Recap:

7/7/15

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\Theta \quad \sin^2 \frac{\Delta m^2 L}{4E} \qquad \text{for } \alpha \neq \beta$$

requires: $\Theta \neq 0 \Rightarrow$ flavor $\neq$ mass

$\Delta m^2 \neq 0 \Rightarrow$ masses non-zero and different

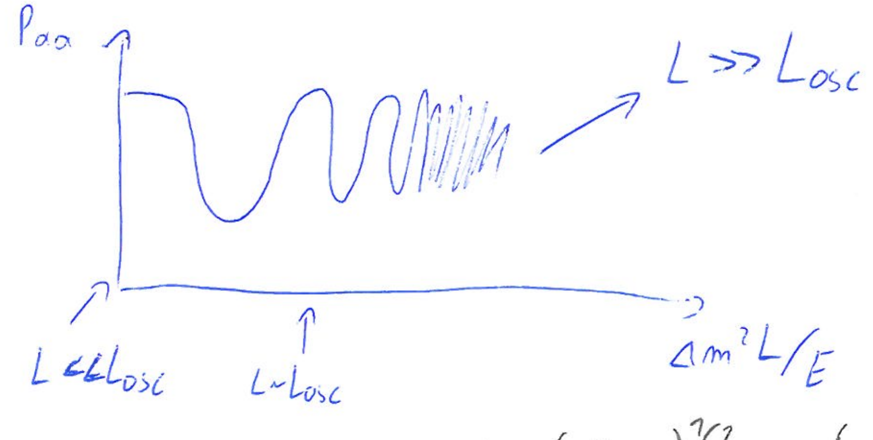
$\Downarrow$

physics beyond the SM!

Formula obtainable from proper quantum-mechanical formalism (coherence of wave packets, localization)

if coherence is lost, or if $L \gg L_{osc} = \dfrac{4\pi E}{\Delta m^2}$:

$$P_{\alpha\beta} = \sum |U_{\alpha i}|^2 \, |U_{\beta i}|^2$$



<u>fast oscillations</u>

$$\left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle = \left\langle \sin^2 \frac{\pi L}{L_{osc}} \right\rangle = \frac{1}{2}$$

Detail: distribution $\Phi(L/E) = \left(\dfrac{1}{2\pi\sigma^2}\right)^{1/2} \exp\left\{ - \dfrac{\left(\frac{L}{E} - \left\langle \frac{L}{E} \right\rangle\right)^2}{2\sigma^2} \right\}$

and e.g. $\left\langle \cos \dfrac{\Delta m^2 L}{2E} \right\rangle = \displaystyle\int d\left(\frac{L}{E}\right) \cos \frac{\Delta m^2 L}{2E} \, \Phi(L/E)$ (e.g. LBL: $\sigma \sim 0.1 \frac{\text{km}}{\text{GeV}}$

$$= \cos \frac{\Delta m^2 L}{2E} \exp\left\{ -\frac{1}{2} \left(\frac{\Delta m^2}{2} \sigma \right)^2 \right\}$$ and $\frac{1}{2}(1 - \cos\alpha) = \sin^2 \frac{\alpha}{2}$

B) Oscillations in Matter

re-obtain ν-mixing and Δm^2 by writing

$$(\not p - m)\psi = 0 \Rightarrow (p^2 - m^2)\psi = 0$$

with $\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}$ $\quad m^2 = \mathrm{diag}(m_1^2, m_2^2, \dots, m_n^2)$

use $p^2 = E^2 + \partial_x^2 = (E + i\partial_x)(E - i\partial_x) \approx (E + i\partial_x)(E + p)$

$$\simeq 2E(E + i\partial_x)$$

$\Rightarrow$ Hamiltonian : $i\partial_x \psi = \left(-E + \frac{m^2}{2E}\right)\psi$

