# Exercises to "Standard Model of Particle Physics"

Summer 2013

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann — Sheet 9 — 24.6.13

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**Exercise 17:** Electroweak interference **[15 Points]**

We haven't calculated a process for a while...
The process $e^-(p_1)\, e^+(p_2) \to \mu^-(k_1)\, \mu^+(k_2)$ has two $s$-channel diagrams, corresponding to photon and $Z$ exchange. Write down both amplitudes (ignore lepton masses) and calculate $d\sigma/d\Omega$. Show in particular that in the center-of-mass system

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(A_0(1+\cos^2\theta) + A_1\cos\theta\right)$$

What was the QED result again?

**Exercise 18:** Scalar fields **[10 Points]**

Take the standard Higgs doublet $\phi = (\phi_+,\, \phi_0)^T$, which has hypercharge $+1$, i.e. transforms under $SU(2)_W \times U(1)_Y$ like

$$\phi \to \phi' = \exp\left(-ig_Y(+1)\alpha(x)\mathbb{1}\right) \exp\left(-ig_W \sum_{a=1}^{3} \beta_a(x) T_a\right) \phi\,,$$

where $T_a = \sigma_a/2$. A doublet with hypercharge $-1$ can be obtained from $\phi$ by the transformation $\tilde{\phi} = -i\sigma_2\phi^*$. The most general gauge-invariant potential for $\phi$ can be written as

$$V = -\mu^2\phi^\dagger\phi + \lambda\phi^\dagger\phi\,\phi^\dagger\phi\,.$$

a) Show that the following terms can be reduced to the ones given in $V$:

$$\tilde{\phi}^\dagger\tilde{\phi}\,, \qquad \tilde{\phi}^\dagger\phi\,\phi^\dagger\tilde{\phi}\,,$$

and also argue why one can not write down an invariant term involving three fields.

b) Assume there was a new scalar $\Delta$ which transformed in the adjoint representation of $SU(2)_W$. This triplet can be conveniently written down as a $2\times 2$ matrix $\Delta \equiv \sum_a \Delta_a\sigma_a$, which transforms under $SU(2)_W$:

$$\Delta \to \exp\left(-ig_W \sum_{a=1}^{3} \beta_a(x) T_a\right)\, \Delta\, \exp\left(-ig_W \sum_{a=1}^{3} \beta_a(x) T_a\right)^\dagger\,.$$

From $2\otimes 2\otimes 3 = 1\oplus\ldots$ we know that there is just one gauge invariant cubic coupling of $\phi\phi\Delta$. Show that the following terms are invariant under $SU(2)_W$

$$\phi^\dagger\Delta\phi\,, \qquad \phi^\dagger\Delta\tilde{\phi}\,, \qquad \tilde{\phi}^\dagger\Delta\tilde{\phi}\,,$$

expand the expressions in component fields and determine which hypercharge $\Delta$ has to carry to make each term invariant under $SU(2)_W \times U(1)_Y$.

**Tutors:**
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Tutorials homepage: http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html

**Hand-in of sheet:**
during lecture on 1.7.

**Discussion of sheet:**
Thursday, 4.7. 2.15 pm, INF 227 SR 2.402
Friday, 5.7. 2.15 pm, INF 227 SR 1.403