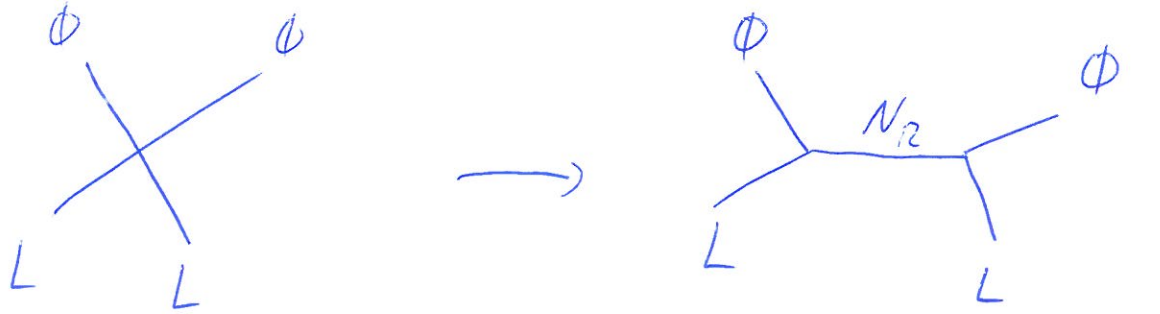
Recap.: How to generate tiny m_ν in $\mathcal{L} = \frac{1}{2} \overline{\nu_L}\, m_\nu\, \nu_L^c$

$\longrightarrow$ effective approach: $\mathcal{L}_5 = \frac{1}{\Lambda} \frac{1}{2} Y_{\alpha\beta}\, \overline{L_\alpha}\, \tilde{\Phi}\, \tilde{\Phi}^T L_\beta^c$

with $L_\alpha = \begin{pmatrix} \nu_\alpha \\ \alpha \end{pmatrix}_L$ $\quad \alpha = e, \mu, \tau$

and $m_\nu = m_\nu^T = Y_{\alpha\beta} \frac{v^2}{\Lambda}$, $\Lambda \approx 10^{15}$ GeV



$\mathcal{L} = \overline{\nu_L}\, \overset{m_D}{} N_R + \frac{1}{2} \overline{N_R^c}\, \underset{M_R}{} N_R$

m_D : 3×3 matrix
M_R : n×n matrix...
[at least two ($\leftrightarrow 2\,\Delta m^2$), no upper limit...]

integrate out N_R:

$\frac{\partial \mathcal{L}}{\partial N_R} = 0 \Rightarrow \overline{\nu_L}\, m_D = -\overline{N_R^c}\, M_R$

$\Rightarrow \overline{N_R^c} = -\overline{\nu_L}\, m_D\, M_R^{-1}$ $\qquad$ use: $\overline{N_R^c} = N_R^T C$; $C^T = C^{-1}$

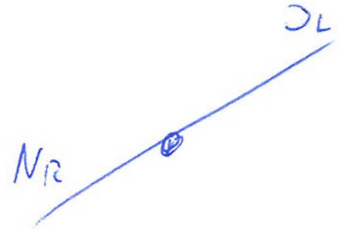
$\Rightarrow N_R = -M_R^{-1} m_D^T C\, \overline{\nu_L}^T = -M_R^{-1} m_D^T \nu_L^c$; insert back in $\mathcal{L}$:

$$\mathcal{L} = -\overline{\nu_L}\, m_D M_R^{-1} m_D^T \nu_L^c + \frac{1}{2} \overline{\nu_L}\, m_D M_R^{-1} M_R M_R^{-1} m_D^T \nu_L^c$$

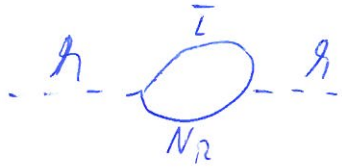
$$= -\frac{1}{2} \overline{\nu_L}\, m_\nu\, \nu_L^c \quad \text{with } m_\nu = -m_D M_R^{-1} m_D^T$$

the larger M_R, the smaller m_ν

„Seesaw Mechanism"

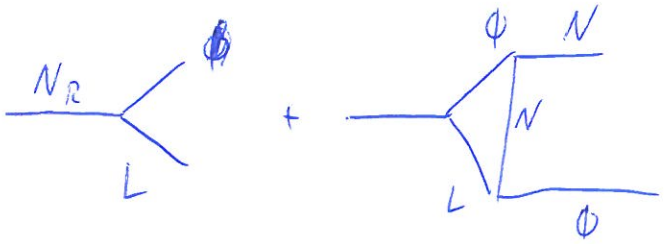


<u>predictions</u> : → coupling to Higgs ↔ vacuum stability worsened by fermionic contribution to $\dot{\lambda}$ (m_D has same effect as m_{top}...)

