Recap.: $L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ create neutrino mass by adding: ν_R

$\nu_R \sim (1,0)$; $L \sim (2,-1)$; $\phi \sim (2,1)$

$$\mathcal{L}_D = g_\nu \bar{L} \tilde{\Phi} \nu_R \xrightarrow{SSB} \frac{v}{\sqrt{2}} g_\nu \bar{\nu}_L \nu_R$$ mass for neutrino!

But: $g_\nu \simeq 10^{-12}$ $\boxed{\text{Why?}}$

Solution possibly connected to Majorana-nature!

$\Longleftrightarrow$ particle-antiparticle conjugation: $\psi^c = C \bar{\psi}^T$; $C = i\gamma_2 \gamma_0$

flips charge-like quantum numbers and chirality

$$(\psi_L)^c = (\psi^c)_R \quad ; \quad (\psi_R)^c = (\psi^c)_L$$

recall that for a mass term LH and RH component are necessary: $m \bar{\psi} \psi = m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$

$\Rightarrow$ A fermion has only 2 possibilities:

(i) ψ_R independent of ψ_L : Dirac particle

(ii) $\psi_R = (\psi_L)^c$: Majorana particle

$\Rightarrow \boxed{\psi^c = \psi}$

recall that one can write $\psi = \begin{pmatrix}\phi \\ \xi\end{pmatrix} \Rightarrow \psi_L = \begin{pmatrix}\phi \\ 0\end{pmatrix}$; $\psi_R = \begin{pmatrix}0 \\ \xi\end{pmatrix}$

$$\Rightarrow (\not{p} - m)\psi = 0 \longrightarrow \begin{cases} (i\partial_0 - i\vec{\sigma}\vec{\nabla})\phi - m\xi = 0 \\ (i\partial_0 + i\vec{\sigma}\vec{\nabla})\xi - m\phi = 0 \end{cases}$$

[ϕ, ξ are „Weyl spinors"]

$\Rightarrow$ with $\psi = \psi^c$ one can write Majorana-fermion

as $\psi = \begin{pmatrix}\phi \\ -i\sigma_2\phi^*\end{pmatrix}$ $\Rightarrow$ only 2 degrees of freedom

[Dirac have 4 d.o.f.]

or: $$\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s \left[\underbrace{b_s(\vec{p})}_{\text{annihilaton}} u_s(\vec{p}) e^{-ipx} + \underbrace{b_s^\dagger}_{\text{creator}} v_s e^{ipx} \right]$$

where $v_s = C\bar{u}_s^T$ and $u_s = C\bar{v}_s^T$, thus $\psi^c = \psi$

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mass term for Majoranas: $\mathcal{L} = \frac{1}{2} m_\nu \bar{\nu}\nu = \frac{1}{2} m_\nu (\bar{\nu}_L + \overline{\nu_L^c})(\nu_L + \nu_L^c)$

$$= \frac{m_\nu}{2}(\nu_L \nu_L^c + \overline{\nu_L^c}\nu_L) = \frac{m_\nu}{2}\bar{\nu}_L \nu_L^c + h.c.$$

and total $\mathcal{L} = \frac{1}{2}(\bar{\nu} i \gamma \nu - m\bar{\nu}\nu)$

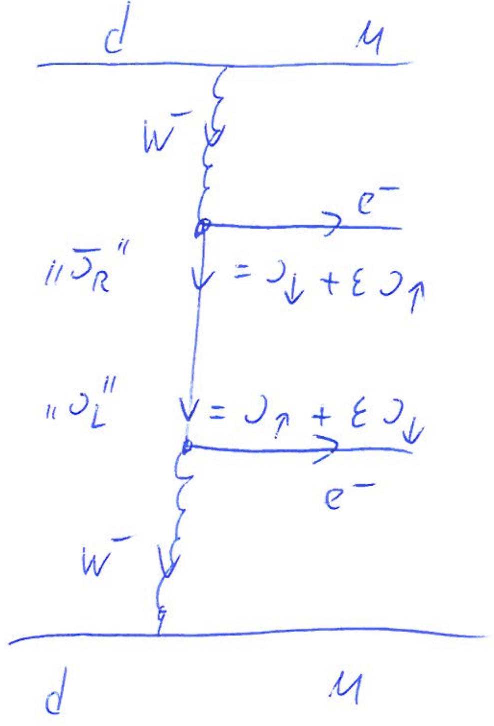
Important: $\mathcal{L} \sim \nu_L^\dagger \nu_L^*$ not invariant under

$\nu_L \to e^{i\alpha}\nu_L \Leftrightarrow$ Lepton number

$\Longrightarrow$ $\boxed{\text{Lepton Number Violation}}$ / $\Delta L = 2$

How to test $\Delta L \neq 0$: $(A, Z) \to (A, Z+2) + 2e^-$

„neutrinoless double beta decay"
$(0\nu\beta\beta)$



$\nu_D = (\nu_\uparrow, \nu_\downarrow, \bar{\nu}_\uparrow, \bar{\nu}_\downarrow)$ 4 d.o.f

[helicity is conserved]

$\nu_M = (\nu_\uparrow, \nu_\downarrow)$ 2 d.o.f.

$\Rightarrow$ can be absorbed if $\nu = \bar{\nu}$ and $\varepsilon \neq 0$

e.g. $^{76}Ge \to {}^{76}Se + 2e^-$: $T_{1/2} \geq 2 \times 10^{25}$ yrs (!)

