Recap: •) massive gauge bosons from SSB of local gauge theories (+ gauge fixing)

•) N generators, vacuum invariant under $M < N$ generators $\Rightarrow$ $N-M$ massless GB end up in massive gauge bosons.

$\Rightarrow$ remaining M gauge bosons stay massless

•) $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$

consider neutral ($\bar{u}\gamma_\mu u$ etc.) part of

$\bar{\Psi}(\not{\partial} + ig\frac{\sigma_i}{2}W^i + \frac{i}{2}g'\hat{Y}B)\Psi$

$\Rightarrow$ $\boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}}$ $\Rightarrow$ vacuum should be invariant under $\hat{Q}$ $\Rightarrow$ $Y_\phi = +1$

i.e. $\hat{Q}\begin{pmatrix}0\\v\end{pmatrix} = 0$

Note: $\left.\begin{array}{l}\tau_i\begin{pmatrix}0\\v\end{pmatrix} \neq 0 \\ \hat{Y}\begin{pmatrix}0\\v\end{pmatrix} \neq 0\end{array}\right\}$ all broken!

also: $\bar{\Psi}_2 D_\mu \gamma^\mu \Psi_2 \to 0 \cdot \bar{u}_R \gamma_\mu u_R W^{\mu,3} g + \frac{Y_2}{2} \bar{u}_R \gamma_\mu u_R B^\mu g'$

compare with QED: $\bar{u} \gamma_\mu u A^\mu = (\bar{u}_L \gamma_\mu u_L + \bar{u}_R \gamma_\mu u_R) A^\mu$

$$\Rightarrow \boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}} \qquad \boxed{g W_\mu^3 + g' B_\mu = e A_\mu}$$

$\Rightarrow$ vacuum should be invariant under $\hat{Q}$; then the gauge boson associated to $\hat{Q}$ will remain massless.

$$e^{i\hat{Q}} \begin{pmatrix} 0 \\ v \end{pmatrix} = (1 + i\hat{Q}) \begin{pmatrix} 0 \\ v \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \hat{Q} \begin{pmatrix} 0 \\ v \end{pmatrix} \overset{!}{=} 0 \Rightarrow -\frac{1}{2} + \frac{Y_\Phi}{2} \overset{!}{=} 0$$

$$\Rightarrow \boxed{Y_\Phi = +1}$$

(Note: $\tau_i \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$ and $\hat{Y} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \Rightarrow$ broken)

$\Rightarrow$ masses of gauge bosons: 19.11.14

$(D_\mu \Phi)^\dagger (D^\mu \Phi)$ with $D_\mu = \partial_\mu + \frac{1}{2}(i g W_\mu^i \sigma^i + i g' Y_\Phi B_\mu)$

with $W_\mu^\pm = \sqrt{\frac{1}{2}}(W_\mu^1 \mp i W_\mu^2)$

this gives:

$$\mathcal{L} = \frac{1}{4} v^2 g^2 W_\mu^+ W^{\mu,-}$$

$$+ \frac{v^2}{8} (W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{\mu,3} \\ B^\mu \end{pmatrix}$$

$$\Rightarrow \boxed{m_{W^\pm}^2 = \frac{1}{4} v^2 g^2}$$

as expected, W_μ^3 and B_μ mix: diagonalize „mass matrix"

$$(W_\mu^3, B_\mu) R \underbrace{R^T \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} R}_{\text{diagonal}} \underbrace{R^T \begin{pmatrix} W^{\mu,3} \\ B^\mu \end{pmatrix}}_{\text{„physical states"}}$$

