The Standard Model of Particle Physics II: Theory

0) Preliminaries + Organization

- English / German ?

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- Exercises: Pascal Humbert (MPIK)

  Thursdays, $9^{15} - 10^{45}$ kHS (Phil 12)
  (time and date okay?) → bring solutions to exercises!

- 10 - 12 exercise sheets, solve ≥ 50% of exercises, be regularly in lecture + exercise

  ⇒ win 4 CP for this MVSpec

- literature: webpage, lecture; books, review articles

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- depth and content adjusted to audience
  ⇒ fill out sheet „related lectures"

- ∄ script... Volunteers ?

Goal of Lecture

→ repeat / recall / introduce structure of the Standard Model (SM)

→ explore some details, in particular Higgs

→ explain why we are not happy with SM

→ explore necessary or well-motivated extensions

→ get glimps on how current research in particle theory works

⇒ preliminary outline:

I The Standard Model

structure / Gauge theories / Problems

II Electroweak Symmetry Breaking and the Higgs-Boson

Spontaneous Symmetry Breaking / Higgs properties + interpretation / Alternatives?

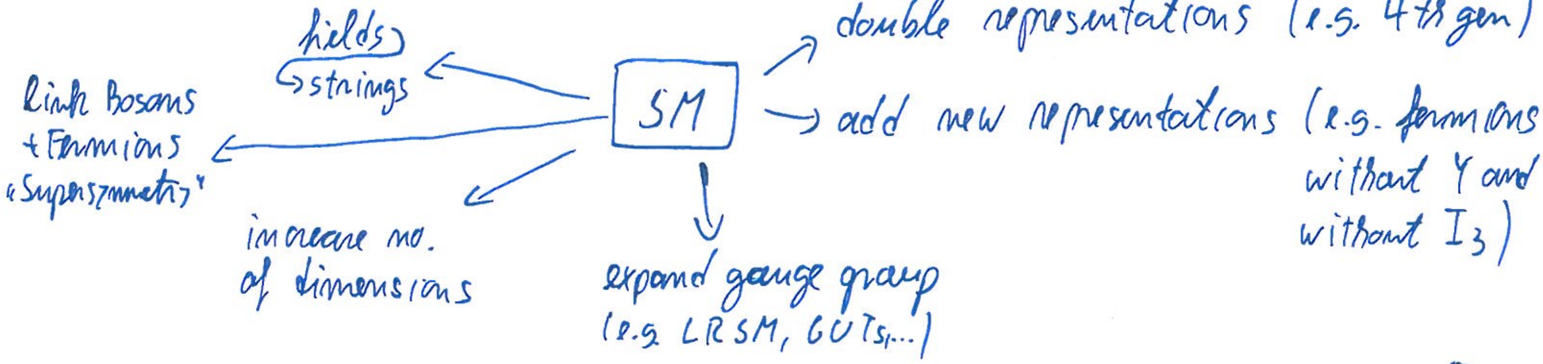
III Neutrino Masses

Oscillations / Dirac vs. Majorana / Flavor Symmetries

IV Extensions of the SM

Two Higgs Doublets / Z' / Left-right symmetric models / ...

(Dark matter)



$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H\partial_\mu H -$$

$$\frac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] -$$

$$igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2}g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$

$$g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$

$$gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$

$$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$

$$W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial +$$

$$m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right] + \frac{ig}{4c_w}Z^0_\mu\left[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2-\gamma^5)d^\lambda_j)\right] +$$

