

# Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

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Sheet 10

21.01.15

## Exercise 22: Seesaw I [10 Points]

The Higgs mechanism generates Dirac masses for the active neutrinos, when right-handed sterile neutrinos are introduced. The following term appears in the Lagrange density:

$$\mathcal{L}_{\text{Dirac}} = -\bar{\nu}_L M_D N_R + \text{h.c.},$$

where  $\nu_L = (\nu_L^1, \nu_L^2, \nu_L^3)^T$  is the column vector of the active neutrinos and  $N_R$  the corresponding vector for the sterile neutrinos. The matrix  $M_D$  is – in general – a complex  $3 \times 3$  matrix. The right-handed sterile neutrinos can, furthermore, have a Majorana mass with the Lagrange density:

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} \overline{(N_R)^c} M_R N_R + \text{h.c.},$$

where  $M_R$  is a symmetric  $3 \times 3$  matrix.

Let  $m_D$  and  $m_R$  denote the mass scale of  $M_D$  and  $M_R$ , respectively. Suppose the entries of  $M_R$  are much larger than the ones of  $M_D$ , “ $m_R \gg m_D$ ”.

- a) Show that it is possible to rewrite the whole mass matrix in the flavour basis in the following way

$$\mathcal{L}_{\text{mass}} \equiv \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} = -\frac{1}{2} \bar{\Psi}^c M \Psi + \text{h.c.},$$

with

$$\Psi \equiv \begin{pmatrix} (\nu_L)^c \\ N_R \end{pmatrix} \quad \text{and} \quad M \equiv \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

For the solution prove and use the identity  $\bar{\nu}_L M_D N_R = \overline{(N_R)^c} M_D^T (\nu_L)^c$ .

- b) Using the (unitary) transformation  $\Psi = U \chi$  with  $U = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}$  (change of basis) it is possible to convert the  $6 \times 6$  matrix  $M$  into a block diagonal form, i.e. that it takes on the following form

$$U^T M U \simeq \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad (1)$$

with symmetric  $3 \times 3$  matrices  $M_1, M_2$ . The matrix  $\rho$  in the transformation matrix  $U$  is assumed to be proportional to the scale  $m_R^{-1}$  and terms of order  $m_R^{-2}$  (and smaller) can be neglected in the calculation.

Determine  $\rho$ , and  $M_1$  and  $M_2$  from eq. (1). What is the connection between the fields  $\chi_1, \chi_2$  and the original fields  $\nu_L, N_R$ ? (You may assume that  $M_R$  is invertible.)

- c) Where does the name “seesaw” come from?

**Exercise 23:** Seesaw II [10 Points]

We consider the lepton sector of the Standard Model and expand it by adding a Higgs triplet  $\Delta$ . The particles considered have the following  $SU(2)_L \otimes U(1)_Y$  transformation properties:

$$L_a \sim (2, -1); \quad l_{aR} \sim (1, -2); \quad \phi \sim (2, 1); \quad \Delta \sim (3, 2),$$

where the fields denote (in order of appearance) the left-handed lepton doublet, the right-handed lepton singlet, the SM Higgs doublet and the (non-SM) Higgs triplet, and  $a$  is a flavour index. For the triplet use the representation as  $2 \times 2$  matrix with the charge eigenstates

$$\Delta = \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}.$$

The mass terms for the leptons stem from the Yukawa Lagrangian

$$\mathcal{L}_Y = \sum_{a,b} \left[ -y_{ab} \bar{l}_{aR} \phi^\dagger L_b + \frac{1}{2} \tilde{y}_{ab} \bar{L}_a^c i\tau_2 \Delta L_b \right] + \text{h.c.}$$

- a) Convince yourself that  $\mathcal{L}_Y$  is a singlet under  $SU(2)_L \otimes U(1)_Y$ .
- b) The introduction of the triplet changes the Higgs potential to

$$\begin{aligned} V(\phi, \Delta) = & a\phi^\dagger\phi + \frac{b}{2}\text{Tr}[\Delta^\dagger\Delta] + c(\phi^\dagger\phi)^2 + \frac{d}{4}(\text{Tr}[\Delta^\dagger\Delta])^2 + \frac{e-h}{2}\phi^\dagger\phi\text{Tr}[\Delta^\dagger\Delta] \\ & + \frac{f}{4}\text{Tr}[\Delta^\dagger\Delta^\dagger]\text{Tr}[\Delta\Delta] + h\phi^\dagger\Delta^\dagger\Delta\phi + (t\phi^\dagger\Delta(i\tau_2\phi^*) + \text{h.c.}). \end{aligned}$$

Use the condition that only neutral components of the Higgs fields can develop non-zero vacuum expectation values (vev's)  $\langle\phi^0\rangle = v$  and  $\langle\Delta^0\rangle = v_\Delta/\sqrt{2}$  and find  $V(\langle\phi\rangle, \langle\Delta\rangle)$ .

- c) Define  $t = |t|e^{i\omega}$ ,  $v_\Delta = |v_\Delta|e^{i\gamma}$  and minimize the potential with respect to  $v$ ,  $|v_\Delta|$ , and  $\gamma$ . *Hint:* Start with  $\gamma$ .
- d) Show that, under the assumptions  $a, b \propto v^2$ , and  $c, d, e, f, h \propto 1$ , together with  $|t| \ll v$ , the conditions for the minimum are approximately equivalent to

$$v^2 \approx -\frac{a}{2c} \quad \text{and} \quad |v_\Delta| = \frac{2|t|v^2}{b + (e-h)v^2}.$$

- e) What do your findings imply for the masses in the lepton sector?
- f) Assume that  $\sqrt{b} = m_\Delta$  is very large compared to the electroweak scale  $v$ , and  $|v_\Delta|$ , and that  $t^2 \propto b$ . Find the relations between the vev's under these conditions. What does it mean for the lepton masses?
- g) What is the difference between this scenario (the so-called “type II” seesaw) and the type I seesaw from Ex. 22?

**Tutor:**

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Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

**Hand-in and discussion of sheet:**

during tutorial on Thursday, 29.01.15, 9.15 am, kHs, Philosophenweg 12