From: det $= 0$ and Tr $= g^2 + g'^2$: one massless state

one massive state $\frac{v^2}{4}(g^2+g'^2)$

$$\boxed{M_Z^2 = \frac{v^2}{4}(g^2+g'^2)}$$

$$M_A = 0$$

$$R = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \qquad \begin{array}{l} c_W = \cos\Theta_W \\ s_W = \sin\Theta_W \end{array} \qquad \text{Weinberg angle}$$

set $[R^T(\ )R]_{12} = 0 \Rightarrow \tan\Theta_W = g'/g$

and physical fields are:

$$\boxed{\begin{aligned} A_\mu &= c_W B_\mu + s_W W_\mu^3 \\ Z_\mu &= c_W W_\mu^3 - s_W B_\mu \end{aligned}} \quad \perp A_\mu$$

Note that $\dfrac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$ „ρ Parameter"

$M_W = (80.385 \pm 0.015)$ GeV

$M_Z = (91.1876 \pm 0.0021)$ GeV

$s_W^2 = 0.231$ $(29°)$

$v = 246$ GeV (fixed)

•) Coupling to fermions

write $\bar\psi_i \not{D} \psi_i$ in terms of $W_\mu^\pm$, Z_μ, A_μ

e.g. $Z_\mu \bar f \gamma^\mu (v_f - a_f \gamma_5) f \frac{e}{2 c_W s_W}$

	u	d	ν	e
$2v_f$	$1-\frac{8}{3}s_W^2$	$-1+\frac{4}{3}s_W^2$	1	$-1+4s_W^2$
$2a_f$	1	-1	1	-1

$W^\pm$ coupling

$$\mathcal{L} = \frac{g}{2\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right] W^{\mu,+}$$

$$+ m_W^2 W_\mu^+ W^{\mu,-}$$

@ low energy: $\partial_\mu W^{\mu,+} = 0$

$\Rightarrow$ Euler Lagrange: $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

$$\Rightarrow W_\mu^- = \frac{g}{2\sqrt{2} m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right]$$

insert back in $\mathcal{L}$:

$$\mathcal{L}_{eff} = - \frac{g^2}{8 m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] \left[\bar{e} \gamma^\mu (1-\gamma_5) \nu_e \right]$$

$\leftrightarrow$ Fermi - theory: $\boxed{\frac{g^2}{8 m_W^2} \equiv \frac{G_F}{\sqrt{2}}}$ „effective theory"

 vs.  have „integrated out W-boson"

•) Fermion Masses

mass term : $m_{up}\, \bar{u}\, u = m_{up}\, \bar{u}_L\, u_R + h.c.$

$$\begin{array}{cc} \swarrow & \searrow \\ I_3 = -\frac{1}{2} & I_3 = 0 \\ Y = 1/3 & Y = 4/3 \end{array}$$

↯ forbidden!

Higgs-doublet helps!

$$\phi \sim 2_L, 1 \quad ; \quad \Psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim 2_L, 1/3 \; ; \; u_R \sim 1_L, 4/3 \; ; \; d_R \sim 1_L, -\frac{2}{3}$$

$\Rightarrow \bar{\Psi}_1\, \phi\, d_R$ is invariant ! [because $\Psi \to U_L \Psi$; $\phi \to U_L \phi$ and Y adds up to zero]

write therefore: $\mathcal{L} = -g_d\, \bar{\Psi}_1\, \phi\, d_R$ Yukawa coupling g_d

$$\mathcal{L}_{SSB} = -g_d\, \bar{\Psi}_1 \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}}\, d_R = \underbrace{-g_d \frac{v}{\sqrt{2}}}_{m_d}\, \bar{d}_L\, d_R \underbrace{- \sqrt{\tfrac{1}{2}}\, g_d}_{\text{coupling to Higgs}}\, h\, \bar{d}_L\, d_R$$

$$g_d = \frac{m_d}{v}$$

what about up-quark: ↔ consider $\bar{\Psi}_1 \tilde{\Phi} u_R$ with $\tilde{\Phi} = i\tau_2 \Phi^*$

Hypercharge adds to zero, what about $SU(2)$?

$$\bar{\Psi}_1 \tilde{\Phi} \to \bar{\Psi}_1' \tilde{\Phi}' = \bar{\Psi}_1 U_L^\dagger \, i\sigma_2 \, U_L^* \, \Phi^*$$

with $U_L \simeq 1 + i\frac{\sigma_i}{2}\alpha_i$; use $-i\sigma_i \; i\sigma_2 \, (-i\sigma_i^*) = i\sigma_2 \;\; \forall i$

$\Rightarrow$ term is invariant!

