Exercises to "Standard Model of Particle Physics II"

Winter 2014/15

Dr. Werner Rodejohann — Sheet 3 — 05.11.14

Exercise 6: Continuous groups [**10 Points**]

a) How many degrees of freedom does a real SO(3) matrix have? (That is a matrix with real entries, which is orthogonal and has a determinant equal to one.)

b) Show that the rotational matrices about the x-, y- and z-axis, denoted by $R_1(\theta_1)$, $R_2(\theta_2)$ and $R_3(\theta_3)$, respectively, are elements of SO(3).

c) Expand $R = R_1(\theta_1) \cdot R_2(\theta_2) \cdot R_3(\theta_3) \approx \mathbb{1} + \theta_1 E_1 + \theta_2 E_2 + \theta_3 E_3$ about small angles θ_i and find the expressions for E_i.

d) The vector space spanned by $\mathbb{1}$ and the E_i shall be called so(3). Check, if the three operators $E_1 \cdot E_3$, $E_3 \cdot E_1$ and $[E_1, E_3]$ are elements of so(3).

e) What is $[E_i, E_j]$?

Exercise 7: Scalar electrodynamics [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu \phi)^* (D^\mu \phi) - \mu^2 (\phi^* \phi) - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &= (D_\mu \phi)^* (D^\mu \phi) - V(\phi^* \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu \phi \equiv \partial_\mu \phi - \mathrm{i} e A_\mu \phi.$$

a) Sketch the potential $V(\phi^* \phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0 \vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = 2e^2 v^2 \vec{A}$, and that thence we have $\Delta \vec{B} = -2e^2 v^2 \vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and thus the system is superconductive.

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Tutorials homepage: http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html

Hand-in and discussion of sheet:
during tutorial on Thursday, 13.11.14, 9.15 am, kHs, Philosophenweg 12