b) Local Case of SSB

$$\mathcal{L} = \left[(\partial^\mu + ieA^\mu)\phi^*\right]\left[(\partial_\mu - ieA_\mu)\phi\right] - \mu^2\phi^*\phi - \lambda(\phi^*\phi$$

Symmetry: $\phi \to \phi' = e^{i\alpha(x)}\phi$

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}(\partial_\mu \alpha)$$

as usual, choose $\phi = \sqrt{\frac{1}{2}}(v + \eta + i\xi)$ and insert in $\mathcal{L}$

$$\Rightarrow \mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - evA_\mu(\partial^\mu \xi) - v^2\lambda\eta^2 + \ldots$$

kin. term for GB ξ

mass $m_A = ev$

WTF?

mass $m_\eta = \sqrt{2v^2\lambda}$

$$= \frac{1}{2}e^2v^2\left(A_\mu - \frac{1}{ev}\partial_\mu \xi\right)^2 \qquad (*)$$

Note: before SSB: ϕ, A_μ : $2+2=4$ degrees of freedom

after SSB: η, ξ, A_μ : $1+1+3=5$ degrees of freedom

Can't be, reformulation cannot increase d.o.f.

key observations: (*) looks like a gauge transformed A

furthermore: $\phi = \sqrt{\frac{1}{2}}\,(v+\eta+i\xi) \simeq \sqrt{\frac{1}{2}}\,(v+h)\exp\left\{ i\,\xi(x)/v \right\}$

indeed, inserting new ϕ in $\mathcal{L}$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - v^2\lambda h^2 + \frac{e^2v^2}{2}\left(A_\mu - \frac{1}{ev}\partial_\mu \xi\right)^2 + \dots$$

$$= \frac{1}{2}(\partial_\mu h)^2 - v^2\lambda h^2 + \frac{e^2v^2}{2} A'^2_\mu + \dots$$

This particular gauge choice implies $\phi \to e^{-i\xi/v}\phi$, i.e. it makes ξ disappear from $\mathcal{L}$ (shows up in A_μ)

„Unitary gauge"

Higgs-mechanism!

- would-be-Goldstone boson ends up as long. d.o.f. of gauge field
- $N-M$ [+] would-be-GB end up as long. d.o.f.; Gauge field associated with unbroken generator of gauge group remains massless

- $m_A \propto ev$, $m_h = \sqrt{2\lambda v^2}$ all $\propto v$ (is the energy scale of theory)

+: vacuum invariant under M generators

Massive photon without a Higgs particle: Stückelberg mechanism

QED: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{\partial} - e\not{A} - m_\psi)\psi$

$$\psi \to e^{i\theta(x)}\psi(x)$$

$$A_\mu \to A_\mu - \partial_\mu \theta$$

mass term forbidden? Try adding a new field σ,

such that $\sigma \to \sigma' = \sigma + m\,\theta(x)$ is gauge trafo

$\Rightarrow \mathcal{L}' = \frac{1}{2}(m A^\mu + \partial^\mu \sigma)(m A_\mu + \partial_\mu \sigma)$ is allowed

proof: $\cancel{\tfrac{1}{2}}\big(m(A^\mu - \partial^\mu\theta) + \partial^\mu(\sigma + m\theta)\big) = m A^\mu + \partial^\mu\sigma - m\partial^\mu\theta + m\partial^\mu\theta$

$= m A^\mu + \partial^\mu \sigma$ ☺

$\Rightarrow \mathcal{L}'$ contains kinetic term for σ

$\Rightarrow \sigma^n$ terms forbidden

$\Rightarrow$ coupling to ψ forbidden

The gauge freedom can be used to make $\sigma(x) = 0$ for all x

$\Rightarrow$ disappears from spectrum

this gauge corresponds to $\Theta = -\sigma/m$; or $A_\mu \to A_\mu + \partial_\mu \sigma/m$

$\Rightarrow$ it shows up in previously massless A_μ

$\leftrightarrow$ different gauge behavior of transversal / longitudinal d.o.f.

