Exercises to "Standard Model of Particle Physics"

Summer 2013

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Exercise 16: Stückelberg mechanism [**20 Points**]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method contains a real scalar field σ together with the Z'-boson associated to the $U(1)'$.

a) Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} + \frac{1}{2}(M_{Z'} Z'_\mu + \partial_\mu \sigma)(M_{Z'} Z'^\mu + \partial^\mu \sigma) + i\overline{\psi}\gamma^\mu(\partial_\mu - ig' Y' Z'_\mu)\psi - m\overline{\psi}\psi\,.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge Y') and gauge boson are given by

$$\psi \to e^{-ig'Y'\theta(x)}\psi\,, \qquad Z'_\mu \to Z'_\mu - \partial_\mu \theta(x)\,.$$

Calculate the gauge transformation of the real scalar σ that makes the Lagrangian invariant, and show the invariance of the other terms, too. Can you fix a gauge to eliminate σ from the theory? Count degrees of freedom in both gauges.

b) Consider an extension of the SM gauge group $G_{\rm SM} \equiv SU(3) \times SU(2) \times U(1)_Y \to G_{\rm SM} \times U(1)'$ with Lagrangian

$$\begin{aligned}\mathcal{L} \supset\ & -\frac{1}{4}\hat{B}^{\mu\nu}\hat{B}_{\mu\nu} - \frac{1}{4}\hat{W}_3^{\mu\nu}\hat{W}_{3\mu\nu} + \frac{1}{2}\hat{M}_Z^2\left(\hat{c}_W \hat{W}_3^\mu - \hat{s}_W \hat{B}^\mu\right)^2 \\ & -\frac{1}{4}\hat{Z}'^{\mu\nu}\hat{Z}'_{\mu\nu} - \frac{\sin\chi}{2}\hat{B}^{\mu\nu}\hat{Z}'_{\mu\nu} + \frac{1}{2}\left(\partial_\mu\sigma + \hat{M}_{Z'}\hat{Z}'_\mu + \hat{M}_B \hat{B}_\mu\right)^2\,,\end{aligned}$$

where σ is charged under both $U(1)'$ (with gauge boson $\hat{Z}'$) and $U(1)_Y$ (with gauge boson $\hat{B}'$). Find again how σ has to transform under $U(1)_Y \times U(1)'$ for the Lagrangian to be invariant and show that the $\sin\chi$ term is gauge invariant. In the SM, you find the Z boson $Z = \hat{c}_W \hat{W}_3^\mu - \hat{s}_W \hat{B}^\mu$ and the massless photon $\gamma = \hat{s}_W \hat{W}_3^\mu + \hat{c}_W \hat{B}^\mu$, but the extra neutral gauge boson $\hat{Z}'$ changes these results. To find the gauge boson mass eigenstates, we need to diagonalize the kinetic terms first (if the "kinetic mixing angle" χ is nonzero). To do this, find a (non-unitary) transformation matrix R that transforms $(\hat{B}, \hat{W}_3, \hat{Z}')$ into $(\tilde{B}, \tilde{W}_3, \tilde{Z}')$ with the properly normalized "diagonal" kinetic terms

$$\mathcal{L} \supset\ -\frac{1}{4}\tilde{B}^{\mu\nu}\tilde{B}_{\mu\nu} - \frac{1}{4}\tilde{W}_3^{\mu\nu}\tilde{W}_{3\mu\nu} - \frac{1}{4}\tilde{Z}'^{\mu\nu}\tilde{Z}'_{\mu\nu}\,.$$

Hint: $\hat{W}_3 = \tilde{W}_3$.

c) The fields $\tilde{B}$, $\tilde{W}_3$ and $\tilde{Z}'$ share a non-diagonal mass matrix. Write it down and show that it has at least one zero eigenvalue, i.e. there is still a massless photon in the model. Calculate the other two masses for the case $\chi = 0$ and expand in $\hat{M}_B \ll \hat{M}_Z < \hat{M}_{Z'}$.

Exercise 17: Effects of Z–Z' mixing [**10 Points**]

The general effective Lagrange density after breaking the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ symmetry to $SU(3)_C \times U(1)_{\text{EM}}$ can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{Z'} + \mathcal{L}_{\text{mix}} \,,$$

where the relevant part of the Standard Model Lagrangian is

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{a\mu\nu} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{\hat{e}}{\hat{c}_W} j_B^\mu \hat{B}_\mu - \frac{\hat{e}}{\hat{s}_W} j_W^{a\mu}\hat{W}^a_\mu \,,$$

and the hats merely denote that the fields are not mass eigenstates. The Z' part reads

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_Z'^2\hat{Z}'_\mu\hat{Z}'^\mu - \hat{g}' j'^\mu Z'_\mu \,,$$

and the kinetic- and mass-mixing terms can be parameterized as

$$\mathcal{L}_{\text{mix}} = -\frac{\sin\chi}{2}\hat{Z}'^{\mu\nu}\hat{B}_{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu \,.$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. Set the kinetic mixing angle χ to zero for simplicity.

b) Since the mass of the physical Z boson changes compared to the SM, the so-called ρ parameter $\rho = M_W^2/M_Z^2 c_W^2$ is no longer equal to one. Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z–Z' mixing.

Tutors:
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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html`

Hand-in of sheet:
during lecture on 24.6.

Discussion of sheet:
Thursday, 27.6. 2.15 pm, INF 227 SR 2.402
Friday, 28.6. 2.15 pm, INF 227 SR 1.403