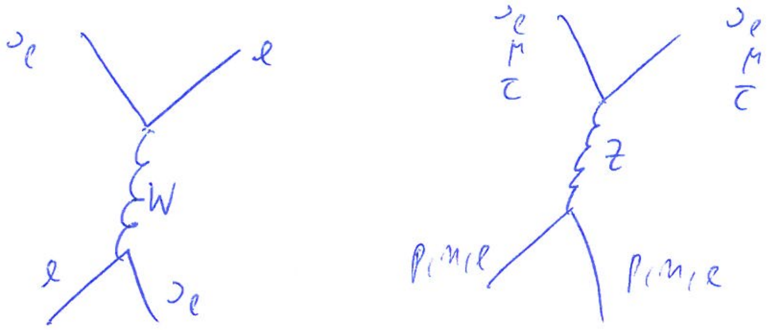
or $\boxed{i\partial_x \psi = \mathcal{H}\psi = \frac{m^2}{2E}\psi}$ (global phase plays no role)

go to flavor basis: $\psi_{fl} = \mathcal{U}\psi$; $\mathcal{H}_{fl} = \mathcal{U}\mathcal{H}\mathcal{U}^\dagger$

$$= \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

diagonalizing $\mathcal{H}_{fl}$ gives mixing angle and eigenvalues $\pm \frac{\Delta m^2}{4E}$

coherent scattering of neutrinos with matter creates potential $V \sim G_F m_e = O\left(\Delta m^2 / E\right)$



$V_{NC}(e) = V_{NC}(p)$

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}} \left[\bar{e}\, \gamma_\mu (1-\gamma_5)\, e\right] \left[\bar{\nu}_e\, \gamma^\mu (1-\gamma_5)\, \nu_e\right] \qquad \text{(after Fierz)}$$

integrate over e such that $\bar{\nu}_{e_L} V \nu_{e_L}$ survives

$\langle \ldots \rangle = \int \sum_{s_e} \frac{d^3p}{(2\pi)^3} f(p) \langle e(p,s_e) | \mathcal{H} | e(p,s_e) \rangle$

$\langle \bar{e}\, \gamma_\mu \gamma_5\, e \rangle = 0$ unpolarized matter

$\langle \bar{e}\, \gamma_i\, e \rangle = 0$ zero velocity matter

$\langle \bar{e}\, \gamma_0\, e \rangle = n_e$

$\Rightarrow V_{ee} = \sqrt{2}\, G_F n_e$ (factor 2 because ν_{e_L})

$$\Rightarrow \mathcal{H} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\Theta + 2\frac{A}{\Delta m^2} & \sin 2\Theta \\ \sin 2\Theta & \cos 2\Theta \end{pmatrix}$$

$A = 2\sqrt{2}\, G_F n_e E \simeq 10^{-5}\, eV^2 \frac{E}{MeV}$ for Sun (core)

diagonalize to find $\Theta_m, \Delta m^2_m$ in matter!

$$\Rightarrow \boxed{\sin^2 2\Theta_m = \frac{\sin^2 2\Theta}{\left(\cos 2\Theta - A/\Delta m^2\right)^2 + \sin^2 2\Theta}}$$

- resonance possible!
- depends on octant of Θ and sign (Δm^2)
- adiabatic$^{(*)}$ propagation through Sun: $n_e(x)$

$\rightarrow \frac{A}{\Delta m^2} \gg 1$: $\nu_e \simeq \nu_2^m$; $\nu_\mu \simeq -\nu_1^m$

$\nu_{1,2}^m(x) \simeq \nu_{1,2}^m$

no level crossing

$\updownarrow$

adiabicity

$\rightarrow$ resonance : $\nu_2^m = \sqrt{\frac{1}{2}}(\nu_e + \nu_\mu)$

$\rightarrow$ exits Sun : $\Theta_m = \Theta \Rightarrow \nu_1^m = \nu_e \cos\Theta \mp \nu_\mu \sin\Theta$

$\nu_2^m = \nu_\mu \cos\Theta + \nu_e \sin\Theta$

$\Rightarrow P_{e\mu} = \cos^2\Theta$

$\Rightarrow$ full conversion for small Θ

<u>M</u>ikheev

<u>S</u>mirnov MSW-effect

<u>W</u>olfenstein

$*$: $\gamma = \frac{\Delta m^2}{2E} \frac{\sin 2\Theta}{\cos 2\Theta} \left(\frac{1}{n_e}\frac{dn_e}{dr}\right)^{-1} \gg 1$

at resonance, n_e constant over several L_{osc}

c) Current Situation in Neutrino Physics

recall: $U = R_{23} \, R_{13}^{\delta} \, R_{12}$ and $\sin^2 \frac{\Delta m^2 L}{4E}$ behavior,

with $L \ll L_{osc}$ and $L \gg L_{osc}$ cases

1) Solar Neutrinos

$$4p + 2e^- \to {}^4He + 2\nu_e + 27 \text{ MeV}$$

$$\Phi_\nu \simeq 7 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1} \quad (!)$$

