Recap.:

•) $\phi \sim 2_L, 1$ gives masses to

*) gauge bosons $\propto v \cdot g$

*) fermions $\propto v \cdot g_f$

*) itself $\propto \lambda v^2$

•) important parameter : $\rho = \dfrac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$

and corrections $\Delta\rho$ are small.

$\Delta\rho = 0$ for $m_t = m_b$ and $g' = 0$

$\leftrightarrow$ custodial symmetry

(Number: $\rho = 1.00040 \pm 0.00024$ )

26.11.

Replace $\Phi = \sqrt{\frac{1}{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix}$ with $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}$

$V = V(\Phi^+\Phi)$ and $\Phi^+\Phi = \varphi^T\varphi$

$\Rightarrow$ Higgs potential has global $O(4)$ symmetry!

Further, write $\mathbb{\Phi} = (\tilde{\Phi}, \Phi)$ with $\tilde{\Phi} = i\tau_2\Phi^*$

$$= \sqrt{\frac{1}{2}} \begin{pmatrix} \Phi_3 - i\Phi_4 & \Phi_1 + i\Phi_2 \\ -\Phi_1 + i\Phi_2 & \Phi_3 + i\Phi_4 \end{pmatrix}$$

Since $\mathrm{Tr}\{\mathbb{\Phi}^+\mathbb{\Phi}\} = 2\Phi^+\Phi$ it follows that

Higgs potential has $SU(2)_L \times SU(2)_R$ invariance

SSB: $\langle \mathbb{\Phi} \rangle = \sqrt{\frac{1}{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$

under symmetry, $\langle \mathbb{\Phi} \rangle \Rightarrow \langle \mathbb{\Phi}' \rangle = \langle U_L \mathbb{\Phi} U_R \rangle$

only invariant for $U_L = U_R^+$ "diagonal subgroup" $SU(2)_{L+R}$

$SU(2)_L$ is global version of SM-gauge group

$SU(2)_R$ is called <u>custodial symmetry</u>

(sometimes $SU(2)_{L+R}$ is called like this)

## Yukawa couplings:

with $L = \begin{pmatrix} t \\ b \end{pmatrix}_L$ and $R = \begin{pmatrix} t \\ b \end{pmatrix}_R$ :

$$\mathcal{L} = -g_t \bar{L} \tilde{\Phi} t_R - g_b \bar{L} \Phi b_R = -\bar{L} \, \Phi \begin{pmatrix} g_t & 0 \\ 0 & g_b \end{pmatrix} R$$

$$= -\underbrace{\frac{g_t + g_b}{2} \bar{L} \, \Phi R}_{\text{invariant under } SU(2)_L \text{ and } SU(2)_R} - \underbrace{\frac{g_t - g_b}{2} \bar{L} \, \Phi \sigma_3 R}_{\text{not invariant under } SU(2)_R}$$

$\Rightarrow$ if $g_t \neq g_b$ : custodial symmetry broken

## Hypercharge:

$\mathcal{L} = \mathrm{Tr}\left\{ (D_\mu \Phi)^\dagger (D^\mu \Phi) \right\}$ is proportional to $(D_\mu \phi)^\dagger (D^\mu \phi)$

$$D_\mu \Phi = \partial_\mu \Phi + i g \vec{W}_\mu \vec{\sigma} \, \Phi + i g' B_\mu \Phi \sigma_3$$

($\sigma_3$ appears because $Y(\phi) = +1$
$Y(\tilde{\phi}) = -1$)

transform $\tilde{W}_\mu \to \tilde{W}_\mu' = U_L \tilde{W}_\mu U_L^\dagger$

$\Phi \to \Phi' = U_L \Phi U_R$

$\Rightarrow$ $\mathcal{L}$ is invariant only if $g' = 0$;

$g' \neq 0$ breaks $SU(2)_R$

<u>All in all:</u> $m_t \neq m_b$ and $g' \neq 0$ break custodial symmetry and cause $\Delta\rho \neq 0$

Application: „New physics" in Higgs sector dangerous if it breaks custodial symmetry,

e.g. $\mathcal{L} = \frac{1}{\Lambda^2}\left(\Phi^+ D_\mu \Phi\right)^2 = \frac{1}{\Lambda^2} \mathrm{Tr}\{\sigma_3 \Phi^+ D_\mu \Phi\}^2$

or additional Higgs particles which are triplets under $SU(2)_L$

etc.

