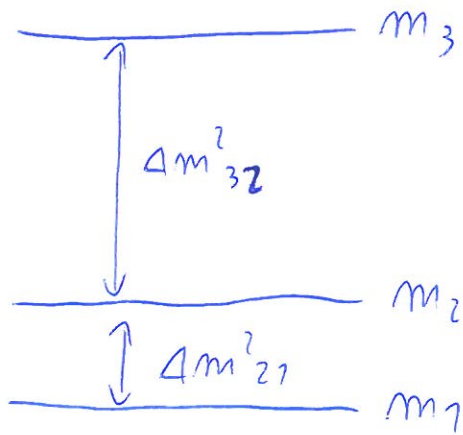


Neutrino masses

we know $\Delta m_{21}^2 > 0$ and $|\Delta m_{31}^2|$

•) if $\Delta m_{31}^2 > 0$: "normal ordering" $m_3 > m_2 > m_1$

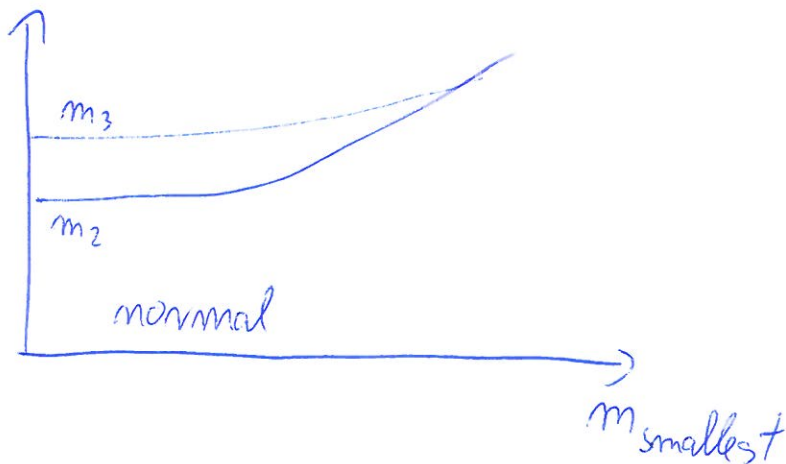


$$|U_{e3}|^2 \approx 0.02$$

$$|U_{\mu 3}|^2 \approx 0.49$$

$$|U_{\tau 3}|^2 \approx 0.49$$

•) if $\Delta m_{31}^2 < 0$: "inverted ordering" $m_2 > m_1 > m_3$

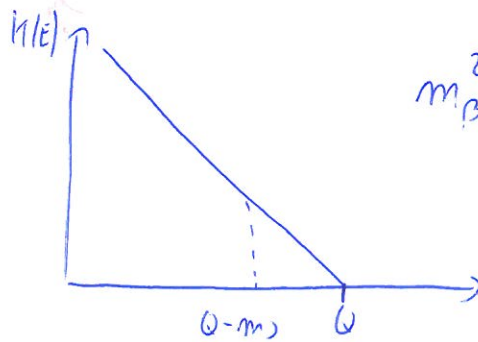


cf. with m_ν : if normal ordering: $m_\nu \approx \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

inverted ordering: $m_\nu \approx \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

What do we know about mass scale?

•) KATRIN-plot:



$$m_\beta^2 = \sum |U_{ei}|^2 m_i^2 \leq (2 \text{ eV})^2$$

•) Cosmology: Neutrinos are hot dark matter, but observation of large scale structure of Universe requires cold dark matter

\Rightarrow constrain amount of HDM: $\sum_i m_i \leq \text{eV}$

•) Neutrinoless double beta decay (later): $|\sum U_{ei}^2 m_i| \leq 0.3 \text{ eV}$

Why is neutrino mass so tiny?