5 sources of neutrinos; $1/3 \dots 1/2$ of expected rate observed $\Rightarrow$ solar neutrino problem

$P(\nu_e \to \nu_e)$ depends on energy of generated neutrino!

~~[illegible]~~ depends on matter effects!

turns out $\Delta m^2_{31} \gg \Delta m^2_{21}$

Δm^2_{31}: $\simeq \Delta m^2_{32}$, averages out for solar ν

Δm^2_{21}: relevant for ν_{sol}

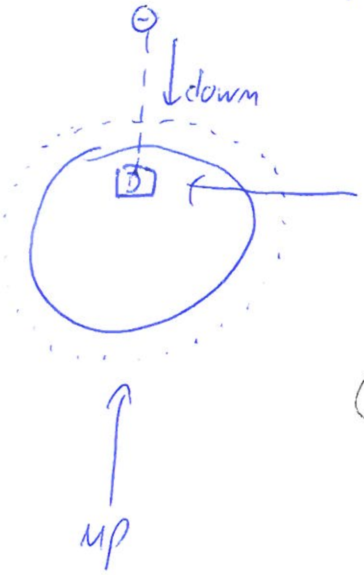
Result of fits: $\boxed{\begin{aligned} \sin^2\Theta_{12} &\simeq 0.31 \\ \Delta m^2_{21} &= 8\cdot 10^{-5}\ eV^2 \end{aligned}}$

can be tested by using <u>reactor neutrinos</u>.

with $L \simeq 10^2$ km, $E \simeq$ MeV $\Rightarrow \frac{L}{E}\Delta m^2_{21} \sim 1$
(KamLand)

2) Atmospheric Neutrinos

cosmic ray + atmosphere $\to \pi^\pm \to \overset{(-)}{\nu}_\mu, \overset{(-)}{\nu}_e$



$\Theta = 0$: down-going $\quad L = 10$ km

$\Theta = \frac{\pi}{2}$, $\quad L = 500$ km

$\Theta = \pi$: up-going $\quad L = 10^4$ km

$\frac{\Delta m^2_{21} L}{E}$ negligible: $P_{\mu\tau} = \sin^2 2\Theta_{23}\ \sin^2 \frac{\Delta m^2_{31}}{4E} L$

Result of fits: $\boxed{\begin{aligned} \sin^2\Theta_{23} &\simeq 1/2 \\ \Delta m^2_{31} &\simeq 2.5\cdot 10^{-3}\ eV^2 \end{aligned}}$ maximal!?

14/1/15

Recap.:

general 3-flavor oscillation probability:

$$\Delta m^2_{21} \simeq 8 \times 10^{-5}\ eV^2$$

$$|\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2 \times 10^{-3}\ eV^2 \simeq 30 \times \Delta m^2_{21}$$

$\Rightarrow$ depending on $\frac{L}{E}$ of experiment, general

$$P(\nu_\alpha \to \nu_\beta)^{(3\ \text{flavor})} \longrightarrow P(\nu_\alpha \to \nu_\beta)^{(2\ \text{flavor})}$$

solar ν + LBL reactor: $\theta_{12} \simeq 33°$, Δm^2_{21} $[P(\nu_e \to \nu_e)]$

atm. ν + LBL accelerator: $\theta_{23} \simeq 45°$, $|\Delta m^2_{31}|$ $[P(\nu_\mu \to \nu_\tau)]$

can be tested with man-made <u>accelerator neutrinos</u>

with $\frac{L}{E} \simeq 100$ km/GeV $\Rightarrow \frac{L}{E} \Delta m^2_{31} \simeq 1$ $[P(\nu_\mu \to \nu_e)]$

