Exercises to "Standard Model of Particle Physics"

Summer 2013

Prof. Dr. Andre Schöning, Dr. Werner Rodejohann **FINAL SHEET!!** 1.7.13

Exercise 20: Z-decay **[15 Points]**

The Lagrangian for the coupling of a fermion pair f with the Z-boson is

$$\mathcal{L} = \frac{g}{2c_W} \overline{f} \gamma^\mu \left(v_f - a_f \, \gamma_5\right) f \, Z_\mu \, .$$

For neutrinos, we have $v_\nu = a_\nu = \frac{1}{2}$.

a) Calculate the decay width for $Z \to \bar{\nu}\nu$, keep a possible neutrino mass in the expression.

b) Neutrinos could be Majorana particles, which obey the relation $\nu^c = \nu$. Here the superscript c denotes charge conjugation,

$$\nu^c = C(\bar{\nu})^T \, ,$$

with $C = i\gamma_2\gamma_0$. Show the following properties

$$\begin{gathered} -C = C^T = C^{-1} = C^* = C^\dagger \, , \\ C^{-1}\gamma_\mu C = -\gamma_\mu^T \, , \quad C^{-1}\gamma_5 C = \gamma_5^T \, , \\ \overline{\psi^c} = -\psi^T C^{-1} \, , \quad (\psi_L)^c = (\psi^c)_R \, . \end{gathered}$$

c) Show with the above results that for Majorana neutrinos the vector current $\bar{\nu}\gamma_\mu\nu$ vanishes. What about $\bar{\nu}\gamma_\mu\gamma_5\nu$, $\bar{\nu}\gamma_5\nu$ and $\bar{\nu}[\gamma_\mu, \gamma_\nu]\nu$?

d) Using c), calculate the decay width $Z \to \nu\nu$ (now identical particles!) for Majorana neutrinos and compare with the result from a).

Exercise 21: Quark Flavors **[10 Points]**

The quark part of the QCD Lagrangian is

$$\mathcal{L}_Q = \bar{q}_\alpha^A(x) \, \left(i \not{D} - m_A\right) \, q_\alpha^A(x) \, .$$

One sums over the flavor index A and the Color index α. Consider the global transformations

$$\begin{aligned} q_\alpha^A(x) \quad &\to \quad q_\alpha'^A(x) = \left(e^{-i\theta^A T^A}\right)_{AB} \, q_\alpha^B(x) \, , \\ q_\alpha^A(x) \quad &\to \quad q_\alpha'^A(x) = \left(e^{-i\theta^A T^A \gamma_5}\right)_{AB} \, q_\alpha^B(x) \, . \end{aligned}$$

The T^A are generators of the (flavour–)$SU(N)$. When is $\mathcal{L}_Q$ invariant under these transformations and what are the conserved currents?

Exercise 08/15: FINAL EXERCISE!! [**−100 Points**]

Consider the following relation of charged lepton masses:

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} \,.$$

What is in general the smallest and largest possible value of Q? Calculate the value of Q with the charged lepton masses from the Particle Data Group. What's going on?

Tutors:

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Tutorials homepage: http://www.mpi-hd.mpg.de/manitop/StandardModel/exercise.html

Hand-in of sheet:

during lecture on 8.7.

Discussion of sheet:

Thursday, 11.7. 2.15 pm, INF 227 SR 2.402
Friday, 12.7. 2.15 pm, INF 227 SR 1.403