Exercises to "Standard Model of Particle Physics"

Summer 2013

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann   Sheet 6   3.6.13

Exercise 13: $SU(N)$ [**10 Points**]

Let $U \in SU(N)$, i.e. $\det U = 1$ and $U^\dagger U = \mathbb{1}$. Any element of $SU(N)$ can be written as $U = \exp(-i\theta^a T^a)$, where the T^a are generators of the group with normalization $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$.

a) Show that the T^a are traceless hermitian matrices.

b) How many linear independent generators are there?

c) The structure constants, d_{abc} and f_{abc}, are defined through

$$[T^a, T^b] = i f_{abc} T^c, \qquad \{T^a, T^b\} = \frac{1}{N}\delta_{ab} + d_{abc} T^c \ .$$

Show that

$$\mathrm{Tr}(T^a T^b T^c) = \frac{1}{4}(d_{abc} + i f_{abc}) \ ,$$
$$\left[\sum_a T^a T^a, T^b\right] = 0 \ .$$

d) Calculate the f_{abc} for

$$T_a = \frac{\sigma_a}{2} \ ,$$

where σ_a are Pauli matrices.

Exercise 14: Higgs sector and gauge bosons [**10 Points**]

a) Suppose a more exotic Higgs sector, namely that the Higgs field has isospin 3 and hypercharge -4. If the neutral component (which has $I_3 = 2$) acquires a vev $v/\sqrt{2}$, show that

$$m_W^2 = \frac{g^2}{2}\Phi^\dagger(T^+T^- + T^-T^+)\Phi = 4g^2v^2$$

and that $\rho = 1$ (T^+ and T^- are the "ladder operators" used in the lecture).

b) If there are several representations of Higgs scalars, the ρ-parameter is given by (as a bonus you can try, using a), to calculate this expression)

$$\rho = \frac{\Sigma v_i^2 \left(T_i(T_i+1) - \frac{1}{4}Y_i^2\right)}{\Sigma \frac{1}{2} v_i^2 Y_i^2}$$

Here T_i is the isospin of the scalar, Y_i the hypercharge and v_i the vev of the neutral component. Show that $\rho = 1$ if only Higgs doublets with $Y_i = 1$ exist. From the observational constraint $|\rho - 1| < \epsilon \ll 1$, give a constraint on the vev of a scalar triplet.

c) From the kinetic term of the Standard Model Higgs one can obtain the hW^+W^- and hhW^+W^- couplings. Show that they are given as igm_W and $\frac{1}{4}ig^2$, respectively. If you wish, derive the hZZ and $hhZZ$ couplings.

Bonus Exercise: Gauge invariance [**5 Points**]

Show that the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |(\partial_\mu - igA_\mu)\Phi|^2 - V(\Phi^\dagger \Phi)$$

is invariant under the transformations

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \alpha(x)$$
$$\Phi \to \Phi' = e^{ig\alpha(x)} \, \Phi \, .$$

Tutors:
Julian Heeck, email: julian.heeck@mpi-hd.mpg.de
He Zhang, email: he.zhang@mpi-hd.mpg.de

Tutorials homepage: http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html

Hand-in of sheet:
during lecture on 10.6.

Discussion of sheet:
Thursday, 13.6. 2.15 pm, INF 227 SR 2.402
Friday, 14.6. 2.15 pm, INF 227 SR 1.403