Exercises to "Standard Model of Particle Physics"
Summer 2012

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann   Sheet 4   13.5.13

Exercise 7: γ-matrices, Trace theorems, etc. [**10 Points**]

Using the relations $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, show that

a) $\gamma_5^2 = 1$

b) the trace of an odd number of γ-matrices vanishes

c) $\text{Tr}\{\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\} = 4(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})$

d) $\text{Tr}\{\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\} = 4i\,\epsilon_{\mu\nu\rho\sigma}$

e) $\gamma_\mu\gamma_\nu\gamma^\mu = -2\gamma_\nu$

f) $\gamma_\mu\gamma_\nu\gamma_\sigma\gamma^\mu = 4g_{\nu\sigma}$

g) $\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma^\mu = -2\gamma_\sigma\gamma_\rho\gamma_\nu$

Exercise 8: Muon Decay [**10 Points**]

Calculate the decay width for muon decay, $\mu(p) \rightarrow e(p_1) + \bar{\nu}_e(p_2) + \nu_\mu(p_3)$. Neglect all masses except for the muon mass, and see Exercise 3c) for the phase space. Estimate from the result the decay width of $\tau \rightarrow \mu\nu\nu$ and compare with the experimental result (check our for instance the "particle data group").

Exercise 9: Dirac spinors, helicity, spin, chirality, yadayadayada [**10 Points**]

The Hamiltonian of the Dirac equation can be written as

$$i\,\partial_t\Psi = H\,\Psi \text{ with } H = \gamma_0\,\vec{\gamma}\vec{p} + \gamma_0\,m$$

a) Show that for a free and massive fermion, chirality is not a good quantum number.

b) Show that helicity is a good quantum number. For this you need the helicity operator

$$h = \gamma_5\,\gamma_0\,\vec{\gamma}\,\frac{\vec{p}}{|\vec{p}|}$$

c) Show that for massless particles helicity and chirality coincide.

d) Do 20 sit-ups!

Tutors:
Julian Heeck, email: `julian.heeck@mpi-hd.mpg.de`
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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html`

Hand-in of sheet:
during lecture on 22.5.

Discussion of sheet:
The Thursday group discusses it on Friday, 24.05. 2.15 pm, INF 227 SR 1.403
The Friday group has no tutorial on 24.5.
Friday, 31.05. 2.15 pm, INF 227 SR 1.403