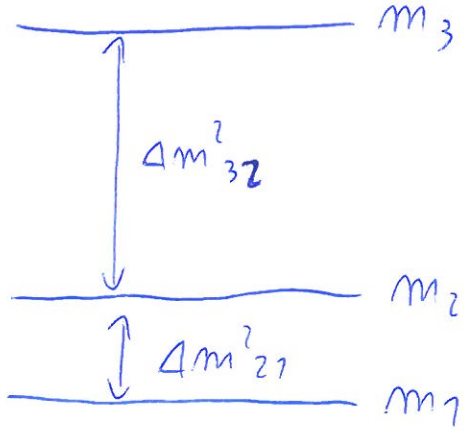
Neutrino masses

we know $\Delta m^2_{21} > 0$ and $|\Delta m^2_{31}|$

- if $\Delta m^2_{31} > 0$: „normal ordering" $m_3 > m_2 > m_1$

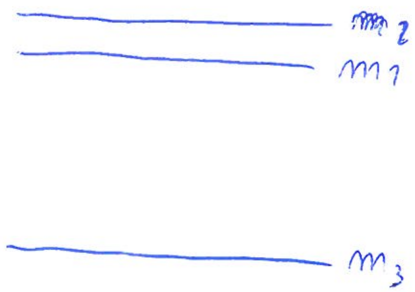


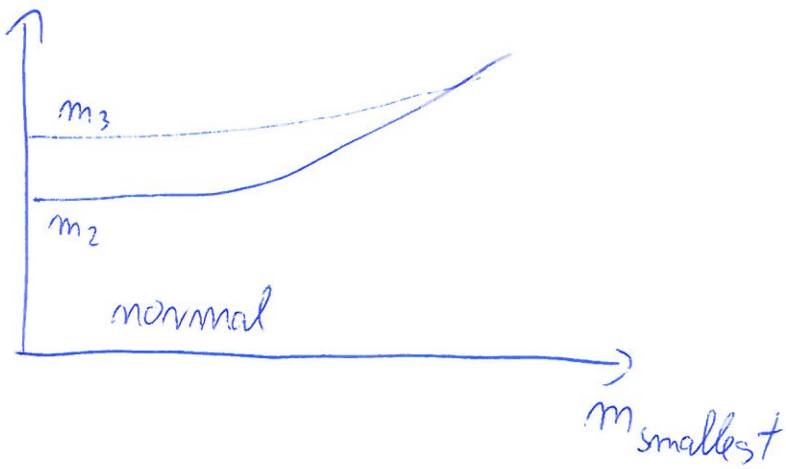
$|U_{e3}|^2 \simeq 0.02$

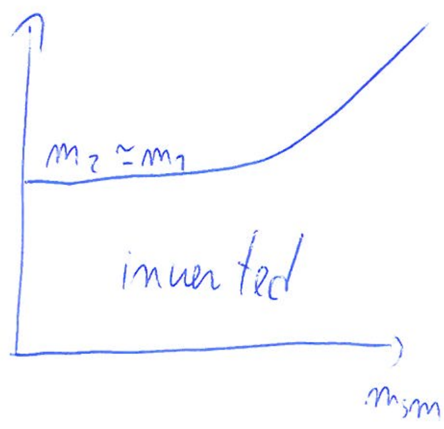
$|U_{\mu 3}|^2 \simeq 0.49$

$|U_{\tau 3}|^2 \simeq 0.49$

- if $\Delta m^2_{31} < 0$: „inverted ordering" $m_2 > m_1 > m_3$







cf. with m_ν: if normal ordering : $m_\nu \simeq \begin{pmatrix} 0 & 0 & 0 \\ & 1 & 1 \\ & 1 & 1 \end{pmatrix}$

inverted ordering : $m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{pmatrix}$

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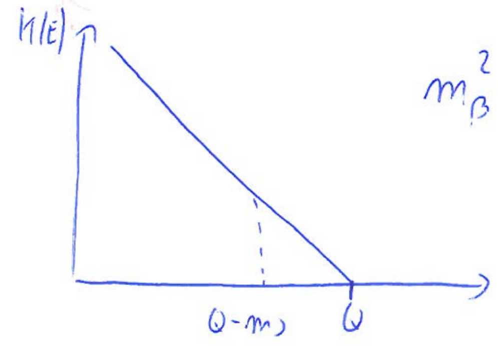
what do we know about mass scale ?