What is ε? E.g., $u_\downarrow(p) = u_L^{(m=0)}(p) + \frac{m}{2E} u_R^{(m=0)}$

$$(\underset{\text{conserved, not Lorentz-invariant}}{\text{helicity}} = \underset{\text{Lorentz invariant, not conserved}}{\text{chirality}} \text{ for } m=0)$$

$\Rightarrow \varepsilon = O(m/E) \Rightarrow$ probability for observing Majorana nature is always suppressed by $\left(\frac{m}{E}\right)^2$

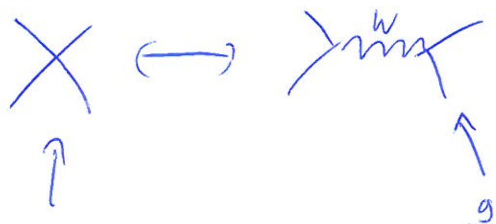
$$\Gamma(0\nu\beta\beta) \propto \frac{(U_{ei}^2 m_i)^2}{E^2}$$ with $|\Sigma U_{ei}^2 m_i| = \langle m \rangle$

$E \sim 100$ MeV ($\leftrightarrow$ 1 fm distance between n)

III 4) Generating tiny neutrino masses

a) Nonrenormalizable Operators

$G_F \leftrightarrow \frac{1}{m_W^2}$:



$G_F \bar f_i \gamma_\mu f_i \bar f_i' \gamma^\mu f_i''$

non-renormalizable
higher-dimensional
operator
„effective dim-6 operator"

$\frac{g}{\sqrt{2}} \bar f_i \gamma_\mu f_i' W^\mu$

„renormalizable theory

$\Rightarrow$ use all SM-fields and try to write down all possible gauge-invariant and Lorentz-invariant terms of dimension 5, 6, 7, ...

Turns out: $\exists$ dim-5 term:

$$\boxed{\mathcal{L} = \frac{1}{\Lambda}\frac{1}{2}\bar L \tilde\Phi \tilde\Phi^T L^c \xrightarrow[\text{EWSB}]{} \frac{1}{2}\frac{v^2}{\Lambda}\bar\nu_L \nu_L^c}$$

Weinberg-operator

Majorana mass term !

$(\tilde\Phi = i\tau_2 \Phi^* ;\ L \to UL ;\ \Phi \to U\Phi)$

$\Rightarrow m_\nu = \frac{v^2}{\Lambda}$ with $v \simeq 10^2$ GeV ; $m_\nu \simeq 0.1$ eV

$\Lambda = 10^{15}$ GeV !

b) Seesaw-Mechanism(s)

↔ renormalizable realizations of Weinberg operator
only 3 possibilities :

Masterformula: $2 \otimes 2 = 3 \oplus 1$ (SU(2) multiplication)

$L \tilde{\Phi} \sim (2,1) \otimes (2,-1) = (3,0) \oplus (1,0)$

$\Rightarrow$ couple to $(1,0)$ or to $(3,0)$ $[3 \otimes 3 = 5 \oplus 3 \oplus 1]$

alternatively: $L L^c \sim (2,1) \otimes (2,1) = (3,2) \oplus (1,2)$

$\Rightarrow$ couple to $(1,-2)$ or $(3,-2)$

[however: $(1,-2)$ does not work; gives $\bar{\nu} \ell^c - \bar{\ell} \nu^c$]

$(1,0)$	type I
$(3,-2)$	type II
$(3,0)$	type III

(i) Type I Seesaw

introduce $N_R \sim (1,0)$ RH neutrino, total singlet

$$\mathcal{L} = m_D \overline{\nu_L} N_R + \frac{1}{2} M_R \overline{N_R^c} N_R \qquad (*)$$

↙ usual Dirac-mass after EWSB: $m_D = y \cdot v$

↘ Majorana mass for RH neutrinos

note: M_R is not given by „Yukawa × vev"

<u>new</u> energy scale! not related to SM scale!

„naked" mass term, „bare" mass term

rewrite (*) as: $\mathcal{L} = \frac{1}{2} (\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$

! Majorana mass term ! → $0\nu\beta\beta$!

⇒ diagonalize! assumption: $M_R \gg m_D$

↙ should be larger than SM-scale not protected by SM-Higgs

$$\mathcal{L} = \frac{1}{2}\left(\bar\nu_L , \overline{N_R^c}\right) U U^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} U^* U^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

with $U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$ $\quad c = \cos\theta$

$s = \sin\theta$

$$U^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} U^* = \mathrm{diag}(m_\nu , M)$$

mixing: $\tan 2\theta = \frac{2 m_D}{M_R - 0} \ll 1$

eigenvalues: $\frac{1}{2}\left[(0 + M_R) \mp \sqrt{(0 - M_R)^2 + 4 m_D^2}\right]$

$$\simeq \begin{cases} -m_D^2/M_R \\ M_R \end{cases}$$

$\Rightarrow$ in diagonal basis: $\mathcal{L} = \frac{1}{2} m_\nu \bar\nu_L \nu_L^c + \frac{1}{2} M_R \overline{N_R^c} N_R$

with $m_\nu = -\frac{m_D^2}{M_R} \Rightarrow \Lambda \simeq M_R$ in Weinberg-operator