Exercises to "Standard Model of Particle Physics II"

Winter 2016/17

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 10 11.01.17

Problem 19: *Galaxy rotation curves* [**10 Points**]

NGC2998 is a spiral galaxy in Ursa Major. You can download Its rotation curve data from `http://astroweb.case.edu/ssm/620f03/n2998.dat` (by Stacy McGaugh). The first column gives the radius, the second the observed/inferred circular velocity with its 1σ uncertainty in the third column. The next few columns provide rotation curves that would arise from the stellar disk, gaseous disk, and the bulge alone.

a) Using Newtonian mechanics, derive the expression for the circular velocity of a star orbiting a central mass on a circular orbit as a function of its distance from the center of the Galaxy. (assume that the central mass is a function of the distance, too.)

b) Interpolate the data for the disk, gas, and bulge distribution of the circular velocity. Plot it against the distance from the center of the galaxy.

c) Using the result from a), what are possible explanations for the discrepancy between the observed and the expected rotation curve?

d) Include an additional source of matter with a density distribution

$$\rho(r) = \frac{\rho_0}{(1 + r/r_0)^\alpha} \tag{1}$$

and determine a set of parameters that fits the data. What does this choice imply for the properties of the DM?

e) In Modified Newtonian Dynamics (MOND), a modified version of Newton's first law is assumed,

$$\mu(a/a_0)\, a = \frac{M_\odot G_N}{R^2}, \text{ where } \mu(a/a_0) = \begin{cases} \frac{a}{a_0} & , \ a \ll a_0 \, , \\ 1 & , \ a \gg a_0 , \end{cases} \tag{2}$$

where a_0 is a constant. Derive the circular velocity distribution of the galactic disk in MOND and find a good fit to the observed velocities.

Problem 20: *WIMP miracle* [**10 Points**]

Dark Matter (DM) is often assumed to be a thermal relic which was in thermal equilibrium with the Standard Model particles only in the early phases of the Universe. Weakly interacting massive particles (WIMPs) are thermal relic DM candidates with masses $m_{\rm DM} \sim 100\,\text{GeV}$ and couplings typical for electroweak physics. The fact that the observed relic density

$$\frac{\Omega_{\rm DM}}{0.2} \approx \frac{10^{-8}\,\text{GeV}^{-2}}{\sigma}$$

can be explained by a WIMP is called the *WIMP miracle*.

a) DM is generally believed to be cold, meaning that the temperature at which it thermally decoupled from the Standard Model is much lower than its mass. In this case, its number density is given by

$$n_{\rm DM} \sim (m_{\rm DM}T)^{3/2} \exp\left(-\frac{m_{\rm DM}}{T}\right), \quad \text{for} \quad m_{\rm DM} \gg T.$$

When the DM interaction rate Γ becomes comparable to the Hubble expansion rate H, the WIMP *freezes out*. In terms of the number density one can write $\Gamma = n_{\rm DM} \cdot v \cdot \sigma$ with σ an interaction cross section and v the velocity. In a radiation dominated Universe, the Friedmann equation gives $H \sim T^2/M_{\rm Pl}$ with the Planck Scale $M_{\rm Pl} = 10^{19}\,\text{GeV}$. Use the freeze-out condition $\Gamma = H$ to show that this implies $m_{\rm DM} \cdot \sigma \cdot M_{\rm Pl} > 1$.

b) Use the previous results to derive a lower bound on the DM mass, assuming a cross section as suggested by the relic density, $\sigma = 10^{-8}\,\text{GeV}^{-2}$.

c) Unitarity constraints provide an upper bound on the annihilation cross section. For a velocity of $v = 0.3$, one has

$$\sigma \lesssim \frac{4\pi}{m_{\rm DM}^2 v^2}.$$

Use this constraint to derive an upper bound on the Dark Matter mass.

Tutor:
Moritz Platscher
e-mail: moritz.platscher@mpi-hd.mpg.de

Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106