Exercises to "Standard Model of Particle Physics"

Summer 2013

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann Sheet 3 6.5.13

Exercise 5: It's a photon! [**20 Points**]

a) as a start, recall that the action is given as

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

where the Lagrangian $\mathcal{L}$ depends on some field ϕ and its derivative. By varying the action and setting $\delta S = 0$, derive the Euler-Lagrange equation

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial \mathcal{L}}{\partial \phi}$$

b) Using the QED Lagrangian given in the lecture, apply the Euler-Lagrange equation to the photon field A_μ and find that

$$\partial^\mu F_{\mu\nu} = j_\nu$$

Show that this current is conserved.

c) Since Maxwell's equations are unchanged when A_μ is replaced with $A_\mu + \partial_\mu \chi$, where χ is an arbitrary function of x, we can further simplify the result from b). Show that one can always find a χ to enforce the Lorenz condition

$$\partial^\mu A_\mu = 0$$

so that the Maxwell equations reduce to $\partial^\mu \partial_\mu A^\nu = j^\nu$.

d) Show that one can still make the (gauge) transformation $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$, as long as $\partial^\mu \partial_\mu \Lambda = \Box \Lambda = 0$.

e) A free photon is described by $A^\mu = \epsilon^\mu(q) e^{-iqx}$. Using c) and d), show that the polarization vector ϵ^μ has only two independent entries. In particular, the zeroth component of ϵ_μ can be set to zero. Show further that the replacement

$$\epsilon_\mu \to \epsilon'_\mu = \epsilon_\mu + a q_\mu$$

with constant a, leaves physics unchanged.

f) a poor men's definition of a propagator is that it is the inverse of the equation of motion in momentum space. So, for the equation of motion from b), $\partial^\mu \partial_\mu A^\nu = \Box A^\nu = j^\nu$, we can write $g^{\rho\nu} \Box A_\rho = j^\nu$ and can get the propagator $X_{\nu\mu}$ from the condition

$$-g^{\rho\nu} q^2\, X_{\nu\mu} = \delta^\rho_\mu$$

Write $X_{\nu\mu}$ as a linear combination of $g_{\nu\mu}$ and $q_\nu q_\mu$ and derive the propagator. If we wouldn't have put in the Lorenz condition, what would have happened?

g) actually, the most general possible equation of motion goes like

$$\left(g^{\rho\nu}\Box - (1 - \frac{1}{\xi})\partial^\rho \partial^\nu \right) A_\rho = j^\nu$$

Derive the propagator. Comment on the term proportional to ξ.

h) Massive vector bosons can be described by replacing $\Box$ with $\Box + M^2$, thus

$$\left(g^{\rho\nu}(\Box + M^2) - \partial^\rho \partial^\nu \right) A_\rho = j^\nu$$

Derive the propagator.

Exercise 6: Moller scattering [**10 Points**]

Consider $e^-(p_1) + e^-(p_2) \to e^-(p_3) + e^-(p_4)$ and evaluate the spin-averaged matrix element $|\bar{\mathcal{M}}|^2$. What are the crossed processes?

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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html`

Hand-in of sheet:
during lecture on 13.5.

Discussion of sheet:
Thursday, 23.05. 2.15 pm, INF 227 SR 2.402
Friday, 17.05. 2.15 pm, INF 227 SR 1.403