Exercises to “Standard Model of Particle Physics II”

Winter 2016/17

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Problem 24: *Z' physics* [**15 Points**]

The general effective Lagrange density after breaking the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ symmetry to $SU(3)_C \times U(1)_{EM}$ can be written as

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Z'} + \mathcal{L}_{mix},$$

where the relevant part of the Standard Model Lagrangian is

$$\mathcal{L}_{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j_B^\mu\hat{B}_\mu - \frac{e}{s_W}j_W^{\mu,a}\hat{W}^a_\mu$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z'}^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z'}_\mu\hat{Z'}^\mu - g'j'_\mu\hat{Z}'_\mu$$

and the kinetic- and mass-mixing terms can be parameterized as

$$\mathcal{L}_{mix} = -\frac{\sin\chi}{2}\hat{Z'}_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. Set the kinetic mixing angle χ to zero for simplicity.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. Assume $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged $B-L$ symmetry (baryon minus lepton-number). Write down the corresponding current for the SM fermions explicitly:

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu(B_\psi - L_\psi)\psi\,.$$

d) To cancel quantum anomalies in the $B-L$ model, one also has to introduce three right-handed neutrinos $N_{1,2,3}$ which are then part of the current: $\Delta j'_\mu = -\sum_i \bar{N}_i\,\gamma_\mu P_R N_i$. Due to Z-Z' mixing, the SM-like Z_1 will also couple to these new neutrinos. Derive a constraint on the Z-Z' mixing from the well-measured invisible Z-width, $\Gamma_{\text{inv}} = (499.0 \pm 1.5)\,\text{MeV}$. Assume that $M(N_i) \ll M_Z/2$, and use that in the Standard Model $\Gamma(Z_1 \to \nu_i\overline{\nu}_i) = 166\,\text{MeV}$ (no sum over i) and the fact that decay rates differ only by the coupling constants of Z_1 to ν_i and N_i.

Problem 25: *$e - \overline{\nu}$ scattering and W_R* **[10 Points]**

In the Standard Model, the $SU(2)_L$ gauge fields couple only to left-handed fermions, giving rise to the so-called $V - A$ structure of Fermi theory,

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \overline{\nu}\gamma_\mu(1-\gamma_5)e\,\overline{e}\gamma^\mu(1-\gamma_5)\nu.$$

a) Calculate the differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}E}$ for the scattering process $\overline{\nu}e \to \overline{\nu}e$ in Fermi theory, assuming that the electron is initially at rest and neutrinos are massless. Use the identity

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{16\pi}\frac{\left|\overline{\mathcal{M}}\right|^2}{(s-m_e^2)^2} \tag{1}$$

to obtain the spin-averaged differential cross section in terms of the final electron energy E, where t denotes the Mandelstam t-variable.

b) If an aditional W_R boson, which couples to right-handed currents, is included, it will induce an interaction of the type $V + A$. The effective theory can be described by

$$\mathcal{L}_R = \varepsilon\frac{G_F}{\sqrt{2}} \overline{\nu}\gamma_\mu(1+\gamma_5)e\,\overline{e}\gamma^\mu(1+\gamma_5)\nu,$$

where ε parametrises the different mass and coupling of the W_R. Note that now *two* diagrams contribute to the amplitude for $\overline{\nu}e \to \overline{\nu}e$, $|\mathcal{M}|^2 = |\mathcal{M}_L + \mathcal{M}_R|^2$. Since the squares of $\mathcal{M}_L$ and $\mathcal{M}_R$ will have the form calculated in a), we only need to worry about the interference term $\mathcal{M}_L\mathcal{M}_R^* + c.c.$. Calculate the part of the cross section that corresponds to the interference of the two contributions and show that it scales with the neutrino mass squared.

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Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106