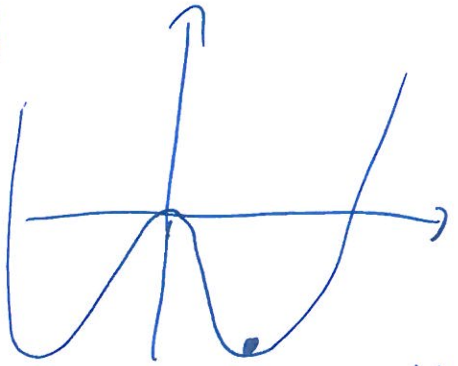
5.11.14

Recap: SSB



$\mathcal{L}$ is symmetric under symmetry with N generators (global!).

Vacuum symmetric under $M < N$ of those generators.

$\Rightarrow$ $\underline{N - M \text{ massless Goldstone-bosons}}$

If in addition to SSB an <u>explicit</u> breaking exists, so-called <u>Pseudo-Goldstone bosons</u> appear.

Let's discuss the example from last week again:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi^*) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{2}\varepsilon^2 \phi_1^2 \quad (*)$$

as usual, $\mu^2 < 0$; $\phi = \sqrt{\frac{1}{2}}(\phi_1 + i \phi_2)$

$$\Rightarrow V = \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 + \frac{1}{2}\varepsilon^2 \phi_1^2$$

minimum:

$$\frac{\partial V}{\partial \phi_1} = \varepsilon \phi_1 + \mu^2 \phi_1 + \lambda \phi_1 (\phi_1^2 + \phi_2^2) \stackrel{!}{=} 0$$

$$\frac{\partial V}{\partial \phi_2} = \mu^2 \phi_2 + \lambda \phi_2 (\phi_1^2 + \phi_2^2)$$

solution: $\Phi_2 = 0 \qquad \Phi_1^2 = -\frac{(\mu^2+\varepsilon)}{\lambda} \equiv v^2$

$\rightarrow$ insert $\Phi_1 = v + \eta$ in $(*)$
$\qquad\qquad \Phi_2 = \xi$

$$\Rightarrow V = \frac{1}{2}\mu^2\left[(v+\eta)^2+\xi^2\right] + \frac{\lambda}{4}\left[(v+\eta)^2+\xi^2\right]^2 + \frac{1}{2}\varepsilon(v+\eta)^2$$

interested in terms $\propto \eta^2$ and ξ^2

$$\Rightarrow V = \eta^2\left(\frac{1}{2}\mu^2 + \frac{\lambda}{4}6v^2 + \frac{1}{2}\varepsilon\right) + \xi^2\left(\frac{1}{2}\mu^2 + \frac{\lambda}{4}2v^2\right)$$

$$= \frac{1}{2}\eta^2\left(\mu^2+\varepsilon+\underbrace{3v^2\lambda}_{-3(\mu^2+\varepsilon)}\right) + \frac{1}{2}\xi^2\left(\mu^2+\underbrace{\lambda v^2}_{=-(\mu^2+\varepsilon)}\right)$$

$$= -\eta^2(\mu^2+\varepsilon^2) - \frac{1}{2}\xi^2\varepsilon$$ (note sign change!)

$$\Rightarrow m_\eta^2 = 2(\mu^2+\varepsilon)$$ $(|\varepsilon| < |\mu^2|)$

$$m_\xi^2 = \varepsilon$$ (small) mass of would-be Goldstone-boson

•) an invariant lagrangian, $\delta \mathcal{L} = 0$, leads to a conserved current: $J_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \delta \psi$ (lecture 1)

If we add non-symmetric term to $\mathcal{L}$, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$,

then $\delta \mathcal{L} = \delta \mathcal{L}_1$ and it is easy to show that $\boxed{\partial_\mu J^\mu = \delta \mathcal{L}_1}$

On chiral symmetry breaking

„massless QCD": $\mathcal{L} = G^a_{\mu\nu} G^{\mu\nu a} + \bar{u}\, i \not{D} u + \bar{d}\, i \not{D} d$

define $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ and consider only fermions:

$$\mathcal{L} = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R \qquad D_\mu = \partial_\mu - i g A^a_\mu \lambda^a$$

invariant under:
$$\psi_L \to \psi_L' = \exp\left\{-i \frac{\vec{\tau}}{2} \vec{\theta}_L\right\} \psi_L$$
$$\psi_R \to \psi_R' = \exp\left\{-i \frac{\vec{\tau}}{2} \vec{\theta}_R\right\} \psi_R \qquad (*)$$

