Exercises to "Standard Model of Particle Physics"

Summer 2013

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Exercise 3: Space phase [**20 Points**]

a) Show that

$$\int \mathrm{d}^4 p\, \delta(p^2 - m^2)\, \theta(E) = \int \mathrm{d}^3 p\, \frac{1}{2E}\,,$$

and therefore that the r.h.s. is Lorentz-invariant.

b) Show that for the 2-to-2 scattering $a + b \to 1 + 2$ in the center-of-mass system the following relation for the phase space holds:

$$(2\pi)^2\, \mathrm{d}\Phi_2 = \int \frac{\mathrm{d}^3 p_1}{2E_1} \frac{\mathrm{d}^3 p_2}{2E_2}\, \delta(p_a + p_b - p_1 - p_2) = \int \mathrm{d}\Omega \frac{|\vec{p}_1|}{4\sqrt{s}}\,.$$

c) Consider an unpolarized 3-body decay $p \longrightarrow p_1 + p_2 + p_3$, e.g. β-decay of a muon: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. Show that when $m_\mu \gg m_{e,\nu}$ the phase space in the rest system of the decaying particle is

$$\int d^4 p_1\, d^4 p_2\, d^4 p_3\, \delta(p_1^2 - m_1^2)\, \delta(p_2^2 - m_2^2)\, \delta(p_3^2 - m_3^2)\, \Theta(E_1)\, \Theta(E_2)\, \Theta(E_3)\, \delta(p_1 + p_2 + p_3 - p)$$

$$= \pi^2 \int\limits_0^{m_\mu/2} dE_1 \int\limits_{m_\mu/2 - E_1}^{m_\mu/2} dE_3\,.$$

Exercise 4: Dirac equation stuff [**10 Points**]

Let us recall some properties of the Dirac equation:

a) Show that $\Lambda_\pm = (\pm \not{p} + m)/(2m)$, when acting on a general solution of the Dirac equation, are projection operators for positive and negative energy states.

b) Prove the Gordon decomposition(s)

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f \left[(p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu \right] u_i$$

$$0 = \bar{u}_f \left[(p_f - p_i)^\mu + i\sigma^{\mu\nu}(p_f + p_i)_\nu \right] u_i$$

with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$.

c) if we deal (instead of fundamental fermions) with extended objects like hadrons, the electromagnetic current is no longer $\bar{u}\gamma^\mu u$, but rather $\bar{u}(p')\Gamma^\mu u(p)$, with

$$\Gamma^\mu = A(q^2)q^\mu + B(q^2)P^\mu + C(q^2)\gamma^\mu + D(q^2)i\sigma^{\mu\nu}q_\nu + E(q^2)i\sigma^{\mu\nu}P_\nu$$

where $q = p' - p$ and $P = p' + p$. Use the Gordon decompositions and the conservation of the current to show that only two of the five functions in Γ^μ are independent, and that one can write instead

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)i\sigma^{\mu\nu}q_\nu \,.$$

Tutors:
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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html`

Hand-in of sheet:
during lecture on 6.5.

Discussion of sheet:
Thursday, 16.05. 2.15 pm, INF 227 SR 2.402
Friday, 10.05. 2.15 pm, INF 227 SR 1.403