II 3) Problems of the Standard Model

~~conservation up to Planck scale~~

•) # of parameters:

	#	Σ
Quarks	10	10
Leptons	3	13
Gauge	3	16
Higgs	2	18
(strong CP	1	19)

(ν massless...)

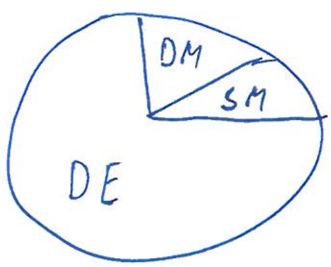
(add ν-mass: 9 or 7 more parameters)

What predicts these parameters?

•) B and L accidentaly conserved. Why?

•) Gravitation?

•) Dark Matter?



SM = 5 %

DM = 27 %

DE = 68 %

•) Dark Energy?

•) Baryon-Asymmetry?

•) NEUTRINO MASS!

III Neutrino Mass and Lepton Mixing

→ Flavour: formalism

→ Neutrino Oscillations (Vacuum, matter, current status)

→ Neutrino mass (Dirac, Majorana, seesaw, double beta decay)

→ flavour symmetries (?)

III 1) Flavour

⟹ SM fermion doublets come in 3 copies:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad + u_R, d_R, c_R, s_R, t_R, b_R$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad + e_R, \mu_R, \tau_R$$

identical except for mass ⟹ can mix!

⟹ violation of flavour;

violation of CP

recall: $-\mathcal{L}_Y = g_d \bar{L} \phi d_R + g_u \bar{L} \tilde{\phi} u_R$

with $L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} / \sqrt{2}$

and $m_{u,d} = \dfrac{g_{u,d} v}{\sqrt{2}}$ mass = Yukawa coupling times vev

generalize to 3 generations:

$$L_1' = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad L_2' = \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \quad L_3' = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$$

$$u'_R, c'_R, t'_R \equiv u'_{i,R} \quad ; d'_R, s'_R, b'_R \equiv d'_{i,R}$$

"Flavor States" (the states that couple to $W^\pm, Z$)

$\rightarrow$ most general $\mathcal{L}$

$$-\mathcal{L}_Y = \sum_{i,j} \bar{L}_i' \left[g_{ij}^{(d)} \phi d'_{j,R} + g_{ij}^{(u)} \tilde{\phi} u'_{j,R} \right]$$

$$\xrightarrow{SSB} \sum \Big[\bar{d}'_{i,L} \underbrace{\frac{g_{ij}^{(d)} v}{\sqrt{2}}}_{M_{ij}^{(d)}} d'_{j,R} + \bar{u}'_{i,L} \cancel{\phi}\, u'_{j,R} \underbrace{\frac{g_{ij}^{(u)} v}{\sqrt{2}}}_{M_{ij}^{(u)}} \Big]$$

$$= \bar{d}'_L M^{(d)} d'_R + \bar{u}'_L M^{(u)} u'_R$$

with $d'_{L,R} = (d', s', b')^T_{L,R}$, $u'_{L,R} = (u', c', t')^T_{L,R}$

with „mass matrices" $M^{(u,d)}$

diagonalization yields states with diagonal mass terms:

<u>physical states</u>, <u>mass states</u> $\Leftrightarrow$ mass $\neq$ flavor

$$\left.\begin{aligned} U_d^\dagger M^{(d)} V_d &= D^d = \mathrm{diag}(m_d, m_s, m_b) \\ U_u^\dagger M^{(u)} V_u &= D^u = \mathrm{diag}(m_u, m_c, m_t) \end{aligned}\right\} \text{bi-unitary transformation}$$

$$U_u U_u^\dagger = V_u V_u^\dagger = U_d U_d^\dagger = V_d V_d^\dagger = 1\!\!1$$

<u>proof:</u> consider $h = m m^\dagger$ (hermitian)

$$U^\dagger h U = D^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2) = m_i^2 \delta_{ij}$$

$$m_i^2 = \sum_j (U^\dagger m)_{ij} (m^\dagger U)_{ji} = \sum_j (U^\dagger m)_{ij} (U^\dagger m)^*_{ij} = \sum_j |(U^\dagger m)_{ij}|^2$$

now write $m = U D V^\dagger$ with $D_{ij} = +\sqrt{m_i^2}\, \delta_{ij}$

with $m \cancel{m} m^\dagger = U D^2 U^\dagger \Leftrightarrow m = U D^2 U^\dagger (m^\dagger)^{-1}$

one shows that $V = m^{-1} U D$

finally, V is unitary: $V V^\dagger = m^{-1} U D D^{-1} U^\dagger m$
$= 1\!\!1$

(if m real: orthogonal U, V)

back to $\mathcal{L}$:

$$-\mathcal{L} = \underbrace{\bar{d}'_L U_d}_{=\bar{d}_L} \underbrace{U_d^\dagger M^{(d)} V_d}_{D^d} \underbrace{V_d^\dagger d'_R}_{\equiv d_R} + \underbrace{\bar{u}'_L U_u}_{\equiv \bar{u}_L} \underbrace{U_u^\dagger M^{(u)} V_u}_{D^u} \underbrace{V_u^\dagger u'_R}_{\equiv u_R}$$

