Exercises to “Standard Model of Particle Physics”

Summer 2013

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann Sheet 5 27.5.13

Exercise 10: W-polarization [**10 Points**]

For a W-boson with 4-momentum $p = (E, \boldsymbol{p})$, moving along the z-axis, the polarization vectors for the helicities $\lambda = 0, \pm 1$ can be written as

$$\epsilon^{\mu}_{\lambda=0} = \tfrac{1}{m_W}(|\boldsymbol{p}|, 0, 0, E)$$
$$\epsilon^{\mu}_{\lambda=\pm 1} = \tfrac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

Check if the “completeness relation“

$$\sum_{\lambda} \epsilon^{*}_{\mu}\, \epsilon_{\nu} = -g_{\mu\nu} + \frac{p_\mu\, p_\nu}{m_W^2}$$

is fulfilled.

Exercise 11: Pion Decay [**5 Points**]

Calculate the decay width for pion decay $\pi^-(p) \to e^-(p_1) + \bar{\nu}_e(p_2)$, and derive the ratio of this decay width with the one for $\pi^- \to \mu^- + \bar{\nu}_\mu$. Comment on the result.

Exercise 12: S-Matrix [**10 Points**]

a) Show that from the unitarity of the S-matrix the following condition follows:

$$T_{fi} - T^{*}_{if} = i(2\pi)^4\, \Sigma_n \delta(P_f - P_n) T_{fn}\, T^{*}_{in}$$

b) for $i = f$, i.e. elastic forward scattering ($\theta = 0$) with 2 particles a and b in the initial state, one can show in scattering theory that:

$$\mathrm{Im}\, \mathcal{M}_{ii} = \sqrt{\lambda(s, m_a^2, m_b^2)}\, \sigma_{\mathrm{tot}}$$

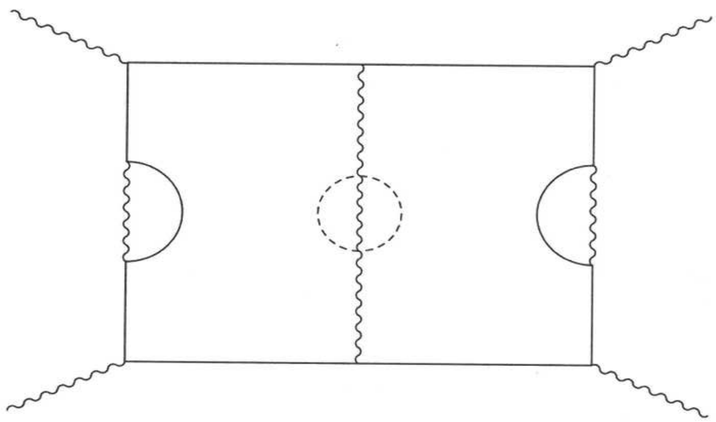
(this is called the optical theorem). With the partial wave decomposition of a matrix element,

$$\mathcal{M} = 16\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(\cos\theta)\, a_l\,,$$

with P_l the Legendre polynomials, θ the scattering angle, and a_l the partial waves, show that $|\mathrm{Re}\{a_0\}| \le \frac{1}{2}$. Repeat the argument from the lecture that Fermi theory runs into problems at a certain energy scale.

Champions League Exercise:

Calculate the following loop-corrected diagram for the process $\gamma\gamma \to \gamma\gamma$:



Tutors:
Julian Heeck, email: `julian.heeck@mpi-hd.mpg.de`
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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html`

Hand-in of sheet:
during lecture on 3.6.

Discussion of sheet:
Thursday, 6.6. 2.15 pm, INF 227 SR 2.402
Friday, 7.6. 2.15 pm, INF 227 SR 1.403