3) Reactor Neutrinos

$L \sim$ km , $E \sim$ MeV $\Rightarrow \frac{\Delta m^2_{31} L}{E} \sim 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\Theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} L$$

Result: $\boxed{\sin^2 \Theta_{13} \simeq 0.02}$; $\Theta_{13} \simeq 9°$

e.g. Double Chooz Experiment

$^{235}_{92}U + n \to X_1 + X_2 + 2n$; mostly $^{94}_{40}Zr$ and $^{140}_{58}Ce$

$\Rightarrow \begin{matrix} Z=92 \\ N=143+1 \end{matrix} \to \begin{matrix} Z=98 \\ N=136+2 \end{matrix} \Rightarrow 6n \to 6p + 6e^- + 6\bar{\nu}_e$

with 200 MeV energy gain per fission

200 MeV $\simeq 2 \cdot 10^6$ eV $= 2 \cdot 10^6 \times 1.6 \times 10^{-19}$ Ws $\simeq 3.2 \times 10^{-21}$ GWs

$\Rightarrow$ per GW reactor power: # 5×10^{20} s^{-1}

---

can be tested with man-made <u>accelerator neutrinos</u>

$P(\nu_\mu \to \nu_e) \propto \sin^2 \Theta_{13}$ in 3-flavor expression

$\hookrightarrow$ also important for future CP studies + mass ordering

$\Rightarrow$ total situation:

$$\mathcal{U} = R_{23}(\theta_{23})\, R_{13}(\theta_{13}, \delta)\, R_{12}(\theta_{12})$$

(very) approximately: $\sin^2\theta_{12} = 1/3$

$\sin^2\theta_{23} = 1/2$

$\sin^2\theta_{13} = 0$

$$\Rightarrow \mathcal{U} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

"tri-bimaximal mixing"

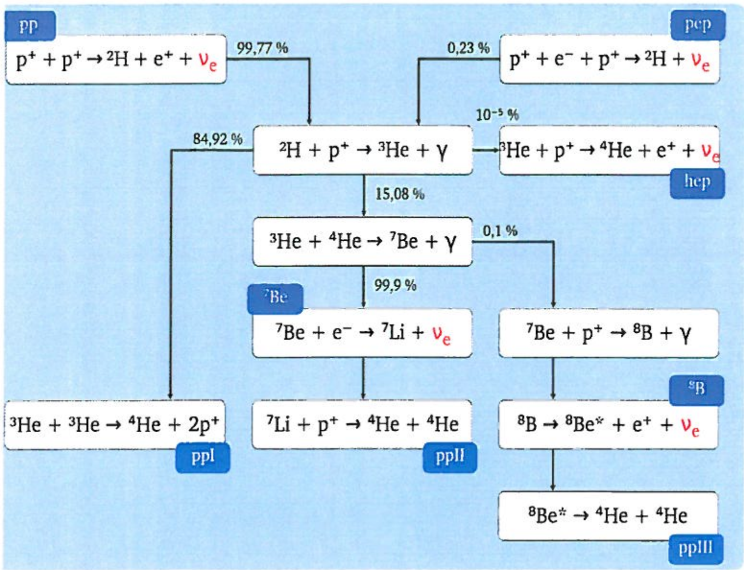
$$m_\nu = \mathcal{U}^* m_\nu^{diag}\, \mathcal{U}^\dagger = \begin{pmatrix} A & B & B \\ & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ & & \frac{1}{2}(A+B+D) \end{pmatrix}$$

