

Recap:

-) massive gauge bosons from SSB of local gauge theories (\leftarrow gauge fixing)
-) N generators, vacuum invariant under $M < N$ generators $\Rightarrow N - M$ massless GB end up ~~as~~ ⁱⁿ massive gauge bosons.
 \Rightarrow remaining M gauge bosons stay massless

•) $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$

consider neutral ($\bar{u} \gamma_\mu u$ etc.) part of

$$\bar{\Psi} \left(\gamma + ig \frac{\sigma_i}{2} W^i + \frac{i}{2} g' \hat{Y} B \right) \Psi$$

$$\Rightarrow \boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}} \Rightarrow \text{vacuum should be invariant under } \hat{Q} \Rightarrow \gamma_\phi = +1$$

i.e. $\hat{Q} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$

Note: $\tau_i \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$
 $\hat{Y} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$ } all broken!

$$\text{also: } \bar{\psi}_2 D_\mu \psi^1 \psi_2 \rightarrow 0 \cdot \bar{u}_R \delta_{\mu M_R} W^{\mu 3} g + \frac{Y_\phi}{2} \bar{u}_R \delta_{\mu M_R} B^\mu g'$$

$$\text{compare with QED: } \bar{u} \delta_{\mu M} A^\mu = (\bar{u}_L \delta_{\mu M_L} + \bar{u}_R \delta_{\mu M_R}) A^\mu$$

$$\Rightarrow \boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}}$$

$$\boxed{g W_\mu^3 + g' B_\mu = e A_\mu}$$

\Rightarrow vacuum should be invariant under \hat{Q} ; then the gauge boson associated to \hat{Q} will remain massless.

$$e^{i\hat{Q}} \begin{pmatrix} 0 \\ v \end{pmatrix} = (1 + i\hat{Q}) \begin{pmatrix} 0 \\ v \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow \cancel{\hat{Q}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \hat{Q} \begin{pmatrix} 0 \\ v \end{pmatrix} \stackrel{!}{=} 0 \Rightarrow -\frac{1}{2} + \frac{Y_\phi}{2} \stackrel{!}{=} 0$$

$$\Rightarrow \boxed{Y_\phi = +1}$$

(Note: $\tau_i \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$ and $\hat{Y} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \Rightarrow \text{broken}$)

\Rightarrow masses of gauge bosons:

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$$(D_\mu \phi)^\dagger (D^\mu \phi) \quad \text{with} \quad D_\mu = \partial_\mu + \frac{i}{2} (ig W_\mu^i \sigma^i + ig' Y_\phi B_\mu)$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

this gives:

$$\mathcal{L} = \frac{1}{4} v^2 g^2 W_{\mu}^{+} W_{\mu}^{-} + \frac{v^2}{8} (W_{\mu}^3, B_{\mu}) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_{\mu}^3 \\ B_{\mu} \end{pmatrix}$$

$$\Rightarrow m_{W^{\pm}}^2 = \frac{1}{4} v^2 g^2$$

as expected, W_{μ}^3 and B_{μ} mix: diagonalize "mass matrix"

$$(W_{\mu}^3, B_{\mu}) R R^T \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} R R^T \begin{pmatrix} W_{\mu}^3 \\ B_{\mu} \end{pmatrix}$$

diagonal
"physical states"

From: $\det = 0$ and $\text{tr} = g^2 + g'^2$: one massless state
 one massive state $\frac{v^2}{4}(g^2 + g'^2)$

$$M_Z^2 = \frac{v^2}{4}(g^2 + g'^2)$$

$$M_A = 0$$

$$R = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}$$

$$c_W = \cos \Theta_W$$

$$s_W = \sin \Theta_W$$

Weinberg angle

$$\text{set } [R^T () R]_{12} = 0 \Rightarrow \tan \Theta_W = g'/g$$

and physical fields are:

$$\begin{aligned}
 A_\mu &= c_W B_\mu + s_W W_\mu^3 \\
 Z_\mu &= c_W W_\mu^3 - s_W B_\mu \quad \perp A_\mu
 \end{aligned}$$

Note that $\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$ "S Parameter"

$$M_W = (80.385 \pm 0.075) \text{ GeV}$$

$$M_Z = (91.1876 \pm 0.0027) \text{ GeV}$$

$$s_W^2 = 0.237 \quad (29^\circ)$$

$$v = 246 \text{ GeV} \quad (\text{fixed})$$

•) Coupling to fermions

write $\bar{\Psi}_i \not{\partial} \Psi_i$ in terms of W_μ^\pm, Z_μ, A_μ

e.g. $Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f \quad \frac{e}{2c_W s_W}$

	u	d	ν	e
$2v_f$	$1 - \frac{8}{3}s_W^2$	$-1 + \frac{4}{3}s_W^2$	1	$-1 + 4s_W^2$
$2a_f$	1	-1	1	-1

W[±] coupling

$$\mathcal{L} = \frac{g}{2\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e \right] W^{\mu,+} + m_W^2 W_\mu^+ W^{\mu,-}$$

① low energy: $\partial_\mu W^{\mu,+} = 0$

⇒ Euler Lagrange: $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

⇒ $W_\mu^- = \frac{g}{2\sqrt{2}m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e \right]$

insert back in \mathcal{L} :

$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{8m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu \right] \left[\bar{e} \gamma^\mu (1 - \gamma_5) \nu_e \right]$$

↔ Fermi - theory: $\frac{g^2}{8m_W^2} \equiv \frac{G_F}{\sqrt{2}}$ "effective theory"

