Exercises to "Standard Model of Particle Physics II"

Winter 2019/20

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 13 (Bonus) January 29, 2020

Tutor: Carlos Jaramillo **e-mail:** carlos.jaramillo@mpi-hd.mpg.de
Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: February 5, 2020

Discussion of solutions: February 5, 2020, 15:45, Philosophenweg 12, kHS

This is a bonus exercise sheet. Hand-in is voluntary.

Problem 26: *Left-right symmetric electroweak model* **[10 Points]**

The left-right symmetric model can be introduced by assuming right-handed fermion doublets in analogy to the left-handed ones. The quark and lepton spectra consist of

$$Q^i_{\mathrm{L,\,R}} = \begin{pmatrix} U^i_{\mathrm{L,\,R}} \\ D^i_{\mathrm{L,\,R}} \end{pmatrix}; \qquad L^i_{\mathrm{L,\,R}} = \begin{pmatrix} \nu^i_{\mathrm{L,\,R}} \\ e^i_{\mathrm{L,\,R}} \end{pmatrix},$$

with the following $\mathrm{SU(2)_L}$, $\mathrm{SU(2)_R}$, $\mathrm{U(1)}_{B-L}$ transformation properties:

$$Q_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1/3); \qquad L_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ -1);$$
$$Q_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1/3); \qquad L_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ -1).$$

The Higgs sector contains a bi-doublet ϕ and two triplets Δ_L and Δ_R with transformation properties

$$\phi : (2_\mathrm{L},\ 2_\mathrm{R},\ 0); \qquad \Delta_\mathrm{L} : (3_\mathrm{L},\ 1_\mathrm{R},\ 2); \qquad \Delta_\mathrm{R} : (1_\mathrm{L},\ 3_\mathrm{R},\ 2).$$

Note that the bi-doublet and the triplets can be expressed by the following 2×2 matrices:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}; \qquad \Delta_{\mathrm{L,\,R}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}.$$

a) Why does this model not work with only the bi-doublet?

b) Construct the Lagrange density for the fermion-Higgs interactions $\mathscr{L}_{\text{Yukawa}}$ (including all possible gauge singlets).

c) Use the assumption that after spontaneous symmetry breaking the vacuum is electrically neutral to derive the fermion mass terms in the broken phase.

d) We leave the quark and lepton sector unchanged, but modify the symmetry-breaking part of the model. In the Higgs sector we still have the bi-doublet ϕ, but instead of the triplets we introduce two scalar doublets $A_{\mathrm{L,\,R}}$ and a fermionic singlet χ with the following transformation properties under $\mathrm{SU(2)_L}$, $\mathrm{SU(2)_R}$ and $\mathrm{U(1)}_{B-L}$:

$$A_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1); \qquad A_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1); \qquad \chi : (1_\mathrm{L},\ 1_\mathrm{R},\ 0).$$

Left-right symmetry implies the invariance of the Lagrange density under the following transformations (where Ψ denotes any fermion field):

$$\Psi_\mathrm{L} \leftrightarrow \Psi_\mathrm{R} \qquad\qquad A_\mathrm{L} \leftrightarrow A_\mathrm{R} \qquad\qquad \phi \leftrightarrow \phi^\dagger.$$

Construct the Lagrange density $\mathscr{L}_\mathrm{Yukawa}$ for the fermion masses in this model. (Again, you have to construct singlets under the whole gauge group.)

Problem 27: *$e - \overline{\nu}$ scattering and W_R* **[10 Points]**

In the Standard Model, the $SU(2)_L$ gauge fields couple only to left-handed fermions, giving rise to the so-called $V - A$ structure of Fermi theory,

$$\mathcal{L}_\mathrm{Fermi} = \frac{G_F}{\sqrt{2}}\,\overline{\nu}\gamma_\mu(1-\gamma_5)e\,\overline{e}\gamma^\mu(1-\gamma_5)\nu.$$

a) Calculate the differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}E}$ for the scattering process $\overline{\nu}e \to \overline{\nu}e$ in Fermi theory, assuming that the electron is initially at rest and neutrinos are massless. Use the identity

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{16\pi}\frac{\left|\overline{\mathcal{M}}\right|^2}{(s-m_e^2)^2} \tag{1}$$

to obtain the spin-averaged differential cross section in terms of the final electron energy E, where t denotes the Mandelstam t-variable.

b) If an aditional W_R boson, which couples to right-handed currents, is included, it will induce an interaction of the type $V + A$. The effective theory can be described by

$$\mathcal{L}_R = \varepsilon\frac{G_F}{\sqrt{2}}\,\overline{\nu}\gamma_\mu(1+\gamma_5)e\,\overline{e}\gamma^\mu(1+\gamma_5)\nu,$$

where ε parametrises the different mass and coupling of the W_R. Note that now *two* diagrams contribute to the amplitude for $\overline{\nu}e \to \overline{\nu}e$, $|\mathcal{M}|^2 = |\mathcal{M}_L + \mathcal{M}_R|^2$. Since the squares of $\mathcal{M}_L$ and $\mathcal{M}_R$ will have the form calculated in a), we only need to worry about the interference term $\mathcal{M}_L\mathcal{M}_R^* + c.c.$. Calculate the part of the cross section that corresponds to the interference of the two contributions and show that it scales with the neutrino mass squared.