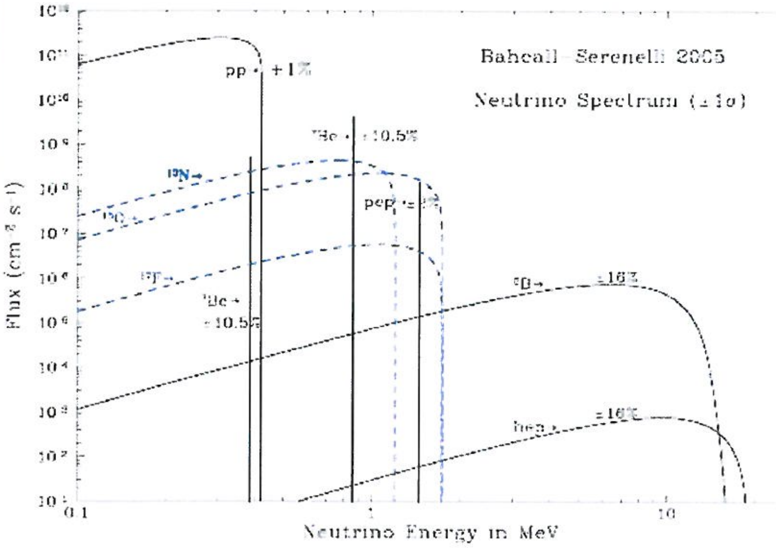
[Majorana-neutrinos have $m_\nu = m_\nu^T$]

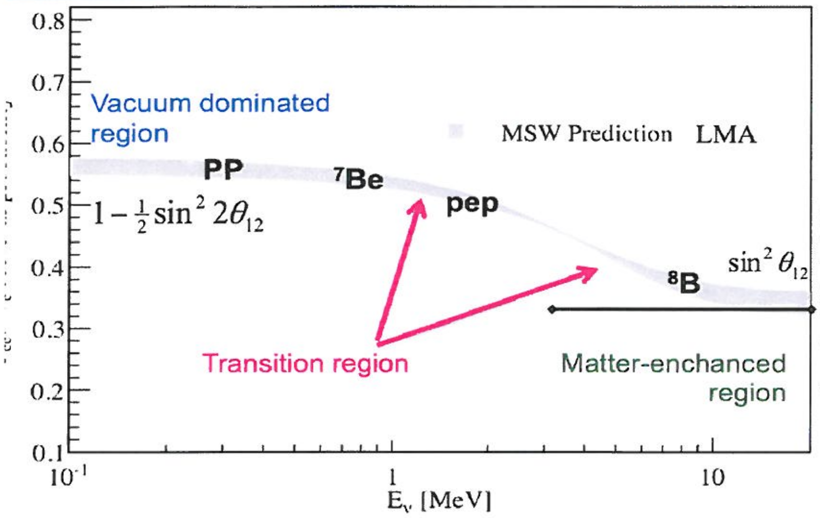
$A = \frac{1}{3}(2m_1 + m_2)$; $B = \frac{1}{3}(m_2 - m_1)$; $D = m_3$

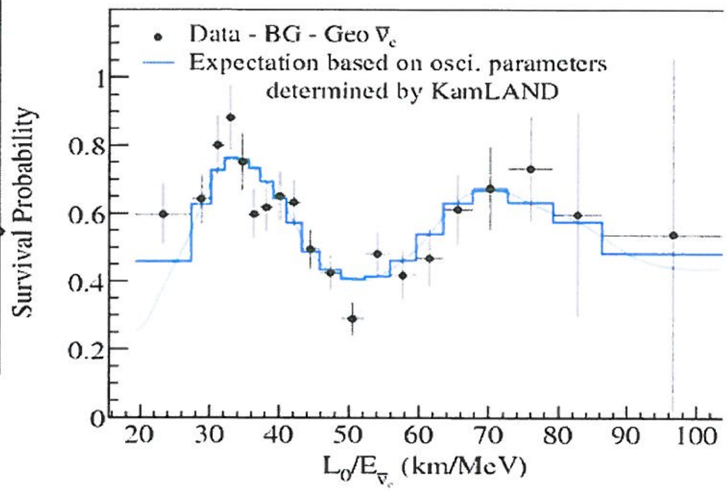
Mixing independent of masses! $\left(\tan\theta_c = \sqrt{m_d/m_s}\right)$