III 3) Dirac and Majorana-Neutrinos

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{vs.} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad : \quad m_u \approx 10^{-6} m_e \\ m_u \approx m_d$$

⇒ most simple explanation connected to Majorana-neutrino

Why not standard mechanism to generate m_ν ?

$$\text{add } \nu_R \sim (1,0) : \Rightarrow \mathcal{L}_D = g_D \bar{L} \tilde{\Phi} \nu_R$$

$$\downarrow \text{SSB} \\ \frac{v}{\sqrt{2}} g_D \bar{\nu}_L \nu_R \equiv m_\nu \bar{\nu}_L \nu_R$$

with $m_\nu \sim \text{eV}$: $g_D = 10^{-12}$... why so tiny?

This would generate a "Dirac mass" for neutrinos

A) Majorana - Masses

need charge-conjugation: (*)

$$\text{electron: } [\gamma_\mu (i\partial_\mu + eA_\mu) - m] \psi = 0 \quad (1)$$

$$\text{positron: } [\gamma_\mu (i\partial_\mu - eA_\mu) - m] \psi^c = 0 \quad (2)$$

Try $\psi^c = S \psi^*$ and evaluate $(S^*)^{-1} (2)^*$:

$$\Rightarrow (S^*)^{-1} \gamma_\mu^* S^* \stackrel{!}{=} -\gamma_\mu \Rightarrow S = i \gamma_2$$

(in standard representation)

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

\Rightarrow the charge-conjugated spinor is:

$$\psi^c = i \gamma_2 \psi^* = i \gamma_2 \gamma_0 \bar{\psi}^T \equiv C \bar{\psi}^T$$

flips charge-like quantum numbers

$$C^\dagger = C^T = C^{-1} = -C$$

$$C \gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$C \gamma_5 C^{-1} = \gamma_5^T$$

$$C \gamma_\mu \gamma_5 C^{-1} = (\gamma_\mu \gamma_5)^T$$

$$(\psi^c)^c = \psi$$

$$\bar{\psi}^c = \psi^T C$$

$$\bar{\psi}_1 \psi_2^c = \bar{\psi}_2^c \psi_1$$

$$(\psi_L)^c = (\psi^c)_R$$

$$(\psi_R)^c = (\psi^c)_L$$

flips chirality!

* strictly speaking: particle-antiparticle transformation...

charge-conjugation:

$$\psi_L \rightarrow \bar{\psi}_L$$

particle-antiparticle:

$$\psi_L \rightarrow \bar{\psi}_R$$

different for chiral states, not for total states...

return to mass terms: $\Psi = \begin{pmatrix} \phi \\ \xi \end{pmatrix} \Rightarrow \Psi_L = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \Psi_R = \begin{pmatrix} 0 \\ \xi \end{pmatrix}$

$$\Rightarrow \left. \begin{aligned} (i\gamma_0 - i\vec{\sigma} \cdot \vec{\nabla}) \phi - m \xi &= 0 \\ (i\gamma_0 + i\vec{\sigma} \cdot \vec{\nabla}) \xi - m \phi &= 0 \end{aligned} \right\} \begin{array}{l} \text{for } m=0, \text{ LH and RH} \\ \text{components evolve inde-} \\ \text{pendently: "Weyl fermions"} \end{array}$$

mass term requires LH and RH terms:

$$m \bar{\Psi} \Psi = m (\bar{\Psi}_L + \bar{\Psi}_R) (\Psi_L + \Psi_R) = m \bar{\Psi}_L \Psi_R + m \bar{\Psi}_R \Psi_L$$

There are now only 2 possibilities:

(i) Ψ_R independent of Ψ_L : Dirac particle

(ii) $\Psi_R = (\Psi_L)^c$: Majorana-particle

$$\Rightarrow \Psi^c = (\Psi_L + \Psi_R)^c = (\Psi_L)^c + (\Psi_R)^c = \Psi_R + \Psi_L = \Psi$$

$$\boxed{\Psi^c = \Psi}$$

identical to its own
antiparticle

\Rightarrow only works for neutral fermions

\Rightarrow has 2 degrees of freedom

e.g. $\Psi = \begin{pmatrix} \phi \\ -i\sigma_2 \phi^* \end{pmatrix}$