Exercises to "Standard Model of Particle Physics II"

Winter 2016/17

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann — Sheet 12 — 25.01.17

Problem 23: *Scalar Portal Dark Matter* [**20 Points**]

By adding an additional scalar particle S to the Standard Model (SM), it is possible to couple dark matter to the Standard Model by so-called Higgs-portal interactions. This leads to a renormalizable theory, since the coupling constant of the term with two Higgs bosons and two new scalars is dimensionless (also coupling constants with a positive mass dimension lead to renormalizable theories).

a) Write down the most general renormalizable potential of the SM Higgs ϕ and the additional scalar S in analogy to the SM Higgs potential, which is given by

$$V_{SM} = \mu_H^2 \phi^\dagger \phi + \lambda_H \left(\phi^\dagger \phi\right)^2 \tag{1}$$

with $\mu_H^2 < 0$ and $\lambda_H < 0$, so that it has a non-zero vacuum expectation value (vev) and is bounded from below. (Hint: You should get 5 additional terms.)

b) To forbid on-shell decays like $S \to HH$ or off-shell decays like $S \to H^* H^* \to 4b$ into four b-quarks we require the Lagrangian to obey a global $\mathbb{Z}_2$-symmetry:

$$S \to -S \qquad\qquad H \to +H. \tag{2}$$

Write down the new potential after eliminating all terms from the general potential of a) that do not comply with this symmetry.

c) Insert for the Higgs-doublet the expansion around the vev $\phi = \left(\begin{smallmatrix} 0 \\ v+H \end{smallmatrix}\right)$ and determine the mass of the scalar m_S and the SHH and $SSHH$ couplings g_{SHH} and g_{SSHH} from the potential. For the part of the potential that is not coupled to the scalar you can use $V \supset -\frac{m_H^2}{2} H^2 + \frac{m_H^2}{2v_H} H^3 + \frac{m_H^2}{8v_H^2} H^4$.

d) The coupling of the Higgs boson to fermions is given by a vertex factor of $-i\frac{m_f}{v_H}$, with the mass of the fermion m_f and the Higgs vev $v_H = 246\,\text{GeV}$. Furthermore, the decay width Γ of a particle with mass m can be taken into account by adding $im\Gamma$ to the denominator of the propagator. With this in mind, draw the tree-level Feynman diagram and determine its squared matrix element $|\mathcal{M}|^2$ averaged over initial spins and summed over final spins while taking the finite decay width of the Higgs boson into account.

e) Summing over color and rewriting the expression in terms of the Mandelstam variable $s = (k_1 + k_2)^2$ we have

$$\overline{\sum_{\text{spin, color}} |\mathcal{M}|^2} = N_C 8 \lambda_3^2 m_b^2 \frac{s - 4m_b^2}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2}. \tag{3}$$

To determine the relic density we need the velocity-averaged cross section. In the non-relativistic limit use the identity

$$m_b v = \sqrt{s - 4m_b^2} \tag{4}$$

(factors of v should cancel since we are only looking at the v^0 term) together with

$$\sigma(SS \to b\bar{b}) = \frac{1}{16\pi s}\sqrt{\frac{1-4m_b^2/s}{1-4m_S^2/s}}\,\overline{\sum|\mathcal{M}|^2} \tag{5}$$

to find an approximate expression for $\sigma v\big|_{SS\to b\bar{b}}$. Simplify your expression by assuming to be at threshold $s = 4m_S^2$ and that the scalar is significantly heavier than the b-quark $m_S \gg m_b$.

f) The simplest parameter point to evaluate this is on the Higgs pole $m_H = 2m_S$. Determine $\sigma v\big|_{SS\to b\bar{b}}$ at the Higgs pole by using $\Gamma_H \approx 4\cdot 10^{-5} m_H$. Additionally, take into account that the Higgs also decays to other particles than to b-quarks - the branching ratio is $\mathrm{BR}(H \to b\bar{b}) = 0.6$ - by calculating

$$\langle \sigma_{ann} v\rangle = \frac{1}{\mathrm{BR}(H\to b\bar{b})}\langle \sigma v\rangle\big|_{SS\to b\bar{b}} \tag{6}$$

From $\langle \sigma_{ann} v\rangle = 4\cdot 10^{-9}\frac{1}{\mathrm{GeV}^2}$, which can be calculated within our model by demanding that there is exactly the right amount of dark matter in the universe today, determine a value for λ_3. This value for λ_3 can finally be compared to the result of the full calculation $\lambda_3 \approx 10^{-3}$ and tested in direct detection experiments like Xenon1T as shown in figure 1.

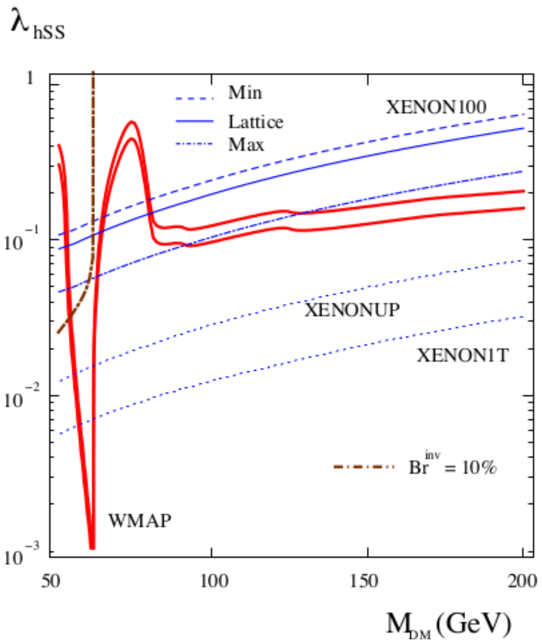

Figure 1: Higgs portal parameter space in terms of the self coupling $\lambda_{HSS} \sim \lambda_3$ and the dark matter mass $m_{DM} = m_S$. The red lines indicate the correct dark matter relic density. Taken from arXiv:1112.3299 [hep-ph].

Tutor:
Moritz Platscher
e-mail: moritz.platscher@mpi-hd.mpg.de

Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106