In general: $$\rho = \sum \frac{V_i^2\left[T_i(T_i+1) - Y_i^2/4\right]}{\frac{1}{2} Y_i^2 V_i^2}$$

with $T_i$ and $Y_i$ $SU(2)_L$ and $U(1)_Y$ quantum numbers

e.g. SM-Higgs: $T_i = \frac{1}{2}$; $Y_i = 1 \Rightarrow \rho = 1$;

other way around: with $T = \frac{1}{2}$: is $Q = T_3 + \frac{Y}{2} = 0$ possible?

~~[illegible]~~

Triplet: $T=1 \overset{g \overset{!}{=} 1}{\Longrightarrow} Y = 2\sqrt{2/3}$ ↯ can't have $Q=0$

Quadruplet: $T = 3/2 \Rightarrow Y = \sqrt{5}$ ↯ "

Quintuplet: $T=2 \Rightarrow Y = 2\sqrt{2}$ ↯ "

Sixtuplet: $T = 5/2 \Rightarrow Y = \sqrt{\frac{35}{3}}$ ↯ "

Septuplet: $T = 3 \Rightarrow Y = 4$ ✓ (⇒ 11 Higgs particles...

⋮ ⋮

$\left[ T = \frac{25}{2} \text{ and } Y = 15 \right]$

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## •) Higgs particle and longitudinal gauge boson scattering

↔ unitarity of scattering processes constraints $m_h$

optical theorem: $\boxed{\sigma_{tot} = \frac{1}{s} \operatorname{Im}\{\mathcal{M}(\theta = 0)\}}$ (1)

with $\sigma_{tot} = \int d\Omega \frac{|\mathcal{M}|^2}{64\pi^2 s}$ (2)

use partial wave decomposition:

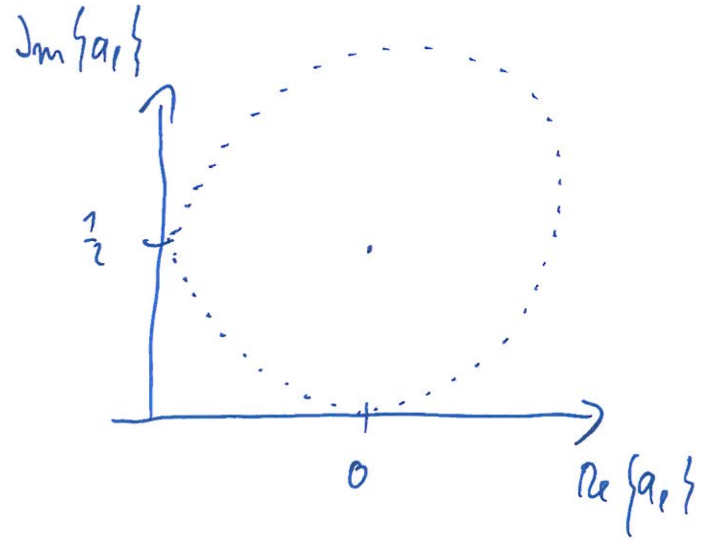
$$\underset{}{\mathcal{M}} = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) \underbrace{P_\ell(\cos\theta)}_{\text{Legendre}} \underbrace{a_\ell}_{\text{partial waves}} \quad (3)$$

insert (3) in (2): $\sigma = \frac{16\pi}{s} \sum (2l+1) |a_l|^2$ $\quad \int_{-1}^{1} P_l P_{l'} = \frac{2}{2l+1} \delta_{ll'}$

$$\overset{(1)}{=} \frac{16\pi}{s} \sum (2l+1)\, \mathrm{Im}\{a_l\}$$

$$\Rightarrow |a_l|^2 = \mathrm{Im}\{a_l\} \quad \text{or} \quad x^2+y^2 = y$$

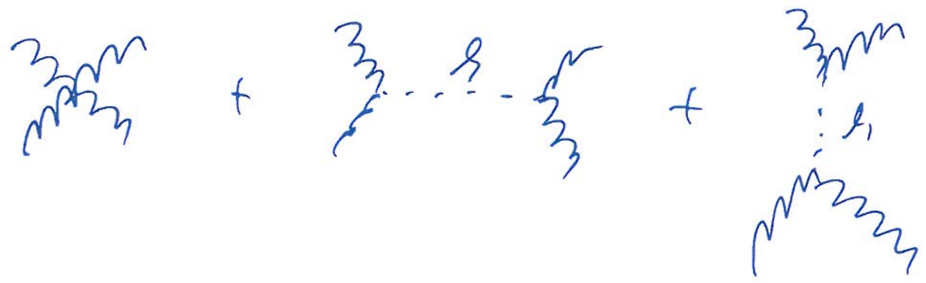
$$\text{or} \quad (y-\tfrac{1}{2})^2 + x^2 = (1/2)^2$$




