Exercises to "Standard Model of Particle Physics II"

Winter 2020/21

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Sheet 13 - February 17, 2020

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
February 24, 2021 - via e-mail, **before 14:00**

Discussion of solutions:
February 24, 2021 - on zoom

Problem 27: *Z' physics* **[16 Points]**

The general effective Lagrange density after breaking the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)'$ symmetry to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{em}$ can be written as

$$\mathscr{L} = \mathscr{L}_\mathrm{SM} + \mathscr{L}_{Z'} + \mathscr{L}_\mathrm{mix},$$

where the relevant part of the Standard Model Lagrangian is

$$\mathscr{L}_\mathrm{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j^\mu_B\hat{B}_\mu - \frac{e}{s_W}j^{\mu,a}_W\hat{W}^a_\mu$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}^2_{Z'}\hat{Z}'_\mu\hat{Z}'^\mu - g' j'_\mu \hat{Z}'_\mu\,,$$

where g' denotes the $\mathrm{U}(1)'$ gauge coupling, and the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_\mathrm{mix} = -\frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. Set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You can assume $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged $B - L$ symmetry (baryon minus lepton-number). Write down explicitly the corresponding current for the SM fermions:

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu(B-L)\psi\,,$$

where $(B - L)$ denotes the $B - L$ charge operator.

d) To cancel quantum anomalies in the $B - L$ model, one also has to introduce three right-handed neutrinos N_i which are then part of the current: $\Delta j'_\mu = -\sum_i \bar{N}_i\,\gamma_\mu P_R N_i$. Due to Z-Z' mixing, the SM-like Z_1 will also couple to these new neutrinos. Can you give a constraint on the Z-Z' mixing from the well-measured (invisible) decay width of Z (for $M(N_i) \ll M_Z/2$)? See the PDG for numbers.