$\boxed{U(1) \text{ is not broken!}}$

<u>Connection to Higgs</u>: $\mathcal{L} = |(\partial_\mu - igqA_\mu)\phi|^2 - \lambda|\phi|^4 - \mu^2|\phi|^2$ (scalar with charge q)

with $\phi = \sqrt{\frac{1}{2}}(v+h)e^{-i\xi/v}$ need all terms...

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + gvqA_\mu \partial^\mu \xi + \frac{1}{2}q^2g^2v^2 A_\mu A^\mu$$

$$+ \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

$$+ \left[(\partial_\mu \xi)(\partial^\mu \xi) + 2gqvA_\mu \partial^\mu \xi + g^2q^2v^2 A_\mu A^\mu\right]\left(\frac{h}{v} + \frac{1}{2}\frac{h^2}{v^2}\right)$$

<u>Limit</u>: $v \to \infty$, $q \to 0$ with qv and g constant (mass of A)

$\Rightarrow h$ becomes infinitely heavy; $\frac{h}{v} \to 0$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + gvqA_\mu \partial^\mu \xi + \frac{1}{2}q^2g^2v^2 A_\mu A^\mu$$

this is the Stückelberg Lagrangian! $\xi \leftrightarrow \sigma$

SSB scale $v \to \infty$: not broken

(Works only for $U(1)$, since q needs to be zero: masses of gauge bosons depend in non-Abelian theories only on g and v; $v \to \infty$ leads to $g \to 0$ decoupling)

c) Local Case of SU(2) SSB

Scalar doublet $\Phi = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \sqrt{\frac{1}{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

gauge trafo: $\phi \to \phi' = \exp\left\{ \frac{i}{2} \theta^a \tau^a \right\} \phi$

$$D_\mu = \partial_\mu + ig \frac{\tau^a}{2} W_\mu^a \equiv \partial_\mu - ig \tilde{W}_\mu$$

$$\tilde{W}_\mu \to \tilde{W}_\mu' = \frac{-i}{g} (\partial_\mu U) \cdot U^{-1} + U \tilde{W}_\mu U^{-1}$$

$$\mathcal{L} = \left[\left(\partial_\mu + ig \frac{\vec{\tau} \vec{W}_\mu}{2} \right) \phi \right]^\dagger \left[\left(\partial_\mu + ig \frac{\vec{\tau} \vec{W}_\mu}{2} \right) \phi \right] - V(\phi^\dagger \phi)$$

(is invariant, see lecture 2, page 11)

Minimum at $\phi^\dagger \phi = \frac{1}{2} \sum_i \phi_i^2 = -\mu^2 / 2\lambda$ (notation)

choose $\phi_1 = \phi_2 = \phi_4 = 0 \; ; \; \phi_3 = v$

$$\Rightarrow \phi = \sqrt{\frac{1}{2}} \begin{pmatrix} \xi_1(x) + i\xi_2(x) \\ v + h(x) + i\xi_3(x) \end{pmatrix}$$ around minimum

<u>Note:</u> $e^{i \vec{\tau} \vec{\theta} / v} \simeq \begin{pmatrix} 1 + i\theta_3/v & (\theta_2 + i\theta_1)/v \\ (-\theta_2 + i\theta_1)/v & 1 - i\theta_3/v \end{pmatrix}$

$$\Rightarrow e^{i \vec{\tau} \vec{\theta} / v} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \frac{1}{\sqrt{2}} \simeq \sqrt{\frac{1}{2}} \begin{pmatrix} \theta_2 + i\theta_1 \\ v + h - i\theta_3 \end{pmatrix}$$

