## Exercises to "Standard Model of Particle Physics II"

Winter 2020/21

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann
Sheet 11 - February 3, 2020

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**Tutor:** Cristina Benso **e-mail:** benso@mpi-hd.mpg.de
**Lecture webpage:** `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

**Hand-in of solutions:** February 10, 2021 - via e-mail, **before 14:00**

**Discussion of solutions:** February 10, 2021 - on zoom

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**Problem 22:** *Stückelberg Mechanism* [**10 Points**]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method involves a real scalar field $\sigma$ together with the $Z'$-boson associated to $U(1)'$.
Consider the Lagrangian

$$\mathscr{L} = -\frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} + \frac{1}{2}(M_{Z'} Z'_\mu + \partial_\mu \sigma)(M_{Z'} Z'^\mu + \partial^\mu \sigma) + i\overline{\psi}\gamma^\mu(\partial_\mu - ig'Y'Z'_\mu)\psi - m\overline{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge $Y'$) and gauge boson are given by

$$\psi \to e^{-ig'Y'\theta(x)}\psi, \qquad Z'_\mu \to Z'_\mu - \partial_\mu \theta(x).$$

a) Calculate the gauge transformation of the real scalar $\sigma$ that makes the Lagrangian invariant and show the invariance of the other terms.

b) Can you fix a gauge to eliminate $\sigma$ from the theory? Show how it is possible. What happens to the number of degrees of freedom?

**Problem 23:** *Seesaw II* [**10 Points**]

We consider the lepton sector of the Standard Model and expand it by adding a Higgs triplet $\Delta$. The particles considered have the following $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ transformation properties:

$$L_a \sim (2,\ -1); \qquad l_{a\mathrm{R}} \sim (1,\ -2); \qquad \phi \sim (2,\ 1); \qquad \Delta \sim (3,\ 2),$$

where the fields denote respectively the left-handed lepton doublet, the right-handed lepton singlet, the SM Higgs doublet and the (non-SM) Higgs triplet in the convention that $Q = I_3 + Y/2$. The flavour index is denoted as $a$. For the triplet use the representation as a $2 \times 2$ matrix with the (electric) charge eigenstates

$$\Delta = \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}.$$

The mass terms for the leptons arise from the Yukawa Lagrangian

$$\mathscr{L}_\mathrm{Y} = \sum_{a,b} \left[ -y_{ab} \overline{l_{a\mathrm{R}}} \phi^\dagger L_b + \frac{1}{2} \tilde{y}_{ab} \overline{L^c_a} i\tau_2 \Delta L_b \right] + \mathrm{h.c.}$$

a) Convince yourself that $\mathscr{L}_\mathrm{Y}$ is a singlet under $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$.

b) The introduction of the triplet changes the Higgs potential to

$$\begin{aligned} V(\phi,\ \Delta) =& a\phi^\dagger\phi + \frac{b}{2}\mathrm{Tr}[\Delta^\dagger\Delta] + c(\phi^\dagger\phi)^2 + \frac{d}{4}(\mathrm{Tr}[\Delta^\dagger\Delta])^2 + \frac{e-h}{2}\phi^\dagger\phi\mathrm{Tr}[\Delta^\dagger\Delta] \\ &+ \frac{f}{4}\mathrm{Tr}[\Delta^\dagger\Delta^\dagger]\mathrm{Tr}[\Delta\Delta] + h\phi^\dagger\Delta^\dagger\Delta\phi + (t\phi^\dagger\Delta(\mathrm{i}\tau_2\phi^*) + \mathrm{h.c.})\,. \end{aligned}$$

Use the condition that only neutral components of the Higgs fields can develop non-zero vacuum expectation values (vev's) $\langle\phi^0\rangle = v$ and $\langle\Delta^0\rangle = v_\Delta/\sqrt{2}$ and find $V(\langle\phi\rangle,\ \langle\Delta\rangle)$.

c) Define $t = |t|e^{i\omega}$, $v_\Delta = |v_\Delta|e^{i\gamma}$ and minimize the potential with respect to $v$, $|v_\Delta|$, and $\gamma$. *Hint*: Start with $\gamma$.

d) Show that, under the assumptions $a,\ b \sim v^2$, and $c,\ d,\ e,\ f,\ h \sim 1$, together with $|t| \ll v$, the conditions for the minimum are approximately equivalent to

$$v^2 \approx -\frac{a}{2c} \qquad \text{and} \qquad |v_\Delta| = \frac{2|t|v^2}{b + (e-h)v^2}\,.$$

e) What do your findings imply for the masses in the lepton sector?

f) Assume that $\sqrt{b} = m_\Delta$ is very large compared to the electroweak scale $v$ and $|v_\Delta|$, and that $t^2 \sim b$, while keeping $c,\ d,\ e,\ f,\ h \sim 1$. Find the relations between the vev's under these conditions. What does it mean for the lepton masses?

g) What is the difference between this scenario (the so-called *type-II* seesaw) and the type-I seesaw from **Problem 21**?