16

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 3 October 28, 2015

Exercise 6: *Continuous groups* [**10 Points**]

a) How many degrees of freedom does a real SO(3) matrix have? (That is a matrix with real entries, which is orthogonal and has a determinant equal to one).

b) Show that the rotational matrices about the x-, y- and z-axis, denoted by $R_1(\theta_1)$, $R_2(\theta_2)$ and $R_3(\theta_3)$, respectively, are elements of SO(3).

c) Expand $R = R_1(\theta_1) \cdot R_2(\theta_2) \cdot R_3(\theta_3) \approx \mathbb{1} + \theta_1 E_1 + \theta_2 E_2 + \theta_3 E_3$ about small angles θ_i and find the expressions for E_i.

d) The vector space spanned by $\mathbb{1}$ and the E_i shall be called $\mathfrak{so}(3)$. Check, if the three operators $E_1 \cdot E_3$, $E_3 \cdot E_1$ and $[E_1, E_3]$ are elements of $\mathfrak{so}(3)$.

e) What is $[E_i, E_j]$?

Exercise 7: *Scalar electrodynamics* [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = 2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = -2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.

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Lecture webpage: `http://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html`

Hand-in and discussion of sheet:
Thursday, 04.11.15, 9.15 am, kHs, Philosophenweg 12