$\Rightarrow$ global $SU(2)_L \times SU(2)_R$ „chiral symmetry"

proof: $\bar{\psi}_L \not{D} \psi_L \to \bar{\psi}_L \underbrace{\left(1 + i \frac{\vec{\tau}}{2}\vec{\theta}_L\right) \not{D} \left(1 - i \frac{\vec{\tau}}{2}\vec{\theta}_L\right)}_{1 + \mathcal{O}(\theta_L^2)} \psi_L = \bar{\psi}_L \not{D} \psi_L$

conserved currents: $j^{\mu, i}_{L,R} = \bar{\psi}_{L,R} \gamma^\mu \frac{\tau_i}{2} \psi_{L,R}$
(from $\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \delta\psi$)

let's rearrange this: $j^{\mu, i}_{V,A} = j^{\mu, i}_R \pm j^{\mu, i}_L$

under $(*)$, any state $|N\rangle \to |N'\rangle = \exp\left\{-i \frac{\tau^i}{2} \theta^i_a P_L\right\} \exp\left\{-i \frac{\tau^i}{2} \theta^i_b P_R\right\}$

$\times |N\rangle$

$$\Rightarrow |N'\rangle = \left(1 - i\frac{\tau^i}{2}\Theta_a^i P_L\right)\left(1 - i\frac{\tau^i}{2}\Theta_b^i P_R\right)|N\rangle$$

$$= \left(1 - \frac{i}{2}\tau^i\Theta_a^i \frac{1}{2}(1-\gamma_5) - \frac{i}{2}\tau^i\Theta_b^i\frac{1}{2}(1+\gamma_5)\right)|N\rangle$$

$$= |N\rangle - \left(\frac{i}{2}\tau^i(\Theta_a^i + \Theta_b^i) + \frac{i}{2}\tau^i(\Theta_b^i - \Theta_a^i)\gamma_5\right)|N\rangle$$

$$\equiv \exp\left\{-i\frac{\vec{\tau}}{2}\vec{\Theta}_V\right\}\exp\left\{-i\frac{\vec{\tau}}{2}\vec{\Theta}_A\gamma_5\right\}|N\rangle$$

$\Rightarrow$ vector and axialvector trafo! $SU(2)_V \times SU(2)_A$

•) Vector-trafo

$\bar{\psi}\,\not{\partial}\,\psi$ with $\psi \to \left(1 - i\frac{\vec{\tau}}{2}\vec{\Theta}_V\right)\psi$

$\bar{\psi} \to \bar{\psi}\left(1 + i\frac{\vec{\tau}}{2}\Theta_V\right)$.

$$\Rightarrow \bar{\psi}\gamma^\mu\psi \to \bar{\psi}\left(1 + i\frac{\vec{\tau}}{2}\vec{\Theta}_V\right)\gamma^\mu\left(1 - i\frac{\vec{\tau}}{2}\vec{\Theta}_V\right)\psi = \bar{\psi}\gamma^\mu\psi$$

conserved current: $\bar{\psi}\gamma_\mu\frac{\tau^i}{2}\psi$ (CVC)

•) Axial vector-trafo

$$\psi \to \left(1 - i\frac{\vec{\tau}}{2}\vec{\Theta}_A\gamma_5\right)\psi$$

$$\bar{\psi} \to \bar{\psi}' = \psi'^{+}\gamma_0 = \psi^{+}\left(1 + i\frac{\vec{\tau}}{2}\vec{\Theta}_A\gamma_5\right)\gamma_0$$

$$= \bar{\psi}\left(1 - i\frac{\vec{\tau}}{2}\vec{\Theta}_A\gamma_5\right)$$

$(\gamma_5^{+} = \gamma_5$
$\gamma_0\gamma_5 = -\gamma_5\gamma_0)$

$$\Rightarrow \quad \bar\Psi \gamma_\mu \Psi \to \bar\Psi (1 - i \tfrac{\vec\tau}{2}\vec\Theta_A \gamma_5)\gamma_\mu(1 - i \tfrac{\vec\tau}{2}\vec\Theta_A \gamma_5)\Psi$$
$$= \bar\Psi(\gamma_\mu - i \tfrac{\vec\tau}{2}\vec\Theta_A \gamma_5\gamma_\mu - i \tfrac{\vec\tau}{2}\vec\Theta_A \gamma_\mu\gamma_5)\Psi = \bar\Psi\gamma_\mu\Psi$$

conserved axial vector current: $\bar\Psi \gamma_\mu \gamma_5 \frac{\tau^i}{2}\Psi$ (PCAC)

mass term: $\bar\Psi\Psi$ breaks $SU(2)_A$ explicitely!!

<u>Hypothesis:</u> (Nambu) $SU(2)_A$ broken spontaneously through condensation of u, d into pions
($\alpha_S \nearrow \Rightarrow$ „<u>dynamical</u>" symmetry breaking)

- pions are interpreted as Goldstone-bosons of this global $SU(2)_A$ broken symmetry!