$\Rightarrow \mathrm{Re}\{a_l\} < 1/2$, and

in particular $\mathrm{Re}\{a_0\} < 1/2$

Now consider $W^+W^- \to W^+W^-$ :



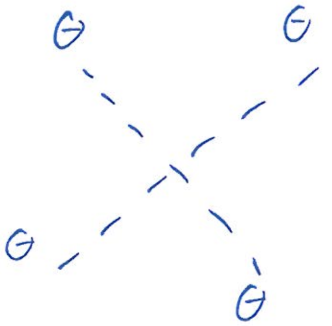

simplify by „equivalence theorem"

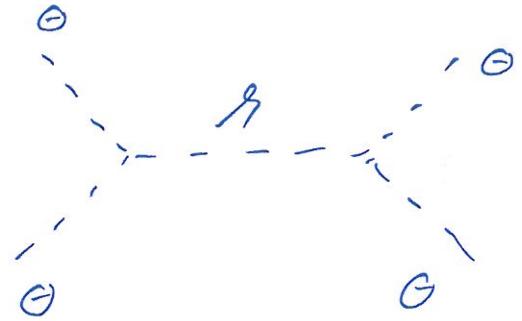
↔ the polarization vectors $\varepsilon_\mu^{\lambda=0} = \frac{1}{m_W}\left(|\vec{p}|, 0, 0, E\right)$

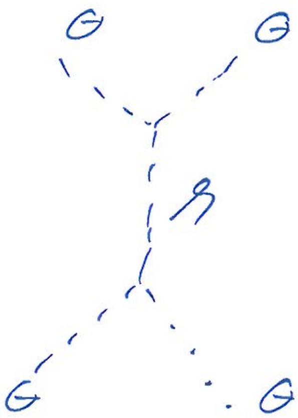
$$\varepsilon_\mu^{\lambda=\pm 1} = \sqrt{\tfrac{1}{2}}\left(0, \mp 1, -i, 0\right)$$

at $E \gg m_W$ : longitudinal mode dominates

⇒ Goldstone bosons dominate !



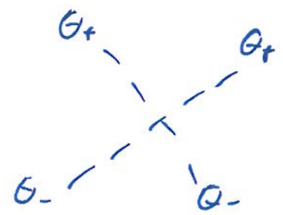




need Feynman rules : $V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$

with $\phi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \theta_1 + i\theta_2 \\ v + h - i\theta_3 \end{pmatrix}$

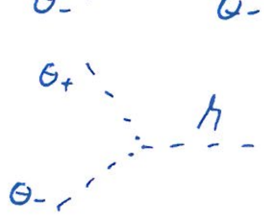
$$\Rightarrow V = \frac{m_h^2}{2v^2}\, \theta_+ \theta_- \theta_+ \theta_- + \frac{m_h^2}{v}\, h\, \theta_+ \theta_- + \dots$$

with $m_h^2 = 2v^2 \lambda$ ; $v^2 = -\mu^2/\lambda$ ; $\theta_\pm = \sqrt{\tfrac{1}{2}}(\theta_1 \pm i\theta_2)$

⇒ 


$-2i\,\frac{m_h^2}{v^2}$

(factor 4 = 2×2 , for each $\theta_+$ there are 2 possibilities to identify them with the fields in $\mathcal{L}$)




$-i\, m_h^2/v$

$$\Rightarrow i\mathcal{M}(W^+W^- \to W^+W^-) = \frac{-2i m_h^2}{v^2} + \left(\frac{-i m_h^2}{v}\right)^2 \left(\frac{i}{s-m_h^2} + \frac{i}{t-m_h^2}\right)$$

I guess $a_0$ dominates: (*)

$$a_0 = \frac{1}{16\pi s}\int_{-s}^{0} dt\, |\mathcal{M}| \simeq \frac{m_h^2}{8\pi v^2} \qquad (s \gg m_h^2)$$

$$\Rightarrow \boxed{m_h^2 < 4\pi v^2 = (870\ \text{GeV})^2}$$

(perturbative) unitarity bound

something at around TeV scale should regularize $W_L^+ W_L^-$ scattering

$\Rightarrow$ LHC

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(*): $t = -\frac{s}{2}(1-\cos\theta) \Rightarrow \left|\frac{dt}{d\cos\theta}\right| = \frac{s}{2}$; $\cos\theta = -1$: $t = -s$; $\cos\theta = +1$: $t = 0$

$$\mathcal{M} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l\, a_l \Leftrightarrow \int_{-1}^{1} d\cos\theta\, \mathcal{M}\, P_{l'}(\cos\theta) = 32\pi\, a_{l'}$$

$$\Rightarrow a_0 = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\, \mathcal{M}$$

$$= \frac{1}{16\pi s}\int_{-1}^{1} dt$$