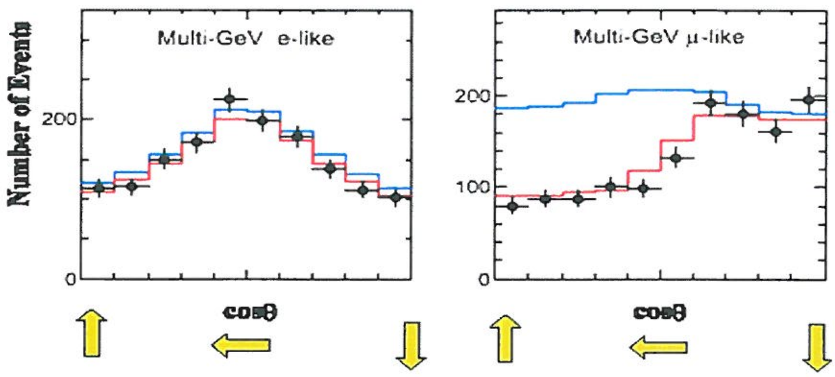
$$\boxed{\text{¿ Why is } V_{CKM} \neq \mathcal{U}_{PMNS} \text{ ?}}$$

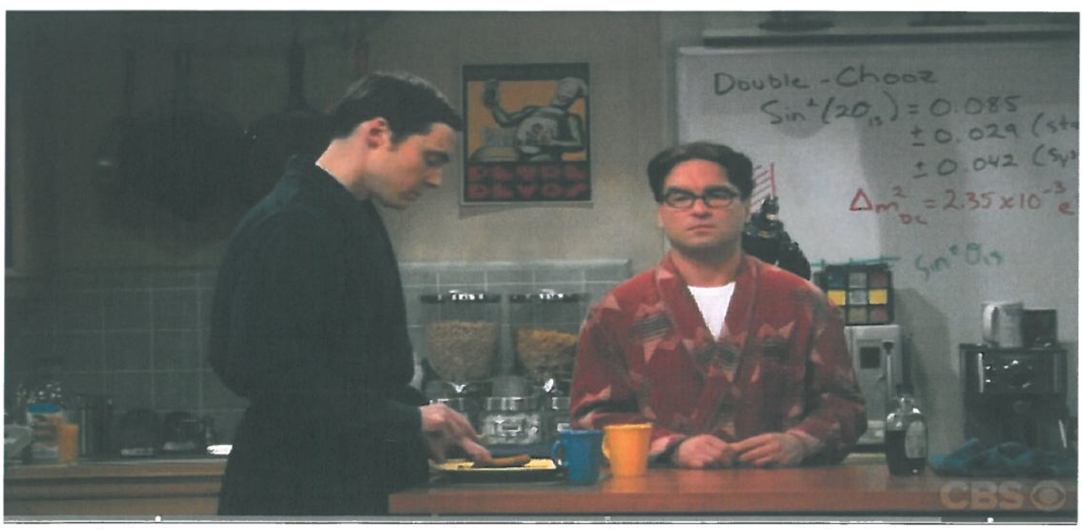

pp
$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$
99,77 %
pep
0,23 %
$p^+ + e^- + p^+ \rightarrow {}^2H + \nu_e$
84,92 %
$^2H + p^+ \rightarrow {}^3He + \gamma$
10^{-5} %
$^3He + p^+ \rightarrow {}^4He + e^+ + \nu_e$
hep
15,08 %
$^3He + {}^4He \rightarrow {}^7Be + \gamma$
0,1 %
99,9 %
^7Be
$^7Be + e^- \rightarrow {}^7Li + \nu_e$
$^7Be + p^+ \rightarrow {}^8B + \gamma$
^8B
$^3He + {}^3He \rightarrow {}^4He + 2p^+$
ppI
$^7Li + p^+ \rightarrow {}^4He + {}^4He$
ppII
$^8B \rightarrow {}^8Be^* + e^+ + \nu_e$
$^8Be^* \rightarrow {}^4He + {}^4He$
ppIII

