

# Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

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Sheet 1

15.10.14

## **Exercise 1:** Global symmetry transformations [5 Points]

- a) Derive the Noether theorem for a global symmetry transformation of the following Lagrange density:

$$\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i)$$

In other words, find an expression for the conserved current under the field transformation  $u_i(x) \rightarrow u'_i(x) = u_j(x) e^{iT_{ij}^k \epsilon_k} \cong u_i(x) + \delta u_i(x)$  with  $\delta u_i(x) = iT_{ij}^k \epsilon_k u_j(x)$ . Here the  $T_{ij}^k$  denote the generators of the algebra of the corresponding symmetry group and  $\epsilon_k$  the infinitesimal, space-time independent coefficients.

- b) Apply your formula to the Lagrange density of a massive complex scalar field:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi$$

What is the conserved current in this case?

*Hint:* Use  $\phi(x) \rightarrow \phi'(x) = e^{i\epsilon} \phi(x)$  as transformation of the scalar field.

## **Exercise 2:** Local symmetry transformations in QED [5 Points]

Show that the Lagrange density of QED

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

with  $D_\mu = \partial_\mu - ieA_\mu$  and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ , is invariant under the local symmetry transformations

$$\begin{aligned}\psi &\rightarrow \psi' = e^{-i\epsilon(x)}\psi & A_\mu &\rightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \epsilon(x) \\ \bar{\psi} &\rightarrow \bar{\psi}' = \bar{\psi}e^{i\epsilon(x)}\end{aligned}$$

**Exercise 3:** General local symmetry transformations [10 Points]

Now, we want the Lagrange density  $\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i)$  to be invariant under a local symmetry transformation. Local symmetry means that, now, the infinitesimal parameters  $\epsilon_k(x)$  depend on the space-time coordinates, and the fields  $u_i$  transform as

$$u_i(x) \rightarrow u'_i(x) = u_j(x) e^{iT_{ij}^k \epsilon_k(x)} \cong u_i(x) + \delta u_i(x)$$

with

$$\delta u_i(x) = iT_{ij}^k \epsilon_k(x) u_j(x).$$

- a) With the transformation properties of the  $u_i$ 's given above, show that we need to introduce a new field to keep  $\mathcal{L}$  invariant under the local symmetry transformation.
- b) Extend the system by a set of additional vector fields  $A_{\mu k}$  (one for each generator  $T_{ij}^k$ ), so that the Lagrange density does not depend on their derivatives:  $\mathcal{L} = \mathcal{L}(u_i, \partial_\mu u_i, A_{\mu k})$ . Assume that the transformation properties of the new fields are of the following form (where  $P$  and  $R$  denote, at first, unspecified matrices):

$$A_{\mu l}(x) \rightarrow A'_{\mu l}(x) = A_{\mu l}(x) + P_{lm}^k A_{\mu m}(x) \epsilon_k(x) + R_l^k \partial_\mu \epsilon_k(x).$$

Derive general conditions for the matrices  $P$  and  $R$  from the variational principle.

- c) Consider the following Lagrange density

$$\mathcal{L} = \frac{1}{2} (D^\mu u_i) (D_\mu u_i),$$

where the covariant derivative of the fields  $u_i$  is given by  $D_\mu u_i = \partial_\mu u_i + g \epsilon_{ijk} A_{\mu j} u_k$  and  $\epsilon_{ijk}$  denotes the totally anti-symmetric tensor. From the results from b) find a relation between the matrix  $R_l^k$  and the generators  $T_{ij}^k$ .

Bonus: Can you find a solution for the matrix  $P$ ?

**Discussion:** Compare global and local symmetries. How would you interpret the physical meaning of locally conserved quantities?

**Tutor:**

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Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

**Hand-in and discussion of sheet:**

during tutorial on Thursday, 30.10.14, 9.15 am, kHs, Philosophenweg 12