

Exercises to “Standard Model of Particle Physics II”

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Sheet 11

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Problem 22: *Seesaw I* [10 Points]

The Higgs mechanism generates Dirac masses for the active neutrinos when right-handed sterile neutrinos are introduced. The following term appears in the Lagrange density:

$$\mathcal{L}_{\text{Dirac}} = -\bar{\nu}_L M_D N_R + \text{h.c.},$$

where $\nu_L = (\nu_L^1, \nu_L^2, \nu_L^3)^T$ is the column vector of the active neutrinos and N_R the corresponding vector for the sterile neutrinos. The matrix M_D is – in general – a complex 3×3 matrix. The right-handed sterile neutrinos can, furthermore, have a Majorana mass with the Lagrange density

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} \overline{(N_R)^c} M_R N_R + \text{h.c.},$$

where M_R is a symmetric 3×3 matrix, and $\psi^c = \mathcal{C} \bar{\psi}^T$ for a general Dirac spinor ψ and the charge conjugation matrix $\mathcal{C} = i\gamma^2 \gamma^0$.

Let m_D and m_R denote the mass scale of M_D and M_R , respectively. Suppose the entries of M_R are much larger than the ones of M_D ($m_R \gg m_D$).

- a) Show that it is possible to rewrite the whole mass matrix in the flavour basis in the following way

$$\mathcal{L}_{\text{mass}} \equiv \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} = -\frac{1}{2} \overline{\Psi^c} M \Psi + \text{h.c.},$$

with

$$\Psi \equiv \begin{pmatrix} (\nu_L)^c \\ N_R \end{pmatrix} \quad \text{and} \quad M \equiv \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}.$$

For the solution prove and use the identity $\bar{\nu}_L M_D N_R = \overline{(N_R)^c} M_D^T (\nu_L)^c$.

- b) Using the (unitary) transformation $\Psi = U \chi$ with $U = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}$ (change of basis) it is possible to convert the 6×6 matrix M into a block diagonal form, i.e. that it takes on the following form

$$U^T M U \simeq \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \tag{1}$$

with symmetric 3×3 matrices M_1 , M_2 . The matrix ρ in the transformation matrix U is assumed to be proportional to the scale m_R^{-1} and terms of order m_R^{-2} (and smaller) can be neglected in the calculation.

Determine ρ , and M_1 and M_2 from Eq. (1). What is the connection between the fields χ_1 , χ_2 and the original fields ν_L , N_R ? (You may assume that M_R is invertible.)

- c) Where does the name “seesaw” come from?

Problem 23: Seesaw II [10 Points]

We consider the lepton sector of the Standard Model and expand it by adding a Higgs triplet Δ . The particles considered have the following $SU(2)_L \otimes U(1)_Y$ transformation properties:

$$L_a \sim (2, -1); \quad l_{aR} \sim (1, -2); \quad \phi \sim (2, 1); \quad \Delta \sim (3, 2),$$

where the fields denote respectively the left-handed lepton doublet, the right-handed lepton singlet, the SM Higgs doublet and the (non-SM) Higgs triplet in the convention that $Q = I_3 + Y/2$. The flavour index is denoted as a . For the triplet use the representation as a 2×2 matrix with the (electric) charge eigenstates

$$\Delta = \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}.$$

The mass terms for the leptons arise from the Yukawa Lagrangian

$$\mathcal{L}_Y = \sum_{a,b} \left[-y_{ab} \bar{l}_{aR} \phi^\dagger L_b + \frac{1}{2} \tilde{y}_{ab} \bar{L}_a^c i\tau_2 \Delta L_b \right] + \text{h.c.}$$

- Convince yourself that \mathcal{L}_Y is a singlet under $SU(2)_L \otimes U(1)_Y$.
- The introduction of the triplet changes the Higgs potential to

$$V(\phi, \Delta) = a\phi^\dagger\phi + \frac{b}{2}\text{Tr}[\Delta^\dagger\Delta] + c(\phi^\dagger\phi)^2 + \frac{d}{4}(\text{Tr}[\Delta^\dagger\Delta])^2 + \frac{e-h}{2}\phi^\dagger\phi\text{Tr}[\Delta^\dagger\Delta] \\ + \frac{f}{4}\text{Tr}[\Delta^\dagger\Delta^\dagger]\text{Tr}[\Delta\Delta] + h\phi^\dagger\Delta^\dagger\Delta\phi + (t\phi^\dagger\Delta(i\tau_2\phi^*) + \text{h.c.}).$$

Use the condition that only neutral components of the Higgs fields can develop non-zero vacuum expectation values (vev's) $\langle\phi^0\rangle = v$ and $\langle\Delta^0\rangle = v_\Delta/\sqrt{2}$ and find $V(\langle\phi\rangle, \langle\Delta\rangle)$.

- Define $t = |t|e^{i\omega}$, $v_\Delta = |v_\Delta|e^{i\gamma}$ and minimize the potential with respect to v , $|v_\Delta|$, and γ . *Hint:* Start with γ .
- Show that, under the assumptions $a, b \propto v^2$, and $c, d, e, f, h \propto 1$, together with $|t| \ll v$, the conditions for the minimum are approximately equivalent to

$$v^2 \approx -\frac{a}{2c} \quad \text{and} \quad |v_\Delta| = \frac{2|t|v^2}{b + (e-h)v^2}.$$

- What do your findings imply for the masses in the lepton sector?
- Assume that $\sqrt{b} = m_\Delta$ is very large compared to the electroweak scale v and $|v_\Delta|$, and that $t^2 \propto b$. Find the relations between the vev's under these conditions. What does it mean for the lepton masses?
- What is the difference between this scenario (the so-called *type-II* seesaw) and the type-I seesaw from **Problem 22**?

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Lecture webpage: <https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html>

Hand-in and discussion of sheet:

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