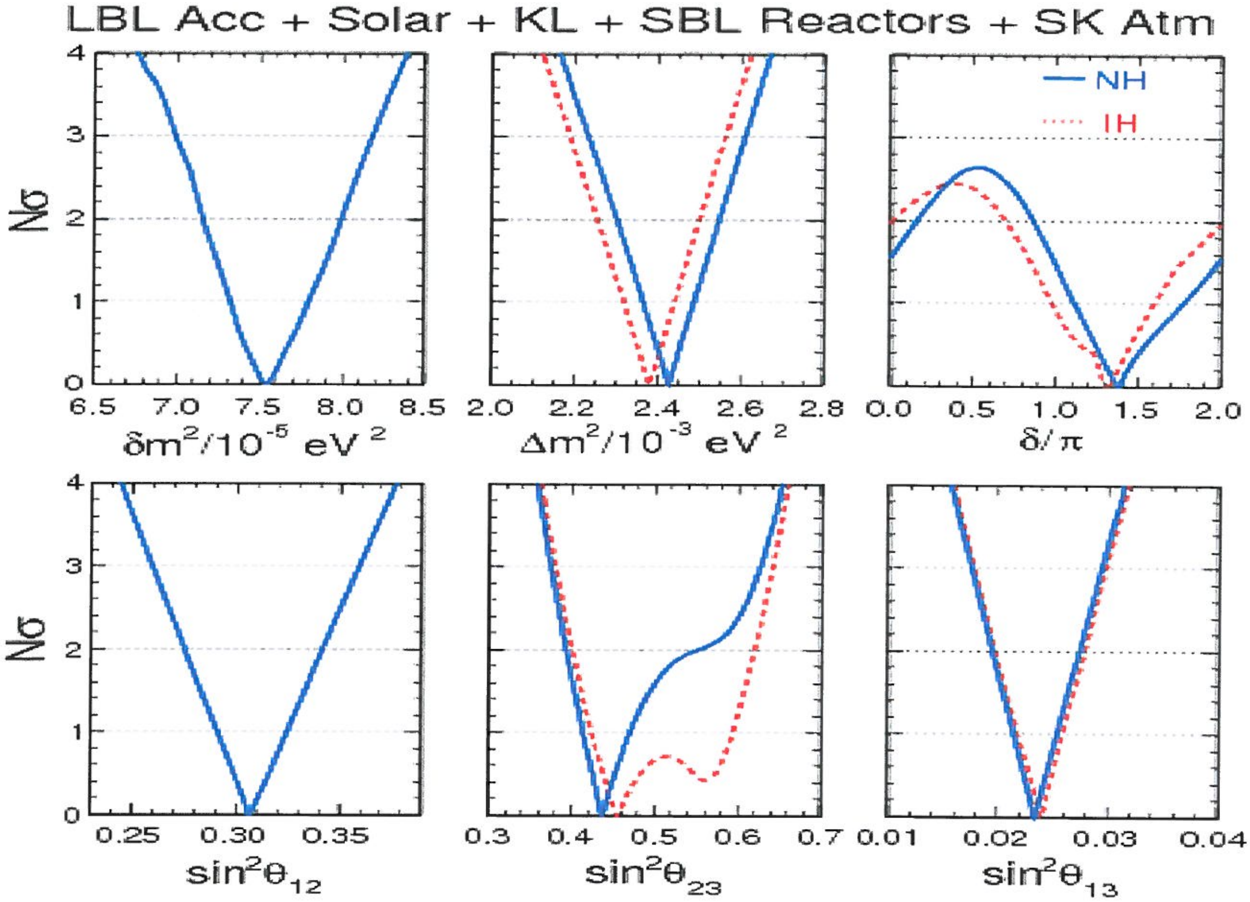

Bahcall–Serenelli 2005
Neutrino Spectrum (±1σ)
pp ±1%
pep
^8B ±16%
hep ±16%
Flux (cm⁻² s⁻¹)
Neutrino Energy in MeV