- Kurie-plot :



$$m_\beta^2 = \sum |U_{ei}|^2 m_i^2 \leq (2\,eV)^2$$

- Cosmology : Neutrinos are <u>hot dark matter</u> , but observation of large scale structure of Universe requires <u>cold dark matter</u>

$\Rightarrow$ constrain amount of HDM: $\sum_i m_i \lesssim eV$

- Neutrinoless double beta decay (later) : $|\sum U_{ei}^2 m_i| \lesssim 0.3\,eV$

$$\boxed{\text{? Why is neutrino mass so tiny ?}}$$

III 3) Dirac and Majorana-Neutrinos

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{vs.} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad : \quad m_\nu \lesssim 10^{-6}\, m_e$$

$$m_u \simeq m_d$$

$\longleftrightarrow$ most simple explanation connected to Majorana-neutrino

Why not standard mechanism to generate m_ν?

add $\nu_R \sim (1,0)$: $\Rightarrow \mathcal{L}_D = g_\nu \bar{L}\, \tilde{\Phi}\, \nu_R$

$$\Big\downarrow \text{SSB}$$

$$\frac{v}{\sqrt{2}} g_\nu \bar{\nu}_L \nu_R \equiv m_\nu \bar{\nu}_L \nu_R$$

with $m_\nu \sim eV$: $g_\nu = 10^{-12}$ .... why so tiny?

This would generate a „Dirac mass" for neutrinos

A) Majorana - Masses

need *charge-conjugation*:(*)

electron: $[\gamma_\mu(i\partial_\mu + eA_\mu) - m]\psi = 0$ (1)

positron: $[\gamma_\mu(i\partial_\mu - eA_\mu) - m]\psi^c = 0$ (2)

Try $\psi^c = S\psi^*$ and evaluate $(S^*)^{-1}\,(2)^*$:

$\Rightarrow (S^*)^{-1}\gamma_\mu^* S^* \stackrel{!}{=} -\gamma_\mu \Rightarrow S = i\gamma_2$

(in standard representation)

$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$

$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow$ the charge-conjugated spinor is:

$$\boxed{\psi^c = i\gamma_2\psi^* = i\gamma_2\gamma_0\bar{\psi}^T \equiv C\bar{\psi}^T}$$

flips charge-like quantum numbers

$C^\dagger = C^T = C^{-1} = -C$

$C\gamma_\mu C^{-1} = -\gamma_\mu^T$

$C\gamma_5 C^{-1} = \gamma_5^T$

$C\gamma_\mu\gamma_5 C^{-1} = (\gamma_\mu\gamma_5)^T$

$(\psi^c)^c = \psi$

$\overline{\psi^c} = \psi^T C$

$\bar{\psi}_1\psi_2^c = \overline{\psi_2^c}\,\psi_1$

$$\boxed{\begin{aligned}(\psi_L)^c &= (\psi^c)_R \\ (\psi_R)^c &= (\psi^c)_L\end{aligned}}$$

flips chirality!

* strictly speaking: particle-antiparticle transformation...

charge-conjugation: $\nu_L \to \bar{\nu}_L$

particle-antiparticle: $\nu_L \to \bar{\nu}_R$

different for chiral states, not for total states...

return to mass terms: $\psi = \begin{pmatrix} \phi \\ \xi \end{pmatrix} \Rightarrow \psi_L = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \psi_R = \begin{pmatrix} 0 \\ \xi \end{pmatrix}$

$$\Rightarrow \left.\begin{array}{l} (i\partial_0 - i\vec{\sigma}\vec{\nabla})\phi - m\xi = 0 \\ (i\partial_0 + i\vec{\sigma}\vec{\nabla})\xi - m\phi = 0 \end{array}\right\}$$

for $m = 0$, LH and RH components evolve independently: „Weyl fermions"

mass term requires LH and RH terms:

$$m\bar{\psi}\psi = m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

There are now only 2 possibilities:

(i) ψ_R independent of ψ_L : Dirac particle

(ii) $\psi_R = (\psi_L)^c$: Majorana-particle

$$\Rightarrow \psi^c = (\psi_L + \psi_R)^c = (\psi_L)^c + (\psi_R)^c = \psi_R + \psi_L = \psi$$

$\boxed{\psi^c = \psi}$ identical to its own antiparticle

$\Rightarrow$ only works for neutral fermions

$\Rightarrow$ has 2 degrees of freedom

e.g. $\psi = \begin{pmatrix} \phi \\ -i\sigma_2\phi^* \end{pmatrix}$