~~any~~ vs. ~~X~~ "low energy" integrated out W-boson"

•) Fermion Masses

mass term: $m_{up} \bar{u} u = m_{up} \bar{u}_L u_R + h.c.$

$$\begin{array}{ccc} & \swarrow & \searrow \\ I_3 = -\frac{2}{3} & & I_3 = 0 \\ \gamma = \frac{1}{3} & & \gamma = \frac{4}{3} \end{array} \quad \Downarrow \text{forbidden!}$$

Higgs-doublet helps!

$$\phi \sim Z_{L,1} \quad ; \quad \Psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim Z_{L,1} \frac{1}{3} \quad ; \quad u_R \sim Z_{L,1} \frac{4}{3} \quad ; \quad d_R \sim Z_{L,1} -\frac{2}{3}$$

$\Rightarrow \bar{\Psi}_1 \phi d_R$ is invariant! [because $\Psi \rightarrow U_L \Psi$; $\phi \rightarrow U_L \phi$ and γ adds up to zero]

write therefore: $\mathcal{L} = -g_d \bar{\Psi}_1 \phi d_R$ Yukawa coupling g_d

$$\mathcal{L}_{SSB} = -g_d \bar{\Psi}_1 \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}} d_R = \underbrace{-g_d \frac{v}{\sqrt{2}}}_{m_d} \bar{d}_L d_R - \underbrace{\sqrt{\frac{1}{2}} g_d h}_{\text{coupling to Higgs}} \bar{d}_L d_R$$

$$g_d = \frac{m_d}{v}$$

what about u -quark: \leftarrow consider $\bar{\Psi}_1 \tilde{\Phi} u_R$ with $\tilde{\Phi} = i\tau_2 \Phi^*$

Hyperechange adds to zero, what about $SU(2)$?

$$\bar{\Psi}_1 \tilde{\Phi} \rightarrow \bar{\Psi}_1 \tilde{\Phi}' = \bar{\Psi}_1 U_L^\dagger i\sigma_2 U_L \Phi^*$$

$$\text{with } U_L \simeq 1 + i \frac{\sigma_i}{2} \alpha_i \text{ use } -i\sigma_i i\sigma_2 (-i\sigma_i^*) = i\sigma_2 \forall i$$

\Rightarrow term is invariant!

$$\Rightarrow \mathcal{L} = -g_u \frac{V}{\sqrt{2}} \bar{u}_L u_R - g_u \sqrt{\frac{1}{2}} h \bar{u}_L u_R$$

\Rightarrow all masses $\propto V$

(+all couplings to h)

•) fermions: $m_f = g_u \frac{V}{\sqrt{2}}$

$m_W = \frac{1}{2} Vg$

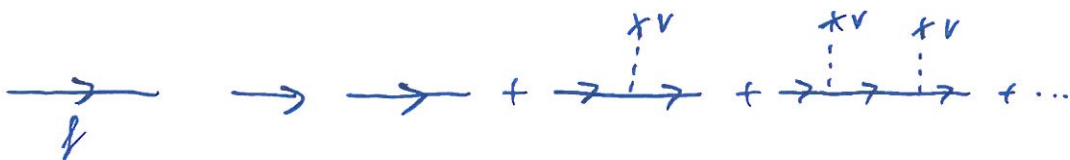
•) gauge bosons:

$m_Z = \frac{1}{2} V \sqrt{g^2 + g'^2}$

•) Higgs

$m_h = \sqrt{2\lambda V^2} (= 125.36 \pm 0.37 \pm 0.18) \text{ GeV}$

\leftarrow interaction with constant background field



$$\frac{1}{p} \rightarrow \frac{1}{p} \frac{g_1 V}{\sqrt{2}} \frac{1}{p} + \frac{1}{p} \frac{g_1 V}{\sqrt{2}} \frac{1}{p} \frac{g_1 V}{\sqrt{2}} \frac{1}{p} + \dots$$

$$= \frac{1}{p} \sum_{k=1}^{\infty} \left(\frac{g_1 V}{\sqrt{2}} \frac{1}{p} \right)^k = \frac{1}{p} \frac{1}{1 - \frac{g_1 V}{\sqrt{2}} \frac{1}{p}} = \frac{1}{p - m_f}$$

with $m_f = \frac{g_1 V}{\sqrt{2}}$

•) Custodial Symmetry

↳ corrections to $S = \frac{m_W^2}{m_z^2 \cos^2 \theta_W} = 1$ are small:

Three Feynman diagrams representing corrections to the S parameter:

- Diagram 1: A top quark loop with two W boson external lines.
- Diagram 2: A top and bottom quark loop with two Z boson external lines.
- Diagram 3: A scalar loop (Higgs boson) with two Z boson external lines.

⇒ $\Delta m_W, \Delta m_z \Rightarrow \Delta S \neq 0$

$$\Delta S = \frac{3G_F^2}{8\pi^2 2\sqrt{2}} \left[m_t^2 + m_b^2 - 2 \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_t^2}{m_b^2} - \frac{11}{9} m_z^2 s_W^2 \log \frac{m_h^2}{m_z^2} \right]$$

→ indirect constraint (together with many many many other observables)

$$m_h = 94^{+29}_{-24} \text{ GeV} \quad \text{or} \quad m_h < 725 \text{ GeV}$$

→ $\Delta S = 0$ for $m_t = m_b$ and $g' = 0$

Why?