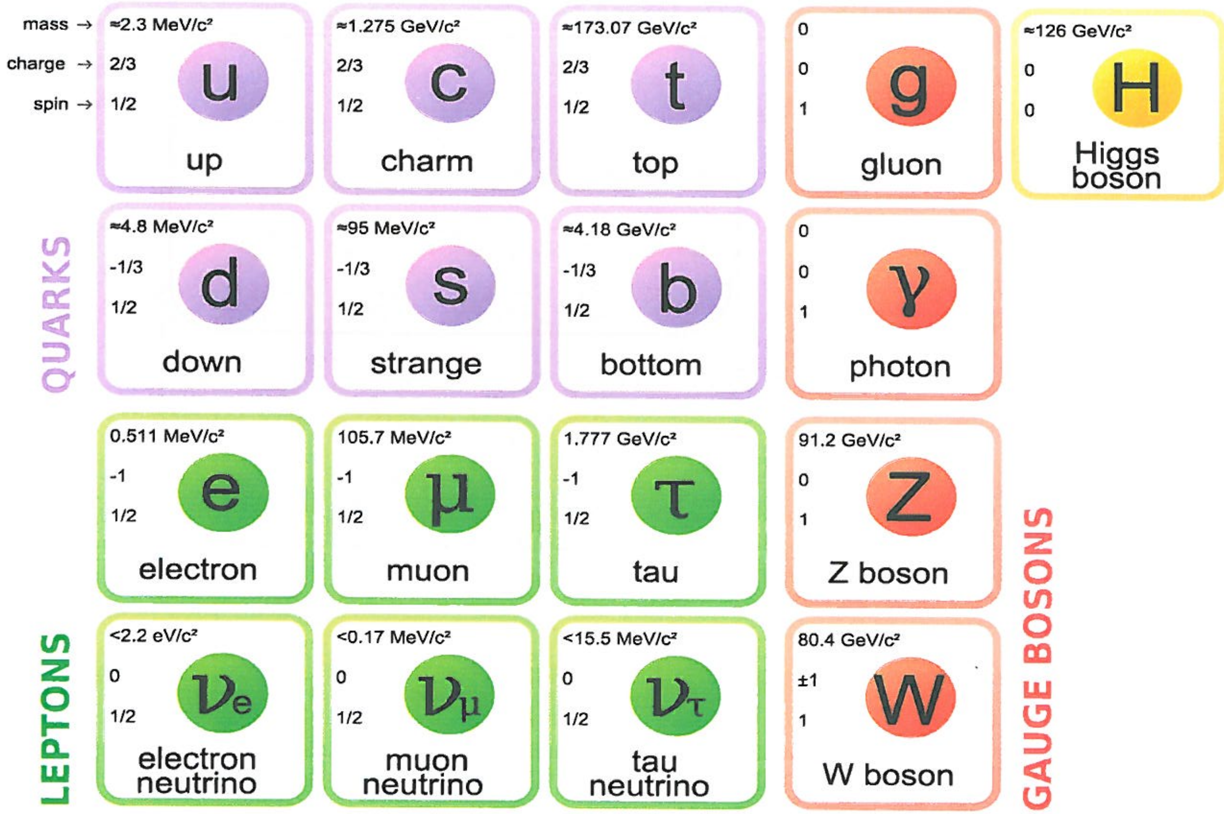
$$\frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) +$$

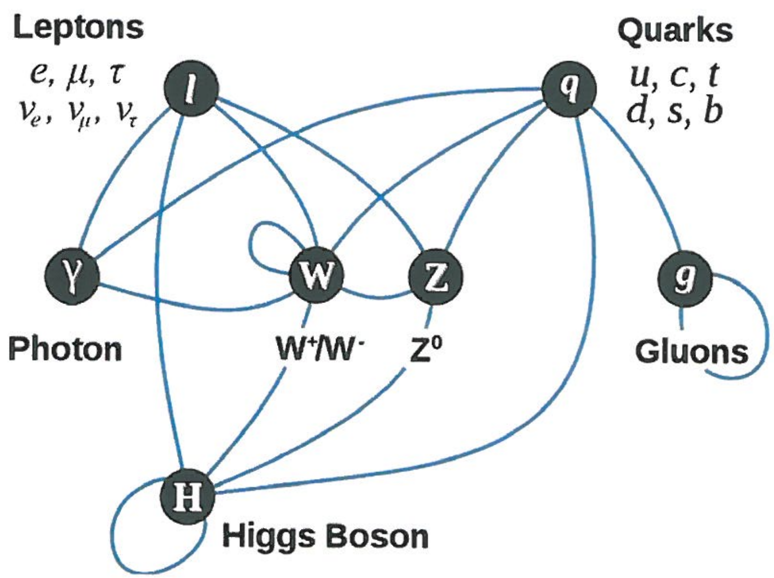
$$i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)] -$$

