

Exercises to “Standard Model of Particle Physics II”

Winter 2015/16

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Sheet 10

13.01.16

Exercise 21: Left-right symmetric electroweak model I [15 Points]

The left-right symmetric model can be introduced by assuming right-handed fermion doublets in analogy to the left-handed ones. The quark and lepton spectra consist of

$$Q_{L,R}^i = \begin{pmatrix} U_{L,R}^i \\ D_{L,R}^i \end{pmatrix}; \quad L_{L,R}^i = \begin{pmatrix} \nu_{L,R}^i \\ e_{L,R}^i \end{pmatrix},$$

with the following $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$ transformation properties:

$$\begin{aligned} Q_L &: (2_L, 1_R, 1/3); & L_L &: (2_L, 1_R, -1); \\ Q_R &: (1_L, 2_R, 1/3); & L_R &: (1_L, 2_R, -1). \end{aligned}$$

The Higgs sector contains a bi-doublet ϕ and two triplets Δ_L and Δ_R with transformation properties

$$\phi : (2_L, 2_R, 0); \quad \Delta_L : (3_L, 1_R, 2); \quad \Delta_R : (1_L, 3_R, 2).$$

Note that the bi-doublet and the triplets can be expressed by the following 2×2 matrices:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}; \quad \Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}.$$

- a) Why does this model not work with only the bi-doublet?
- b) Construct the Lagrange density for the fermion-Higgs interactions $\mathcal{L}_{\text{Yukawa}}$.
- c) Use the assumption that, after spontaneous symmetry breaking the vacuum is electrically neutral, write down the fermion mass terms in the broken phase.
- d) We leave the quark and lepton sector unchanged, but modify the symmetry-breaking part of the model. In the Higgs sector we still have the bi-doublet ϕ , but instead of the triplets we introduce two scalar doublets $A_{L,R}$ and a fermionic singlet χ with the following transformation properties under $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$:

$$A_L : (2_L, 1_R, 1); \quad A_R : (1_L, 2_R, 1); \quad \chi : (1_L, 1_R, 0).$$

Left-right symmetry implies the invariance of the Lagrange density under the following transformations (where Ψ denotes any fermion field):

$$\Psi_L \leftrightarrow \Psi_R \quad A_L \leftrightarrow A_R \quad \phi \leftrightarrow \phi^\dagger.$$

Construct the Lagrange density $\mathcal{L}_{\text{Yukawa}}$ for the fermion masses in this model (you have to build singlets under the whole gauge group).

Exercise 22: Z' physics [15 Points]

The general effective Lagrange density after breaking the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ symmetry to $SU(3)_C \times U(1)_{EM}$ can be written as

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Z'} + \mathcal{L}_{mix},$$

where the relevant part of the Standard Model Lagrangian is

$$\mathcal{L}_{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}_{\mu\nu}^a\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}'_\mu\hat{Z}'^\mu - \frac{\hat{e}}{\hat{c}_W}j_B^\mu\hat{B}_\mu - \frac{e}{c_W}j_W^{\mu,a}\hat{W}_\mu^a$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z}'_\mu\hat{Z}'^\mu - g'j'_\mu\hat{Z}'^\mu$$

and the kinetic- and mass-mixing terms can be parameterized as

$$\mathcal{L}_{mix} = -\frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2 Z'_\mu Z'_\mu \hat{Z}'^\mu$$

- Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B , j_W and j' . Set the kinetic mixing angle χ to zero for simplicity.
- Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008_{-0.0007}^{+0.0017}$ to constrain the Z - Z' mixing.
- A well-motivated extension of the SM is a gauged $B-L$ symmetry (baryon minus lepton-number). Write down explicitly the corresponding current for the SM fermions:

$$j'_\mu = \sum_\psi \bar{\psi} \gamma_\mu (B-L) \psi.$$

- To cancel quantum anomalies in the $B-L$ model, one also has to introduce three right-handed neutrinos N_i which are then part of the current: $\Delta j'_\mu = -\sum_i \bar{N}_i \gamma_\mu P_R N_i$. Due to Z - Z' mixing, the SM-like Z_1 will also couple to these new neutrinos. Can you give a constraint on the Z - Z' mixing from the well-measured Z -width (for $M(N_i) \ll M_Z/2$)? See the PDG for numbers.

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Lectures homepage: https://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html

Discussion and hand-in of sheet:

Tuesday 19.01.16 4.15 pm, INF 501/CIP R. 103