

# Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

Dr. Werner Rodejohann

Sheet 3

05.11.14

## **Exercise 6:** Continuous groups [10 Points]

- a) How many degrees of freedom does a real  $\text{SO}(3)$  matrix have? (That is a matrix with real entries, which is orthogonal and has a determinant equal to one.)
- b) Show that the rotational matrices about the  $x$ -,  $y$ - and  $z$ -axis, denoted by  $R_1(\theta_1)$ ,  $R_2(\theta_2)$  and  $R_3(\theta_3)$ , respectively, are elements of  $\text{SO}(3)$ .
- c) Expand  $R = R_1(\theta_1) \cdot R_2(\theta_2) \cdot R_3(\theta_3) \approx \mathbb{1} + \theta_1 E_1 + \theta_2 E_2 + \theta_3 E_3$  about small angles  $\theta_i$  and find the expressions for  $E_i$ .
- d) The vector space spanned by  $\mathbb{1}$  and the  $E_i$  shall be called  $\text{so}(3)$ . Check, if the three operators  $E_1 \cdot E_3$ ,  $E_3 \cdot E_1$  and  $[E_1, E_3]$  are elements of  $\text{so}(3)$ .
- e) What is  $[E_i, E_j]$ ?

## **Exercise 7:** Scalar electrodynamics [10 Points]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathcal{L} &= (D_\mu \phi)^* (D^\mu \phi) - \mu^2 (\phi^* \phi) - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &= (D_\mu \phi)^* (D^\mu \phi) - V(\phi^* \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu \phi \equiv \partial_\mu \phi - ie A_\mu \phi.$$

- a) Sketch the potential  $V(\phi^* \phi)$ , with  $\mu^2 < 0$ ,  $\lambda > 0$ , and find the value of the scalar field that minimizes  $V$ .
- b) Consider the case, where  $\partial_0 \vec{A} = 0$  and  $A_0 = 0$ . For this case, derive the equations of motion for  $\vec{A}$  and find the current density  $\vec{J}$ , which acts as source for the gauge potential.
- c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and  $\phi$  develops the vacuum expectation value (vev)  $v$ , the current density is given by  $\vec{J} = 2e^2 v^2 \vec{A}$ , and that thence we have  $\Delta \vec{B} = -2e^2 v^2 \vec{B}$ .
- d) The resistance  $R$  of a system is defined by  $\vec{E} = R \vec{J}$ . Show that, after electroweak symmetry breaking, the resistance vanishes ( $R = 0$ ) and thus the system is superconductive.

**Tutor:**

Pascal Humbert, email: `pascal.humbert@mpi-hd.mpg.de`

Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

**Hand-in and discussion of sheet:**

during tutorial on Thursday, 13.11.14, 9.15 am, kHs, Philosophenweg 12