4) Neutrino mass?

$$-\mathcal{L} = \overline{e'_L}\, M^{(\ell)}\, e'_R \quad \text{with } e'_{L,R} = \begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix}_{L,R} \quad \nu'_L = \begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix}_L$$

$$= \underbrace{\overline{e'_L}\, U_\ell}_{\equiv \overline{e_L}}\, \underbrace{U_\ell^\dagger M^{(\ell)} V_\ell}_{D^\ell}\, \underbrace{V_\ell^\dagger\, e'_R}_{\equiv e_R}$$

$$\Rightarrow \mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}}\, U_\ell^\dagger \gamma^\mu U_\nu \underbrace{U^\dagger \nu'_L}_{\nu_L}$$

if no mass term for neutrinos: U_ν arbitrary

$\Rightarrow$ choose $U_\nu = U_\ell$ $\Rightarrow$ no mixing in lepton sector $\Rightarrow$ no analogue of CKM

If there is mass for neutrinos, the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix exists

III 2) Neutrino Oscillations

→ only physics beyond the Standard Model that can be directly tested in lab

→ connected to new representations, and (very possibly) with new energy scales and new concepts

→ ν interact very weakly ↔ challenging expts, but option to do source physics as well (solar interior, earth interior, cosmic rays...)

A) Oscillations in Vacuum

suppose neutrinos have mass:

$$\boxed{\nu_\alpha = U^*_{\alpha i}\, \nu_i}$$

PMNS-matrix

flavor   mass

as usual, $\frac{1}{2}N(N-1)$ angles and $\frac{1}{2}(N-2)(N-1)$ phases

(note: this assumes mass term $\overline{\nu_L}\,\nu_R$; also possible $\overline{\nu_L}\,\nu_L^c$ & $\overline{\nu_L}\,\nu_L^*$ $\Rightarrow$ rephasing does not work, can only rephase N charged lepton fields $\Rightarrow$ $N-1$ additional phases in $\mathcal{U}$, but oscillations are not affected..)

#) How to obtain the oscillation propability

$$|\nu(0)\rangle = |\nu_\alpha\rangle = \mathcal{U}^*_{\alpha j}\,|\nu_j\rangle$$

flavor state produced in CC reaction (e.g.) at time 0

$$|\nu(t)\rangle = \mathcal{U}^*_{\alpha j}\,e^{-iE_j t}\,|\nu_j\rangle$$

is time evolution

amplitude to find state $|\nu_\beta\rangle$:

$$\mathcal{A}(\nu_\alpha \to \nu_\beta, t) = \langle \nu_\beta | \nu(t)\rangle = \mathcal{U}_{\beta i}\,\mathcal{U}^*_{\alpha j}\,e^{-iE_j t}\langle\nu_i|\nu_j\rangle$$

$$= \mathcal{U}^*_{\alpha i}\,\mathcal{U}_{\beta i}\,e^{-iE_i t}$$

probability:

$$P(\nu_\alpha \to \nu_\beta, t) = P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha \to \nu_\beta, t)|^2$$

$$= \sum_{i,j} \underbrace{U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j}}_{J_{ij}^{\alpha\beta}} \underbrace{e^{-i(E_i - E_j)t}}_{e^{-i\Delta_{ij}}}$$

$$= \dots = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{J_{ij}^{\alpha\beta}\} \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{J_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

phase: $\frac{1}{2}\Delta_{ij} = \frac{1}{2}(E_i - E_j)t = \frac{1}{2}\left(\sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2}\right) L$

$$\simeq \frac{1}{2}\left[p_i\left(1 + \frac{m_i^2}{2p_i^2}\right) - p_j\left(1 + \frac{m_j^2}{2p_j^2}\right)\right] L$$

$$\simeq \frac{m_i^2 - m_j^2}{4E} L$$

$$= 1.27 \left(\frac{\Delta m_{ij}^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

$\alpha = \beta$: survival probability

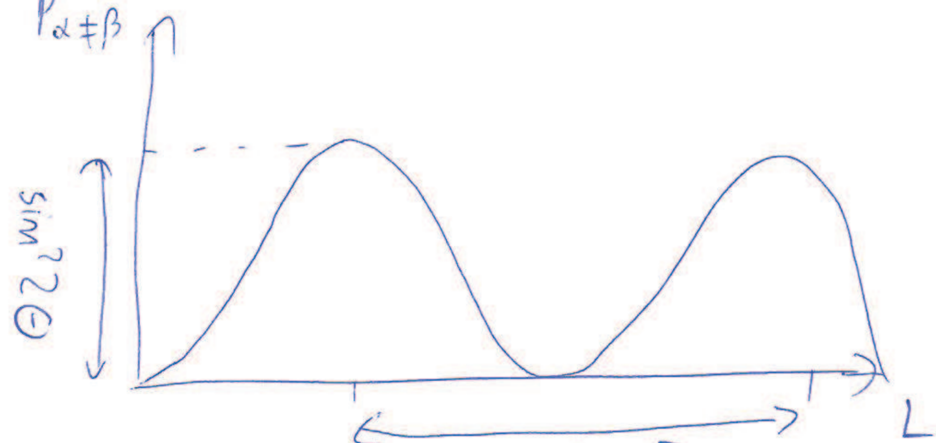
$\alpha \neq \beta$: transition "

requires $U \neq \mathbb{1}$ and $\Delta m_{ij}^2 \neq 0$

2 flavor case:

$$\mathcal{U} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \Rightarrow \begin{array}{l} \nu_e = c\nu_1 + s\nu_2 \\ \nu_\mu = c\nu_2 - s\nu_1 \end{array}$$

$$P_{\alpha \neq \beta} = \sin^2 2\Theta \; \sin^2 \frac{\Delta m^2_{21}}{4E} L$$

