Exercises to “Standard Model of Particle Physics II”

Winter 2019/20

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

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Problem 24: *Majorana neutrinos* [**5 Points**]

The Lagrangian for the coupling of a fermion pair f with the Z-boson is

$$\mathscr{L} = \frac{g}{2\cos\theta_W}\bar{f}\gamma^\mu(v_f - a_f\gamma_5)fZ_\mu,$$

For neutrinos we have that $v_\nu = a_\nu = \frac{1}{2}$.
We already calculated the decay width for $Z \to \bar{\nu}\nu$ in the Standard Model (exercise sheet no. 6).

a) Neutrinos could be Majorana particles, which obey the relation $\nu^c = \nu$. The superscript c denotes charge conjugation,

$$\nu^c = C(\bar{\nu})^T,$$

with $C = i\gamma_2\gamma_0$ in the Dirac basis.
Show the following properties

$$-C = C^T = C^{-1} = -C^* = C^\dagger,$$
$$C^{-1}\gamma_\mu C = -\gamma_\mu^T,$$
$$C^{-1}\gamma_5 C = \gamma_5^T,$$
$$\overline{\psi^c} = -\psi^T C^{-1},$$
$$(\psi_L)^c = (\psi^c)_R,$$

where $(\psi^c)_L = P_L(\psi^c) \equiv \psi^c_L$ and so on.

b) Show that for Majorana neutrinos the vector current $\bar{\nu}\gamma_\mu\nu$ vanishes. What happens with $\bar{\nu}\gamma_5\nu$, $\bar{\nu}\gamma_\mu\gamma_5\nu$ and $\bar{\nu}[\gamma_\mu,\gamma_\nu]\nu$?

c) What does the previous result imply for the couplings v_ν and a_ν?

Problem 25: *Z' physics* **[15 Points]**

The general effective Lagrange density after breaking the $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'}$ symmetry to $\mathrm{SU(3)_C \times U(1)_{em}}$ can be written as

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_{Z'} + \mathscr{L}_{\mathrm{mix}},$$

where the relevant part of the Standard Model Lagrangian is

$$\mathscr{L}_{\mathrm{SM}} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j_B^\mu\hat{B}_\mu - \frac{e}{s_W}j_W^{\mu,a}\hat{W}^a_\mu$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z}'_\mu\hat{Z}'^\mu - g' j'_\mu \hat{Z}'_\mu\,,$$

where g' denotes the $\mathrm{U(1)'}$ gauge coupling, and the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_{\mathrm{mix}} = -\frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. Set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You can assume $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged $B-L$ symmetry (baryon minus lepton-number). Write down explicitly the corresponding current for the SM fermions:

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu(B-L)\psi\,,$$

where $(B-L)$ denotes the $B-L$ charge operator.

d) To cancel quantum anomalies in the $B-L$ model, one also has to introduce three right-handed neutrinos N_i which are then part of the current: $\Delta j'_\mu = -\sum_i \bar{N}_i\,\gamma_\mu P_R N_i$. Due to Z-Z' mixing, the SM-like Z_1 will also couple to these new neutrinos. Can you give a constraint on the Z-Z' mixing from the well-measured (invisible) decay width of Z (for $M(N_i) \ll M_Z/2$)? See the PDG for numbers.