$$\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2-M^2)X^+ + \bar{X}^-(\partial^2-M^2)X^- + \bar{X}^0(\partial^2-\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y +$$

$$igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) -$$

$$\frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]$$





I The Standard Model

I 1) Basic Ingredients

quantum fields in $d=4$ dimensions

fermions ψ ; $[\psi] = 3/2$

vector bosons A_μ ; $[A_\mu] = 1$

scalar bosons ϕ ; $[\phi] = 1$

described by equations of motions, which are obtained from <u>Lagrange functions</u>

$$\boxed{\mathcal{L} = \mathcal{L}(\psi, \partial_\mu \psi)} \quad ; \quad S = \int d^4x \, \mathcal{L} \quad \text{„action"}$$

→ principle of least action: $\delta S = 0$

$$\Rightarrow \boxed{\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = \frac{\partial \mathcal{L}}{\partial \psi}}$$

Euler-Lagrange equation

Examples: •) $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi = \boxed{\bar{\psi}(i\not{\partial} - m)\psi}$ (*)

$= \bar{\psi}(\not{p} - m)\psi$

$\xrightarrow[EL]{} (\not{p} - m)\psi = 0$ Dirac-equation

•) $\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi)(\partial^\mu \phi) - m^2 \phi^2\right]$

$\xrightarrow[EL]{} (\partial_\mu \partial^\mu + m^2)\phi = 0$ Klein-Gordon

•) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\xrightarrow[EL]{} \partial_\mu F^{\mu\nu} = j^\nu$ Maxwell

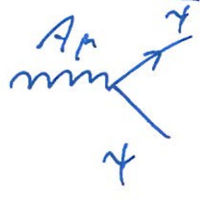
Remarks: •) in $\mathcal{L}$, only field and derivative

•) $\mathcal{L}$ is not unique

•) $\mathcal{L} = T - V$

•) Feynman rules from $\mathcal{L}$, e.g.

$\mathcal{L} = \bar{\psi}(\partial_\mu - ie A_\mu)\gamma^\mu \psi$



$ie\gamma^\mu$

I2) Gauge Invariance

a) Global gauge invariance

(*) on page (2) is invariant under $\psi \to \psi' = e^{i\alpha}\psi$
$\simeq (1+i\alpha)\psi$

$$\Rightarrow \delta\mathcal{L} \overset{!}{=} 0 = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) \quad \left[\text{use } \delta\psi = \psi' - \psi = i\alpha\psi\right]$$

$$= \frac{\partial\mathcal{L}}{\partial\psi} i\alpha\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} i\alpha\,\partial_\mu\psi$$

$$= i\alpha\underbrace{\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]}_{=0}\psi + i\alpha\,\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right]$$

$$\Rightarrow \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right] = 0 \quad \Rightarrow \partial_\mu \bar\psi\gamma^\mu\psi = 0$$

conserved current $\bar\psi\gamma^\mu\psi$

$\hookleftarrow$ Noether theorem

$\updownarrow$

global symmetry

Note
-) use $e^{i\alpha\hat{Q}}$: $\hat{Q}\psi = e\psi$ charge operator: $e\bar\psi\gamma^\mu\psi$
-) infinitesimal trafo is enough
-) α = const : "global gauge trafo"

b) Local gauge invariance

$\alpha = \text{const} \longrightarrow \alpha = \alpha(x) \Rightarrow \psi \to \psi e^{i\alpha(x)}$ leads to terms with $\partial_\mu \alpha$

Solution: $\partial_\mu \to D_\mu$ „covariant derivative"

$$\bar{\psi}(i\partial\!\!\!/ - m)\psi \xrightarrow{\partial \to D} \bar{\psi}(i D\!\!\!\!/ - m)\psi$$

make gauge trafo: $\bar{\psi} e^{-i\alpha}(i D'_\mu \gamma^\mu - m) e^{i\alpha}\psi$