→ 

$$\delta m_h^2 \propto \frac{m_D^2}{V^2} M_R^2 \quad \text{(hierarchy problem)}$$

→ early Universe:



creates more matter than antimatter

⇒ Baryon Asymmetry !
connected to „low energy" m_ν !

(ii) Type II Seesaw

$\mathcal{L} = \bar{L} L^c$ has isospin $I_3 = +1$; $\sim (3, 2)$

$\Rightarrow$ introduce scalar triplet $\sim (3, -2)$

can be written as $\Delta = \begin{pmatrix} \Delta^- & -\sqrt{2}\Delta^0 \\ \sqrt{2}\Delta^{--} & -\Delta^- \end{pmatrix}$

(note: $I_3 = Q - Y/2 \Rightarrow Q = -2, -1, 0$ particles)

$$\boxed{\mathcal{L} = \tfrac{1}{2} y \bar{L} \Delta i \sigma_2 L^c}$$ is gauge invariant and renormalizable

Δ acquires a VEV: $\langle \Delta \rangle = \begin{pmatrix} 0 & v_T \\ 0 & 0 \end{pmatrix}$

$$\Rightarrow \boxed{\mathcal{L} = \tfrac{1}{2} y v_T \bar{\nu}_L \nu_L^c}$$ and $m_\nu = y v_T$

$\Rightarrow v_T = \mathcal{O}(m_\nu)$

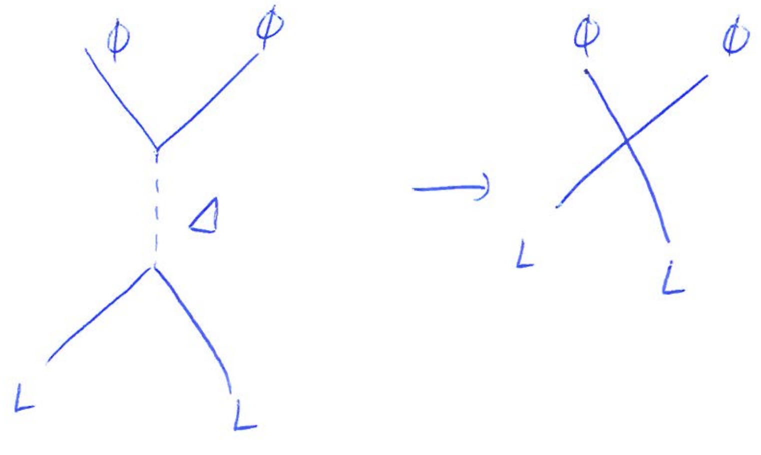
Comments:

#) Scalar potential:

$$V = M_\Delta^2 \, \mathrm{Tr}\{\Delta \Delta^+\} + \mu \, \phi^+ \underset{\sim}{i\sigma_2} \Delta \phi + \ldots$$

with $\frac{\partial V}{\partial \Delta} = 0$: $\Rightarrow$ $v_T \simeq \frac{\mu v^2}{M_\Delta^2}$

„type II seesaw"

#) 

$\Lambda \simeq \frac{M_\Delta^2}{\mu}$
in Weinberg-operator

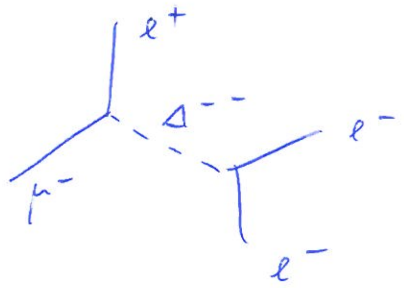
#) Δ couples directly to $W^\pm, Z$

#) breaks custodial symmetry:

$$\rho = \frac{\sum I_3^i (I^i + 1 - Y_i^2/4) v_i^2}{\frac{1}{2} \sum v_i^2 Y_i^2} = \frac{v^2 + 2 v_T^2}{v^2 + 4 v_T^2} \simeq 1 - 2 \, v_T^2/v^2$$

$\Rightarrow v_T \lesssim 8$ GeV

#) can give



$\mu^- \to e^+ e^- e^-$

at tree level

$\propto (m_\nu)_{e\mu} (m_\nu)_{ee}$

$\propto (m_\nu m_\nu^+)_{e\mu}$

(iii) type III Seesaw

$$\mathcal{L} = \bar{L}\, \Sigma^c \tilde{\Phi} \qquad \text{with } \Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} \sim (3,0)$$

„hyperchargeless fermion triplet"

same formula for m_ν, differences in particle content;

needs one set of Σ per massive neutrino...