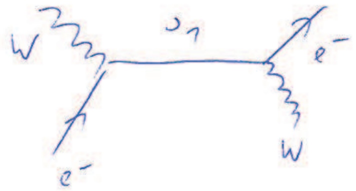


$$L_{osc} = \frac{4\pi E}{\Delta m^2}$$

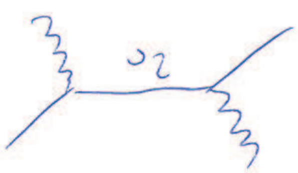
oscillation length

e.g. $E = \text{GeV}$, $\Delta m^2 = 10^{-3}\,\text{eV}^2 \Rightarrow L_{osc} \sim 1000\,\text{km}$

quantum mechanical interference on macroscopic scales!



$: \mathcal{A}_1$



$: \mathcal{A}_2$

$P = |\mathcal{A}_1 + \mathcal{A}_2|^2$ coherence!

This derivation was wrong:

- $E_i - E_j$ not Lorentz-invariant
- different p_i, same E: violates energy-momentum conservation
- definite p: $\psi \sim e^{ipx}$: no localization

$\Rightarrow$ do in quantum mechanics with wave packets: same result.... ☺

o) same E, different p?

$E_j \overset{!}{=} \sqrt{E_j^2 - m_j^2} = p_j$, do Taylor-expansion:

$$\Rightarrow p_j = E + m_j^2 \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0} = E - \xi \frac{m_j^2}{2E} \quad \text{with } \xi = -2E \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0}$$

$$E_j = p_j + m_j^2 \left.\frac{\partial E_j}{\partial m_j^2}\right|_{m_j=0} = p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E}(1-\xi)$$

consider π-decay: $E_j = \frac{m_\pi^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^2}{2m_\pi^2}$

$$\Rightarrow \xi = \frac{1}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8$$

and $E_i - E_j \simeq (1-\xi)\frac{\Delta m^2}{2E}$

phase difference would depend on neutrino source ...

On the correct way to get $P_{\alpha\beta}$:

$$\Psi_i \propto \exp\left\{ -i\,(E_i t - p_i x) - \frac{(x - v_i t)^2}{4\sigma_x^2} \right\}$$

with $\sigma_x \gtrsim 1/\sigma_p$ and group velocity $v_i = \frac{\partial E_i}{\partial p_i} = p_i/E_i$

2 conditions to see oscillations:

1) wave packet separation should be smaller than σ_x !

$$\Rightarrow L\,\Delta v \overset{!}{<} \sigma_x \quad \Rightarrow \quad \frac{L}{L_{osc}} < \frac{p}{\sigma_p}$$

(loss of coherence: interference impossible)

2) m_ν^2 should not be known too precisely !

if known to well: $\Delta m^2 \gg \delta m_\nu^2 = \frac{\partial m_\nu^2}{\partial p_\nu}\,\delta p_\nu$

$$\Rightarrow \delta x_\nu \gg \frac{2p}{\Delta m^2} = \frac{L_{osc}}{2\pi}$$

($\Rightarrow$ know which state ν_i is exchanged)

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proper calculation gives:

$$P \propto \exp\left\{ -i\,\frac{\Delta m_{ij}^2}{2E} L - \left(\frac{L}{L_{ij}^{coh}}\right)^2 - (2\pi^2)(1-\xi)^2 \left(\frac{\sigma_x}{L_{ij}^{osc}}\right)^2 \right\}$$

with $L_{ij}^{coh} = \dfrac{4\sqrt{2}E}{|\Delta m^2_{ij}|}\,\sigma_x$; $L_{ij}^{osc} = \dfrac{4\pi E}{|\Delta m^2_{ij}|}$

$\cancel{\xi = 2E\,p}$ $\quad p_j \equiv E - \xi\, m_j^2/2E$

the last 2 terms suppress oscillation ;

neutrinos still mix ;

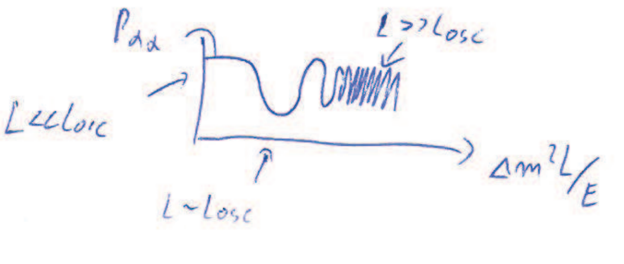
$$P_{\alpha\beta} = \sum |U_{\alpha i}|^2\, |U_{\beta i}|^2$$

Example of such neutrinos loosing coherence:

IceCube: neutrinos as cosmic rays

[the same happens when $L \gg L_{osc}$ „ fast oscillations"

$$\left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle = \left\langle \sin^2 \frac{\pi L}{L_{osc}} \right\rangle = 1/2$$



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