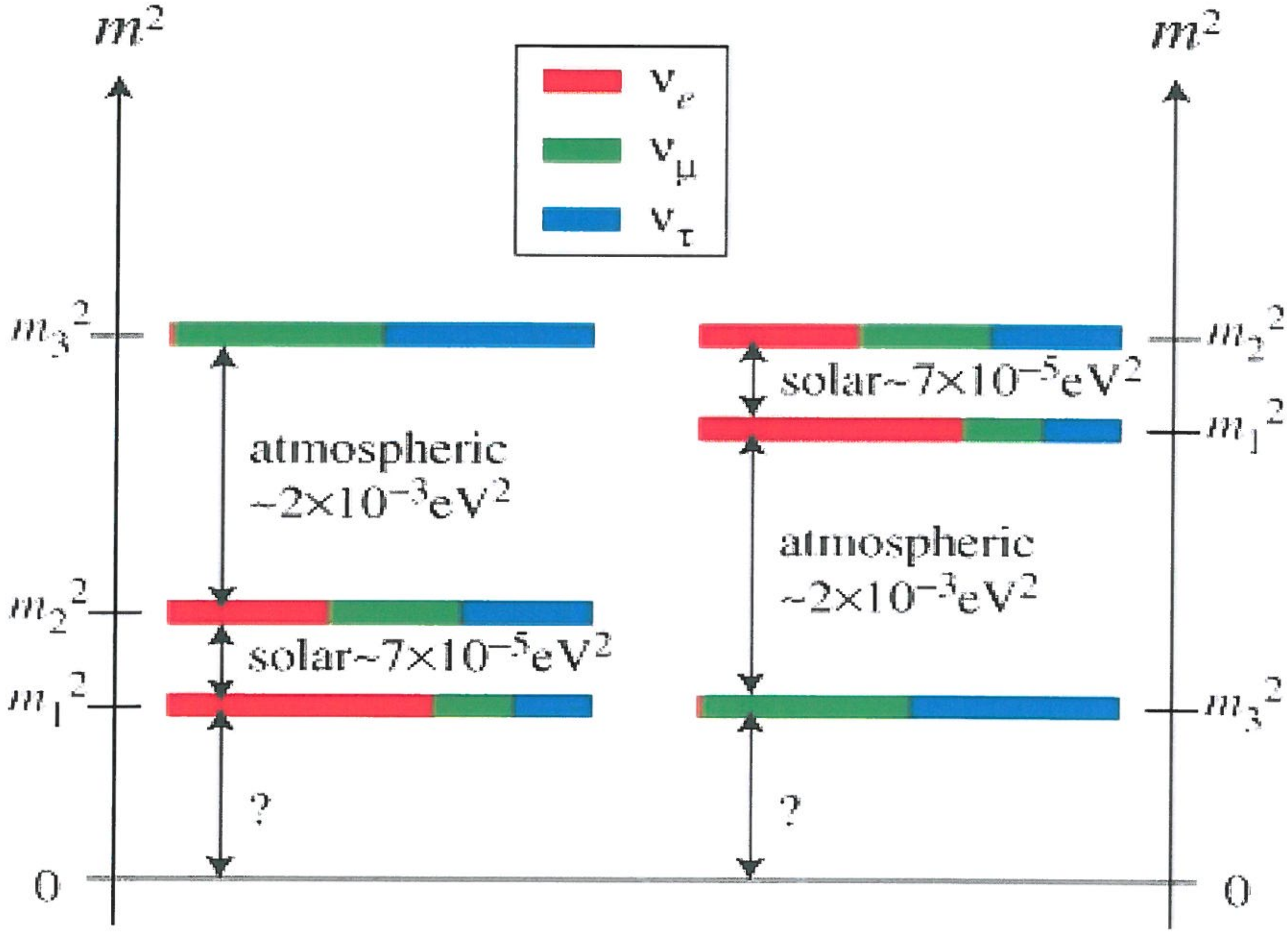

Vacuum dominated region
MSW Prediction LMA
PP
^{7}Be
pep
$1-\frac{1}{2}\sin^2 2\theta_{12}$
$\sin^2\theta_{12}$
^{8}B
Transition region
Matter-enchanced region
E_ν [MeV]


Data - BG - Geo $\bar{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
$L_0/E_{\bar{\nu}_e}$ (km/MeV)


Number of Events
Multi-GeV e-like
Multi-GeV μ-like
cosθ
cosθ


Double-Chooz
$\sin^2(2\theta_{13}) = 0.085$
± 0.029
± 0.042
$\Delta m^2_{DC} = 2.35\times10^{-3}$
CBS


LBL Acc + Solar + KL + SBL Reactors + SK Atm
Nσ
δm²/10⁻⁵ eV²
Δm²/10⁻³ eV²
δ/π
NH
IH
sin²θ₁₂
sin²θ₂₃
sin²θ₁₃


m²
νe
νμ
ντ
m₃²
m₂²
m₁²
atmospheric ~2×10⁻³eV²
solar~7×10⁻⁵eV²
?
0