IV Directions beyond the Standard Model

fields → strings/membranes...

double representations:
4th generation,
2nd Higgs doublet,
⋮

link boson – fermions

SM

increase dimensions

extend gauge group:
additional $U(1)$
$SU(2)$
⋮

$SO(10) \subset SU(3)\times SU(2)\times U(1)$

new representations/fields:
$N_R \sim (1,0)$
$\Delta \sim (3,-2)$
⋮
scalar color octets...
⋮

Let's discuss a few examples... what would change, how do we constrain the physics?

a) double representations

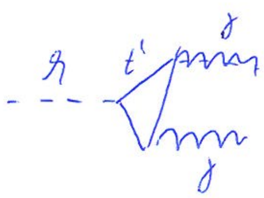
(i) 4th generation

$$Q_4 = \begin{pmatrix} t' \\ b' \end{pmatrix}_L \quad ; \quad m_{t'} = y_{t'} \frac{v}{\sqrt{2}} \Leftrightarrow \text{perturbativity } y_{t'} \leq 4\pi$$
$$\Rightarrow m_{t'} \lesssim 2 \text{ TeV}$$

•) $\Delta \rho_4 = \frac{3 G_F^2}{8\pi^2 2\sqrt{2}} \left[m_{t'}^2 + m_{b'}^2 - 2 \frac{m_{t'}^2 m_{b'}^2}{m_{t'}^2 - m_{b'}^2} \log m_{t'}^2 / m_{b'}^2 \right]$

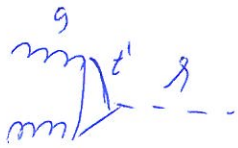
can be evaded if $m_{t'} \simeq m_{b'}$

•) V_{CKM} is 4×4 matrix ⇒ 3×3 part <u>not</u> unitary,
e.g. $|V_{ub'}|^2 = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$

•) 

decreased!

(SM$_3$: W diagram has neg. interferes with smaller top-loop...)



increased!

•) effects for vacuum stability, electroweak baryogenesis, ...

(ii) 2 Higgs-Doublet Models

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \sim (2,1) \qquad v_1$$

$$\Phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_1^- \end{pmatrix} \sim (2,-1) \qquad v_2$$

$\tan\beta \equiv v_1/v_2$

$\sum v_i^2 = v^2 = (246\,\mathrm{GeV}$

$$h^0 = (\mathrm{Re}\,\phi_1^0 - v_1)\cos\alpha - (\mathrm{Re}\,\phi_2^0 - v_2)\sin\alpha$$

α is mixing between 2 Higgses

$$H^0 \perp h^0 \quad \text{with } m_{H^0} > m_{h^0}$$

$$A^0 = \mathrm{Im}\,\phi_2^0 \sin\beta + \mathrm{Im}\,\phi_1^0 \cos\beta$$

(Pseudoscalar...)

$$H^\pm = \phi_2^\pm \sin\beta + \phi_1^\pm \cos\beta$$

- 4 masses, α, β to be determined
- WWh, WWA, ... couplings (modified)
- Φ_1 for up-type, Φ_2 for down-type?
  both contributing to up-quarks?
  $\Rightarrow$ GIM mechanism no longer active;
  off-diagonal hq_1q_2 couplings...;

  ⋮

b) new representations

(i) $N_R \sim (1,0)$ ✓

$\Delta \sim (3,2)$ ✓

simplest example(?) scalar singlet $S \sim (1,0)$ (real!)

$$\mathcal{L} = \partial_\mu S\, \partial^\mu S - V(\Phi, S)$$

$$V(\Phi,S) = -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \lambda_1 (\Phi^\dagger\Phi)^2 + \lambda_2 S^4 + \lambda_3 (\Phi^\dagger\Phi) S^2$$

$$= -m^2 \Phi^\dagger\Phi - \mu^2 S^2 + (\Phi^\dagger\Phi,\ S^2) \begin{pmatrix} \lambda_1 & \lambda_3/2 \\ \lambda_3/2 & \lambda_2 \end{pmatrix} \begin{pmatrix} \Phi^\dagger\Phi \\ S^2 \end{pmatrix}$$

from $\Phi = \begin{pmatrix} 0 \\ \frac{v+\tilde{h}}{\sqrt{2}} \end{pmatrix}$ $\quad S = \frac{h' + w}{\sqrt{2}}$ $\Rightarrow$ mixing of $\tilde{h}, h'$

