

Exercises to “Standard Model of Particle Physics II”

Winter 2018/19

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Sheet 1

October 17, 2018

Problem 1: Global symmetry transformations [5 Points]

- a) Derive the Noether theorem for a global symmetry transformation of the following Lagrange density:

$$\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i)$$

In other words, find an expression for the conserved current under the field transformation $u_i(x) \rightarrow u'_i(x) = u_j(x) e^{iT_{ij}^k \epsilon_k} \cong u_i(x) + \delta u_i(x)$ with $\delta u_i(x) = iT_{ij}^k \epsilon_k u_j(x)$. Here the T_{ij}^k denote the generators of the algebra of the corresponding symmetry group and ϵ_k the infinitesimal, space-time independent coefficients.

- b) Apply your formula to the Lagrange density of a massive complex scalar field:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi$$

What is the conserved current in this case?

Hint: Use $\phi(x) \rightarrow \phi'(x) = e^{i\epsilon} \phi(x)$ as transformation of the scalar field.

Problem 2: Local symmetry transformations in QED [5 Points]

Show that the Lagrange density of QED

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

with $D_\mu = \partial_\mu - ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is invariant under the local symmetry transformations

$$\psi \rightarrow \psi' = e^{-ie\epsilon(x)}\psi, \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}e^{ie\epsilon(x)},$$

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\epsilon(x).$$

Local symmetry means that now the infinitesimal parameter $\epsilon(x)$ depend on the space-time coordinates.

Problem 3: General local symmetry transformations [10 Points]

Now, we want the general Lagrange density $\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i)$ to be invariant under a local symmetry transformation. In analogy with **Problem 1**, the fields u_i transform as

$$u_i(x) \rightarrow u'_i(x) = u_j(x) e^{iT_{ij}^k \epsilon_k(x)} \cong u_i(x) + \delta u_i(x)$$

with

$$\delta u_i(x) = iT_{ij}^k \epsilon_k(x) u_j(x).$$

- a) Considering the transformation properties of the u_i 's given above, show that we need to introduce a new field to keep \mathcal{L} invariant under the local symmetry transformation.
- b) Extend the system by a set of additional vector fields $A_{\mu k}$ (one for each generator T_{ij}^k), so that the Lagrange density does not depend on their derivatives: $\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i, A_{\mu k})$. Assume that the transformation properties of the new fields are of the following form (where P and R denote, at first, unspecified matrices):

$$A_{\mu l}(x) \rightarrow A'_{\mu l}(x) = A_{\mu l}(x) + P_{lm}^k A_{\mu m}(x) \epsilon_k(x) + R_l^k \partial_\mu \epsilon_k(x).$$

Derive general conditions for the matrices P and R from the variational principle.

- c) Consider the following Lagrange density

$$\mathcal{L} = \frac{1}{2} (D^\mu u_i)(D_\mu u_i),$$

where the covariant derivative of the fields u_i is given by $D_\mu u_i = \partial_\mu u_i + g \epsilon_{ijk} A_{\mu j} u_k$ and ϵ_{ijk} denotes the totally anti-symmetric tensor. Taking into account the results from b) find a relation between the matrix R_l^k and the generators T_{ij}^k .

Bonus Exercise: Can you find a solution for the matrix P_{lm}^k ?

Discussion: Compare global and local symmetries. How would you interpret the physical meaning of locally conserved quantities?

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Lecture webpage: <https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html>

Hand-in and discussion of sheet:

October 24, 2018, 16.15, Philosophenweg 12, R106