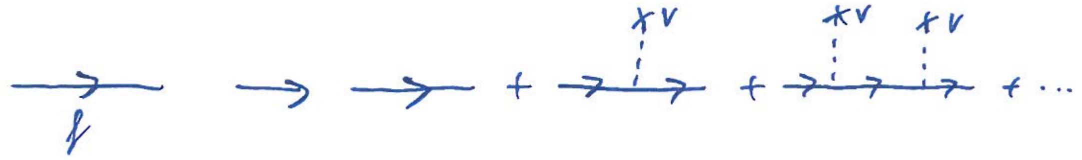
$$\Rightarrow \mathcal{L} = -g_u \frac{v}{\sqrt{2}} \bar{u}_L u_R - g_u \sqrt{\tfrac{1}{2}} \, h \, \bar{u}_L u_R$$

$\Rightarrow$ all masses $\propto v$

(+ all couplings to h)

- fermions: $m_f = g_u \, v/\sqrt{2}$
- gauge bosons: $m_W = \frac{1}{2} v g$, $m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$
- Higgs $m_h = \sqrt{2\lambda v^2}$ (= 125.36 ± 0.37 ± 0.18) GeV

↔ interaction with constant background field



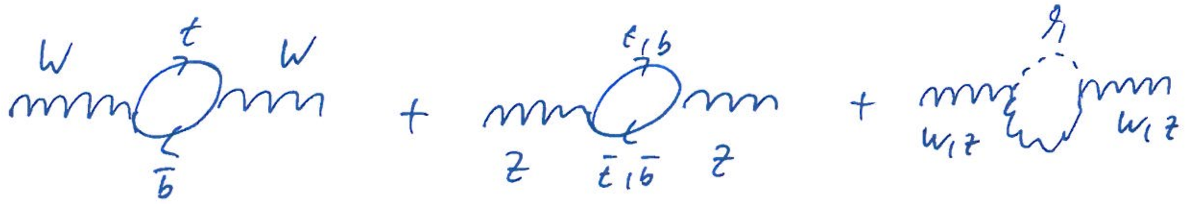
$$\frac{1}{p} \to \frac{1}{p}\frac{g_f}{\sqrt{2}} v \frac{1}{p} + \frac{1}{p}\frac{g_f}{\sqrt{2}} v \frac{1}{p}\frac{g_f}{\sqrt{2}} v \frac{1}{p} + \dots$$

$$= \frac{1}{p}\sum_{k=1}^{\infty}\left(\frac{g_f v}{\sqrt{2}}\frac{1}{p}\right)^k = \frac{1}{p}\frac{1}{1 - \frac{g_f v}{\sqrt{2}}\frac{1}{p}} = \frac{1}{p - m_f}$$

with $m_f = \frac{g_f v}{\sqrt{2}}$

•) Custodial Symmetry

$\leftrightarrow$ corrections to $\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$ are small :



$\Rightarrow \Delta m_W \, , \, \Delta m_Z \Rightarrow \Delta\rho \neq 0$

$$\Delta\rho = \frac{3G_F^2}{8\pi^2 2\sqrt{2}} \Big[m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log m_t^2/m_b^2$$

$$- \frac{11}{9} m_Z^2 s_W^2 \log m_h^2/m_Z^2 \Big]$$

$\rightarrow$ indirect constraint (together with many many many other observables)

$$m_h = 94^{+29}_{-24} \text{ GeV} \quad \text{or} \quad m_h < 125 \text{ GeV}$$

$\rightarrow \Delta\rho = 0$ for $m_t = m_b$ and $g' = 0$

why?