$\downarrow$ diagonalize

h, H

conditions for V: $\lambda_1 > 0,\ \lambda_2 > 0$ (bounded from below for large field values)

$$\mathcal{H} = \begin{pmatrix} \frac{\partial^2 V}{\partial v^2} & \frac{\partial^2 V}{\partial v \partial w} \\ \frac{\partial^2 V}{\partial v \partial w} & \frac{\partial^2 V}{\partial w^2} \end{pmatrix} > 0 \quad \text{for } 4\lambda_1\lambda_2 > \lambda_3^2$$

Note: cubic terms in S forbidden by „Z_2-symmetry"

c) extended gauge sector

(i) additional $U(1)$ $\Rightarrow$ Z' boson

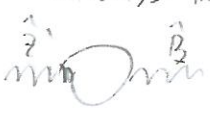
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Z'} + \mathcal{L}_{mix}$$

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^{2}\hat{Z}'_{\mu}\hat{Z}'^{\mu} - \tilde{g}\, j'^{\mu}\hat{Z}'_{\mu}$$

new gauge coupling; new current, unknown structure: e.g. B, L, $B-L$, $L_\mu - L_\tau, \ldots$

$$\mathcal{L}_{mix} = -\frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^{2}\hat{Z}'_{\mu}\hat{Z}^{\mu}$$

always induced:



$U(1)_Y$

if scalars exist that have vevs and are charged under $SU(2)_L \times U(1)_Y$ <u>and</u> $U(1)'$

all terms are hatted, e.g. $\hat{Z}'_\mu$, because mixing is present.

recall:
$$\hat{A}_\mu = \hat{c}_W \hat{B}_\mu + \hat{s}_W \hat{W}^3_\mu$$
$$\hat{Z}_\mu = \hat{c}_W \hat{W}^3_\mu - \hat{s}_W \hat{B}_\mu$$

1) make kinetic terms canonical:

$$\begin{pmatrix} \hat{B}^{\mu} \\ \hat{Z}'^{\mu} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\tan\chi \\ 0 & 1/\cos\chi \end{pmatrix}}_{= \begin{pmatrix} 1 & \sin\chi \\ 0 & \cos\chi \end{pmatrix}^{-1}} \begin{pmatrix} B^{\mu} \\ Z'^{\mu} \end{pmatrix}$$

proof: $-\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} - \frac{1}{4} \hat{Z}'_{\mu\nu} \hat{Z}'^{\mu\nu} - \frac{\sin\chi}{2} \hat{Z}'^{\mu\nu} \hat{B}_{\mu\nu}$

$$= -\frac{1}{4} \left(\hat{B}_{\mu\nu}, \hat{Z}'_{\mu\nu} \right) \begin{pmatrix} 1 & \sin\chi \\ \sin\chi & 1 \end{pmatrix} \begin{pmatrix} \hat{B}^{\mu\nu} \\ \hat{Z}'^{\mu\nu} \end{pmatrix} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu}$$

~~and~~ noting that $\begin{pmatrix} 1 & 0 \\ -t_\chi & 1/c_\chi \end{pmatrix} \begin{pmatrix} 1 & s_\chi \\ s_\chi & 1 \end{pmatrix} \begin{pmatrix} 1 & -t_\chi \\ 0 & 1/c_\chi \end{pmatrix} = \mathbb{1}$

2) for convenience:

undo Weinberg notation

$$B_\mu = \hat{c}_W \tilde{A}_\mu - \hat{s}_W \tilde{Z}_\mu$$
$$W^3_\mu = \hat{s}_W \tilde{A}_\mu + \hat{c}_W \tilde{Z}_\mu$$
$$Z'_\mu = \tilde{Z}'_\mu$$

$\Rightarrow$ mass matrix in $\tilde{A}, \tilde{Z}, \tilde{Z}'$ basis:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & a & b \\ 0 & b & c \end{pmatrix} \qquad a = \hat{M}_Z^2 \qquad b = \hat{s}_W \tan\chi \, \hat{M}_Z^2 + \frac{\delta \hat{M}^2}{\cos\chi}$$

$$c = \frac{1}{c_\chi^2} \left(\hat{M}_Z^2 \hat{s}_W^2 \sin^2_\chi + 2 \hat{s}_W \sin\chi \, \delta \hat{M}^2 + \hat{M}_{Z'}^2 \right)$$

gets diagonalized: $\tan 2\xi = \frac{2b}{a-c}$ and in total:

