Exercises to "Standard Model of Particle Physics II"

Winter 2018/19

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Problem 24: *Left-right symmetric electroweak model* [**10 Points**]

The left-right symmetric model can be introduced by assuming right-handed fermion doublets in analogy to the left-handed ones. The quark and lepton spectra consist of

$$Q^i_{\mathrm{L,\,R}} = \begin{pmatrix} U^i_{\mathrm{L,\,R}} \\ D^i_{\mathrm{L,\,R}} \end{pmatrix}; \qquad L^i_{\mathrm{L,\,R}} = \begin{pmatrix} \nu^i_{\mathrm{L,\,R}} \\ e^i_{\mathrm{L,\,R}} \end{pmatrix},$$

with the following $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$, $\mathrm{U}(1)_{B-L}$ transformation properties:

$$Q_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1/3); \qquad L_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ -1);$$
$$Q_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1/3); \qquad L_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ -1).$$

The Higgs sector contains a bi-doublet ϕ and two triplets Δ_L and Δ_R with transformation properties

$$\phi : (2_\mathrm{L},\ 2_\mathrm{R},\ 0); \qquad \Delta_\mathrm{L} : (3_\mathrm{L},\ 1_\mathrm{R},\ 2); \qquad \Delta_\mathrm{R} : (1_\mathrm{L},\ 3_\mathrm{R},\ 2).$$

Note that the bi-doublet and the triplets can be expressed by the following 2×2 matrices:

$$\phi = \begin{pmatrix} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{pmatrix}; \qquad \Delta_{\mathrm{L,\,R}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}.$$

a) Why does this model not work with only the bi-doublet?

b) Construct the Lagrange density for the fermion-Higgs interactions $\mathscr{L}_\mathrm{Yukawa}$ (including all possible gauge singlets).

c) Use the assumption that after spontaneous symmetry breaking the vacuum is electrically neutral to derive the fermion mass terms in the broken phase.

d) We leave the quark and lepton sector unchanged, but modify the symmetry-breaking part of the model. In the Higgs sector we still have the bi-doublet ϕ, but instead of the triplets we introduce two scalar doublets $A_{\mathrm{L,\,R}}$ and a fermionic singlet χ with the following transformation properties under $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$ and $\mathrm{U}(1)_{B-L}$:

$$A_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1); \qquad A_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1); \qquad \chi : (1_\mathrm{L},\ 1_\mathrm{R},\ 0).$$

Left-right symmetry implies the invariance of the Lagrange density under the following transformations (where Ψ denotes any fermion field):

$$\Psi_\mathrm{L} \leftrightarrow \Psi_\mathrm{R} \qquad A_\mathrm{L} \leftrightarrow A_\mathrm{R} \qquad \phi \leftrightarrow \phi^\dagger.$$

Construct the Lagrange density $\mathscr{L}_\mathrm{Yukawa}$ for the fermion masses in this model. (Again, you have to construct singlets under the whole gauge group.)

Problem 25: *Z' physics* [**10 Points**]

The general effective Lagrange density after breaking the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)'$ symmetry to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{em}$ can be written as

$$\mathscr{L} = \mathscr{L}_\mathrm{SM} + \mathscr{L}_{Z'} + \mathscr{L}_\mathrm{mix},$$

where the relevant part of the Standard Model Lagrangian is

$$\mathscr{L}_\mathrm{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j^\mu_B\hat{B}_\mu - \frac{e}{s_W}j^{\mu,a}_W\hat{W}^a_\mu$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z'}^{\mu\nu} + \frac{1}{2}\hat{M}^2_{Z'}\hat{Z'}_\mu\hat{Z'}^\mu - g' j'_\mu \hat{Z}'_\mu\,,$$

where g' denotes the $\mathrm{U}(1)'$ gauge coupling, and the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_\mathrm{mix} = -\frac{\sin\chi}{2}\hat{Z'}_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. Set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You can assume $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged $B-L$ symmetry (baryon minus lepton-number). Write down explicitly the corresponding current for the SM fermions:

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu(B-L)\psi\,,$$

where $(B-L)$ denotes the $B-L$ charge operator.

d) To cancel quantum anomalies in the $B-L$ model, one also has to introduce three right-handed neutrinos N_i which are then part of the current: $\Delta j'_\mu = -\sum_i \bar{N}_i\,\gamma_\mu P_R N_i$. Due to Z-Z' mixing, the SM-like Z_1 will also couple to these new neutrinos. Can you give a constraint on the Z-Z' mixing from the well-measured (invisible) decay width of Z (for $M(N_i) \ll M_Z/2$)? See the PDG for numbers.

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
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