$\Rightarrow$ is invariant for $D'_\mu \psi' = e^{i\alpha} D_\mu \psi$

this is achieved for

$$\boxed{\begin{aligned} D_\mu &= \partial_\mu - ie A_\mu \\ A_\mu \to A'_\mu &= A_\mu + \tfrac{1}{e}\partial_\mu \alpha \end{aligned}}$$

proof:
$$\begin{aligned} D'_\mu \psi' &= (\partial_\mu - ie A_\mu - i\partial_\mu \alpha) e^{i\alpha}\psi \\ &= i(\partial_\mu \alpha) e^{i\alpha}\psi + e^{i\alpha}(\partial_\mu \psi) - ie A_\mu e^{i\alpha}\psi \\ &\quad - i(\partial_\mu \alpha) e^{i\alpha}\psi \\ &= e^{i\alpha}(\partial_\mu - ie A_\mu)\psi = e^{i\alpha} D_\mu \psi \end{aligned}$$

☺

(also write as $D'_\mu = e^{i\alpha} D_\mu$)

⇒ new Lagrange density

$$\mathcal{L} = i\bar{\Psi} \not{D} \Psi - m\bar{\Psi}\Psi$$

$$= \bar{\Psi}(\not{\partial} - m)\Psi + \underbrace{e\bar{\Psi}\gamma^\mu\Psi}_{j^\mu} A_\mu$$

(use charge operator useful for particles with different charge ...)

*) ∃ another gauge invariant term:

$$F'_{\mu\nu} = (\partial_\mu A'_\nu - \partial_\nu A'_\mu) = \partial_\mu (A_\nu + \tfrac{1}{e}\partial_\nu\alpha) - \partial_\nu (A_\mu + \tfrac{1}{e}\partial_\mu\alpha)$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + \tfrac{1}{e}\underbrace{(\partial_\mu\partial_\nu\alpha - \partial_\nu\partial_\mu\alpha)}_{=0}$$

$$= F_{\mu\nu}$$

is gauge invariant

⇒ include term $F_{\mu\nu}F^{\mu\nu}$

(All terms that can be written down <u>must</u> be written down!)

*) $\frac{i}{e}[D_\mu, D_\nu] = \frac{i}{e}[\partial_\mu - ieA_\mu, \partial_\nu - ieA_\nu]$

$= \frac{i}{e}(-ie)\{[A_\mu, \partial_\nu] + [\partial_\mu, A_\nu]\}$

$= A_\mu\partial_\nu - (\partial_\nu A_\mu) - A_\mu\partial_\nu + (\partial_\mu A_\nu) + A_\nu\partial_\mu - A_\nu\partial_\mu$

$= F_{\mu\nu}$

*) $m_\gamma^2 A_\mu A^\mu \to m_\gamma^2 A'_\mu A'_\mu$ is not gauge invariant

$\Rightarrow$ Photon mass forbidden : $m_\gamma = 0$

($\Rightarrow$ stable ...)

$\Rightarrow$ total Lagrangian

$$\mathcal{L}_{tot} = \bar{\psi}(\not{p} - m)\psi + e\bar{\psi}\gamma_\mu A^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- $\bar{\psi}(\not{p} - m)\psi$ → free field
- $e\bar{\psi}\gamma_\mu A^\mu \psi$ → interaction with "gauge field"
- $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ → kinetic term of gauge field

$\longleftrightarrow$ QED

tested to 10^{-11} precision $\Rightarrow$ works!

⇒ <u>Lesson</u> local symmetry

↓ predicts

massless gauge field

\+ interactions between fermions and gauge field

this was an <u>Abelian</u> gauge group $U(1)$

$$e^{i\alpha(x)} e^{i\beta(x)} = e^{i\beta(x)} e^{i\alpha(x)}$$

⇒ try to generalize this further to describe the more complex weak and strong interactions

turns out: Nature has chosen $SU(2)$, $SU(3)$

„Non-Abelian" gauge groups