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \hat{c}_W \sin\chi \\ 0 & \cos\xi & -\hat{s}_W \cos\xi \sin\chi + \sin\xi \cos\chi \\ 0 & -\sin\xi & \cos\xi \cos\chi + \hat{s}_W \sin\xi \sin\chi \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{Z}' \end{pmatrix}$$

we can re-define Weinberg angle:

$$\hat{s}_W^2 \hat{c}_W^2 = \frac{\pi \alpha}{\sqrt{2}\, G_F \hat{M}_Z^2}$$ because in SM: $s_W^2 c_W^2 = \frac{\pi \alpha}{\sqrt{2}\, G_F M_Z^2}$

μ-decay

$$\Rightarrow \hat{s}_W^2 \simeq s_W^2 \left[1 - \frac{1-s_W^2}{1-2s_W^2} \sin^2\xi \left(\frac{M_2^2}{M_1^2} - 1 \right) \right]$$

small effect

<u>Tests:</u> collider, EWPO, LFV, ...

(ii) $SU(2)_L \times SU(2)_R \times U(1)$

SM: only V−A, Left-handed interactions only.

Make up for this injustice with <u>left-right symmetry</u>

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv L \sim (2,1,-1) \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv R \sim (1,2,-1)$$

$SU(2)_L$ $SU(2)_R$ $U(1)_{B-L}$

Baryon − lepton number

$$Q = I_3^L + I_3^R + \frac{B-L}{2} = I_3^L + Y/2$$

⇒ additional $SU(2)_R$ with gauge coupling g_R

scalar sector: $$\Delta_{L,R} = \begin{pmatrix} \Delta^+_{L,R} & -\sqrt{2}\,\Delta^0_{L,R} \\ \sqrt{2}\,\Delta^{--}_{L,R} & -\Delta^-_{L,R} \end{pmatrix} \sim \begin{matrix} (3,1,2) \\ (1,3,2) \end{matrix}$$

$$\Phi = \begin{pmatrix} \Phi_1^0 & \Phi_1^+ \\ \Phi_2^- & \Phi_2^0 \end{pmatrix} \sim (2,2,0) \text{, bidoublet}$$

$$\Rightarrow \mathcal{L} = \bar{L}\,(f\,\Phi + \tilde{f}\,\tilde{\Phi})\,R + h_L \bar{L}\,\Delta_L\, i\sigma_2\, L^c + h_R \bar{R}\,\Delta_R\, i\sigma_2\, R^c$$

with $\tilde{\Phi} = \sigma_2 \Phi^*_{} \sigma_2$, $f, \tilde{f}, h_L, h_R$ are Yukawas

add discrete LR - symmetry:

$C: L \leftrightarrow R^c \quad \Phi \leftrightarrow \Phi^T \quad \Delta_L \leftrightarrow \Delta_R$

$$\Rightarrow h_L = h_R \;, \; f = f^T \;, \; \tilde{f} = \tilde{f}^T \;, \; g_L = g_R$$

$$\langle \Phi \rangle = \begin{pmatrix} \varkappa_1/\sqrt{2} & 0 \\ 0 & \frac{\varkappa_2}{\sqrt{2}} e^{i\alpha} \end{pmatrix} \;, \; \langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & v_L, v_R \\ 0 & 0 \end{pmatrix}$$

($\langle \Delta_L \rangle$ in general complex)

$$\Rightarrow M_e = \varkappa_2 e^{i\alpha} f + \varkappa_1 \tilde{f}$$

$$m_D = \varkappa_1 f + \varkappa_2 e^{-i\alpha} \tilde{f}$$

$$m_L = v_L h_L$$

$$M_R = v_R h_R$$

($\Rightarrow$ is now vev $*$ Yukawa)

potential gives $\quad v_L v_R = \gamma v^2 = \gamma(\varkappa_1^2 + \varkappa_2^2) \;; \; \gamma = O(1)$

Breaking chain: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$\Big\downarrow \; M_{W_R}, v_R \text{ breaks } SU(2)_R$$

$SU(2)_L \times U(1)_Y$

$$\Big\downarrow \; \text{see below...}$$

$\Rightarrow$ there are RH interactions: suppressed by $\left(\frac{m_W}{m_{W_R}}\right)^2$

$\Rightarrow$ neutrino mass via type I (+ II) seesaw:

if $v_R \to \infty$: no m_ν! no RH interactions

$\Rightarrow$ smallness of neutrino mass

$\Updownarrow$

maximal parity violation