- <u>Pseudo-Goldstone</u> bosons, because $m_u, m_d \neq 0$

$$m_u \bar u u + m_d \bar d d = \tfrac{1}{2}(m_u + m_d)\bar\Psi\Psi + \tfrac{1}{2}(m_u - m_d)\bar\Psi\tau_3\Psi$$

- breaks $SU(2)_A$
- conserves $SU(2)_V$

- breaks $SU(2)_V$ <u>isospin</u>

very small effect

- $SU(2)_L \times SU(2)_R \to SU(2)_V$

- expect $\partial_\mu j^\mu_{A} \propto m_u + m_d \propto m_\pi^2$ PCAC

  $\partial_\mu j^{\mu,V} \propto (m_u - m_d)$ CVC

•) vacuum : $\langle 0| \begin{pmatrix} \bar{u}_L u_R & \bar{u}_L d_R \\ \bar{d}_L u_R & \bar{d}_L d_R \end{pmatrix} |0\rangle$ (L and R in tract, need same trafo for both...)

$$= \underbrace{\tfrac{1}{2}(\bar{u}_L u_R + \bar{d}_L d_R)}_{\sim f_\pi} + \underbrace{\tfrac{1}{2}(\bar{u}_L u_R - \bar{d}_L d_R)\tau_3}_{\pi^0} + \underbrace{\bar{u}_L d_R \tau^-}_{\pi^-} + \underbrace{\bar{d}_L u_R \tau^+}_{\pi^+}$$

•) this is all key observation for "chiral perturbation theory"

write $\mathcal{U} = \exp\left\{ \frac{i}{f_\pi} \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \right\}$; $\chi = \begin{pmatrix} m_\pi^2 & 0 \\ 0 & m_\pi^2 \end{pmatrix}$

$$\mathcal{L} = \frac{c}{4} \mathrm{Tr}\left\{ (\partial_\mu \mathcal{U})(\partial^\mu \mathcal{U})^\dagger \right\} + \frac{\tilde{c}}{4} \mathrm{Tr}\left\{ \chi \mathcal{U}^\dagger + \mathcal{U}\chi^\dagger \right\}$$

with trafo $\underset{SU(2)_R \text{ trafo}}{R}\ \mathcal{U}\ \underset{SU(2)_L \text{ trafo}}{L^\dagger}$

replace QCD by effective theory consisting of hadrons, but consistent with symmetries of original theory (Weinberg)

•) pions have negative parity : ⇒ matrix element of axial current, pion and vacuum,

$$\langle 0 | j_A^{\mu i} | \pi^j(q) \rangle = -i f_\pi q_\mu \delta^{ij} e^{-iqx} ,$$

governs weak decay of pions ($f_\pi \simeq 93$ MeV)

[recall:  $\mathcal{M} \propto G_F \bar{u}(\mu^-) \gamma_\mu (1-\gamma_5) v(\bar{\nu}_\mu) \; (\;)^\mu$

•) must be V or A
•) spinless

$$\Rightarrow (\;)^\mu = q^\mu f_\pi(q^2)$$]

take divergence:
$$\boxed{\langle 0 | \partial^\mu j_A^{\mu i} | \pi^j(q) \rangle = - f_\pi q^2 \delta^{ij} e^{-iqx} = - f_\pi m_\pi^2 \delta^{ij} e^{-iqx}}$$

PCAC-relation

approximately conserved (cf. with m_{proton})

One is tempted to assume that $j_{A,pion}^{\mu i} = f_\pi \partial^\mu \phi^i(x)$

↓ pion field

axial vector current is identified with pion field ($\partial_\mu j_A^\mu \propto \Box \phi^i \propto m_\pi^2$)

•) Axial current of nucleon

$$A_\mu^a = g_A \bar{\Psi}_N \gamma_\mu \gamma_5 \frac{\tau^a}{2} \Psi_N$$

$N = p, n$

$g_A = 1.25$

$$\Rightarrow \partial^\mu A_\mu^a = g_A \bar{\Psi}_N \not{\partial} \gamma_5 \frac{\tau^a}{2} \Psi_N = i g_A M_N \bar{\Psi}_N \gamma_5 \tau^a \Psi_N \neq 0$$

include pion contribution:

$$A_\mu^a = g_A \bar{\Psi}_N \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi_N + f_\pi \partial_\mu \phi^a$$ and require $\partial^\mu A_\mu = 0$

$$\Rightarrow \Box \phi^a = -i g_A \frac{M_N}{f_\pi} \bar{\Psi}_N \gamma_5 \tau^a \Psi_N$$

is Klein-Gordon equation for massless pion coupled to nucleon

include ~~$\partial^\mu A_\mu$~~ pion mass:

$$(\Box + m_\pi^2)\, \phi^a = -g_A i \frac{M_N}{f_\pi} \bar{\Psi}_N \gamma_5 \tau^a \Psi_N$$

$$\Rightarrow \boxed{g_{\pi NN} = g_A \frac{M_N}{f_\pi}}$$

$(\simeq 13)$

Goldberger-Treiman relation

Literature: nucl-th/9706075