$\Rightarrow$ the states in d_L, u_L, u_R, d_R are physical

not done yet: CC term

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \, \bar{u}'_L \gamma^\mu d'_L$$

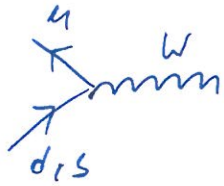
$$= \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\bar{u}'_L U_u}_{\bar{u}_L} U_u^\dagger \gamma^\mu U_d \underbrace{U_d^\dagger d'_L}_{d_L}$$

$$\Rightarrow \boxed{V = U_u^\dagger U_d}$$

Cabibbo
Kobayashi
Maskawa

CKM-matrix

$\Rightarrow$ actually



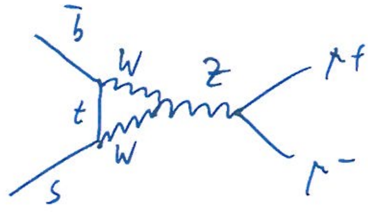
$i g \gamma_\mu (1-\gamma_5) V_{u\,d}$, $V_{u\,s}$

$\Rightarrow$ transitions between families!

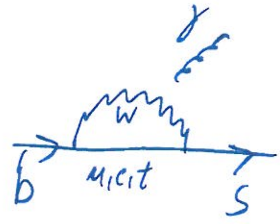
Note: no flavor changing neutral currents (FCNC) (at tree level...): $\mathcal{L}_{NC} \sim \bar{u}' (c_V - c_A \gamma_5) \gamma_\mu u'$
$= \bar{u} (c_V - c_A \gamma_5) \gamma_\mu u$

Glashow - Illiopolus - Maiani (GIM) mechanism

FCNC induced at loop level, e.g.



$B_s \to \mu^+\mu^-$



$b \to s\gamma$

e.g. $b \to s\gamma$: $\mathcal{A} \propto V^\dagger_{su} V_{ub}\, f\left(m_u^2/m_W^2\right) + V^\dagger_{sc} V_{cb}\, f\left(m_c^2/m_W^2\right)$

$$+ V^\dagger_{st} V_{tb}\, f\left(m_t^2/m_W^2\right)$$

with $f(x) = c + \text{small}(x)$

$\Rightarrow \mathcal{A} \propto (VV^\dagger)_{sb} = \delta_{sb} = 0$ at leading order

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$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

all elements have to determined by experiment, checked for consistency, unitarity, etc.

How many parameters are physical?

In general, N families $\Rightarrow$ V is complex $N\times N$

	#	# tot
complex $N\times N$	$2N^2$	$2N^2$
$VV^\dagger = \mathbb{1}$	$-N^2$	N^2
rephase u_i, d_i (mod. one total unphysical phase)	$-(2N-1)$	$(N-1)^2$

if V were real, it would consist of $\frac{1}{2}N(N-1)$ Euler angles

$\Rightarrow$ remaining parameters are phases

families	angles	phases
2	1	–
3	3	1
4	6	3
⋮		
N	$\frac{1}{2}N(N-1)$	$\frac{1}{2}(N-2)(N-1)$

convenient to parametrize those parameters most useful way: $V = R_{23}(\Theta_{23})\, R_{13}(\Theta_{13}, \delta)\, R_{12}(\Theta_{12})$

with $R_{23}(\Theta_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$ $\quad c_{ij} = \cos\Theta_{ij}$, $s_{ij} = \sin\Theta_{ij}$

$$R_{12}(\Theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{13}(\Theta, \delta) = \begin{pmatrix} c & 0 & s e^{-i\delta} \\ 0 & 1 & 0 \\ -s e^{i\delta} & 0 & c \end{pmatrix}$$

more intuitive: (Wolfenstein)

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$\lambda \simeq 0.22357 \pm 0.00061$; $A = 0.814^{+0.023}_{-0.024}$

$\bar{\rho} = \rho(1 - \lambda^2/2) = 0.117 \pm 0.021$; $\bar{\eta} = 0.353 \pm 0.013$

$\Rightarrow$ mixing (1-2) > mixing (2-3) > mixing (1-3)

hierarchical !

is this hierarchy connected to mass hierarchy?

suppose $M^{(u)}$ diagonal and $M^{(d)} = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$

$$V^{\dagger} M^{(d)} V^{*} = D = \mathrm{diag}(m_d, m_s) \Rightarrow V D V^{T} = M^{(d)}$$

with $V = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$, solve for $(V D V^{T})_{11} = 0$

$$\Rightarrow m_d c^2 + m_s s^2 = 0$$

(masses can change sign by chiral trafo $\psi \to e^{i\gamma_5} \psi$ or $\psi \to i\gamma_5 \psi$)

$$\Rightarrow \boxed{\tan\theta = \sqrt{\frac{m_d}{m_s}}}$$

$\simeq \sin\theta_c \simeq 0.23$

$\sqrt{\frac{5}{100}} \simeq 0.22$

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is this always the case?

No, consider $M = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \Rightarrow \sqrt{\tfrac{1}{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \sqrt{\tfrac{1}{2}}$

$$= \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$$

mixing angle (45°) not related to masses

seems to be the case for neutrinos