$\Rightarrow$ do proper gauge to remove ξ_i from $\phi : \to \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$

relevant term in $\mathcal{L}$ for gauge boson masses:

$$\mathcal{L} \subset \left(ig \frac{\vec{\tau}}{2} \vec{W}_\mu \phi \right)^\dagger \left(ig \frac{\vec{\tau}}{2} \vec{W}_\mu \phi \right) \longrightarrow \frac{g^2 v^2}{8} \left[(W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2 \right]$$

$$\Rightarrow M_{W_i}^2 = \frac{1}{4} g^2 v^2$$

for later use: $W_\mu^\pm = \sqrt{\frac{1}{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$

$$\mathcal{L} \subset \frac{1}{4} v^2 g^2 W_\mu^+ W^{\mu -} + \frac{1}{8} g^2 v^2 (W_\mu^3)^2$$

II 2) Higgs-Mechanism and Electroweak Theory

$SU(2)_L \times U(1)_Y$ $\qquad \psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim 2_L, Y_1$

$\qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim 2_L, \tilde{Y}_1$

$\psi_2 = u_R \sim 1_L, Y_2$

$\psi_3 = d_R \sim 1_L, Y_3$

$e_R \sim 1_L, Y_3$

choose doublet Higgs: $\Phi = \sqrt{\frac{1}{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \sim 2_L, Y_\Phi$

•) usual potential

•) $\langle \phi_3 \rangle = v = \sqrt{-\mu^2/\lambda}$

$$D_\mu = \partial_\mu + ig \frac{\sigma_i}{2} W_\mu^i + \frac{i}{2} g' \hat{Y} B_\mu$$

hypercharge operator

isospin operator:

$\left.\begin{aligned} \hat{I}_3 u_L &= +\tfrac{1}{2} u_L \\ \hat{I}_3 d_L &= -\tfrac{1}{2} d_L \end{aligned}\right\} \frac{\sigma_3}{2}$

$\hat{I}_3 u_R = 0$

need to embed electromagnetism:

$$\bar{\psi}_1 D_\mu \gamma^\mu \psi_1 \rightarrow \left(\frac{1}{2} \bar{u}_L \gamma_\mu u_L W^{\mu,3} - \frac{1}{2} \bar{d}_L \gamma_\mu d_L W^{\mu,3}\right) g$$

$$+ \frac{Y_1}{2} \left(\bar{u}_L \gamma_\mu u_L + \bar{d}_L \gamma_\mu d_L\right) B^\mu g'$$

also: $\bar{\Psi}_2 D_\mu \gamma^\mu \Psi_2 \rightarrow 0 \cdot \bar{u}_R \gamma_\mu u_R W^{\mu,3} g + \frac{Y_2}{2} \bar{u}_R \gamma_\mu u_R B^\mu g'$

compare with QED: $\bar{u} \gamma_\mu u A^\mu = (\bar{u}_L \gamma_\mu u_L + \bar{u}_R \gamma_\mu u_R) A^\mu$

$$\Rightarrow \boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}} \qquad \boxed{g W_\mu^3 + g' B_\mu = e A_\mu}$$

$\Rightarrow$ vacuum should be invariant under $\hat{Q}$; then the gauge boson associated to $\hat{Q}$ will remain massless.

$$e^{i\hat{Q}} \begin{pmatrix} 0 \\ v \end{pmatrix} = (1 + i\hat{Q}) \begin{pmatrix} 0 \\ v \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \hat{Q} \begin{pmatrix} 0 \\ v \end{pmatrix} \overset{!}{=} 0 \Rightarrow -\frac{1}{2} + \frac{Y_\Phi}{2} \overset{!}{=} 0$$

$$\Rightarrow \boxed{Y_\Phi = +1}$$

(Note: $\tau_i \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$ and $\hat{Y} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \Rightarrow$ broken)

$\Rightarrow$ masses of gauge bosons:

$(D_\mu \Phi)^\dagger (D^\mu \Phi)$ with $D_\mu = \partial_\mu + \frac{1}{2}(ig W_\mu^i \sigma^i + ig' Y_\Phi B_\mu)$

with $W_\mu^\pm = \sqrt{\frac{1}{2}}(W_\mu^1 \mp i W_\mu^2)$