Introduction to Neutrino (Oscillation) Physics

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

STANDARD MODEL OF PARTICLE PHYSICS

RODEJOHANN/SCHOENING

16/07/12

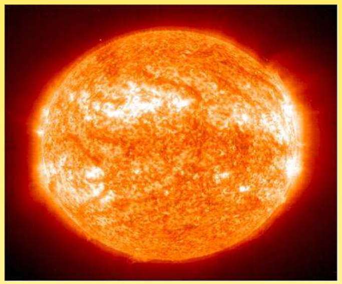
ν

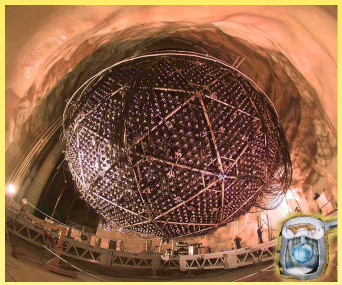
$m_\nu = m_L - m_D^T M_R^{-1} m_D$

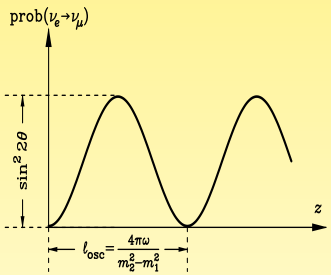
erc

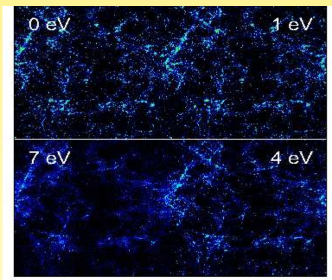
MANITOP

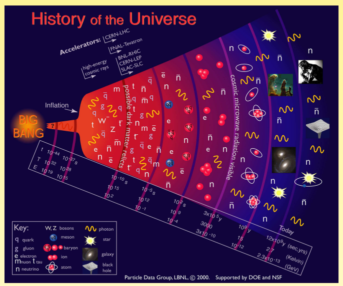
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology









Literature

- ArXiv:
  - Bilenky, Giunti, Grimus: *Phenomenology of Neutrino Oscillations*, `hep-ph/9812360`
  - Akhmedov: *Neutrino Physics*, `hep-ph/0001264`
  - Grimus: *Neutrino Physics – Theory*, `hep-ph/0307149`
- Textbooks:
  - Fukugita, Yanagida: *Physics of Neutrinos and Applications to Astrophysics*
  - Kayser: *The Physics of Massive Neutrinos*
  - Giunti, Kim: *Fundamentals of Neutrino Physics and Astrophysics*
  - Schmitz: *Neutrinophysik*

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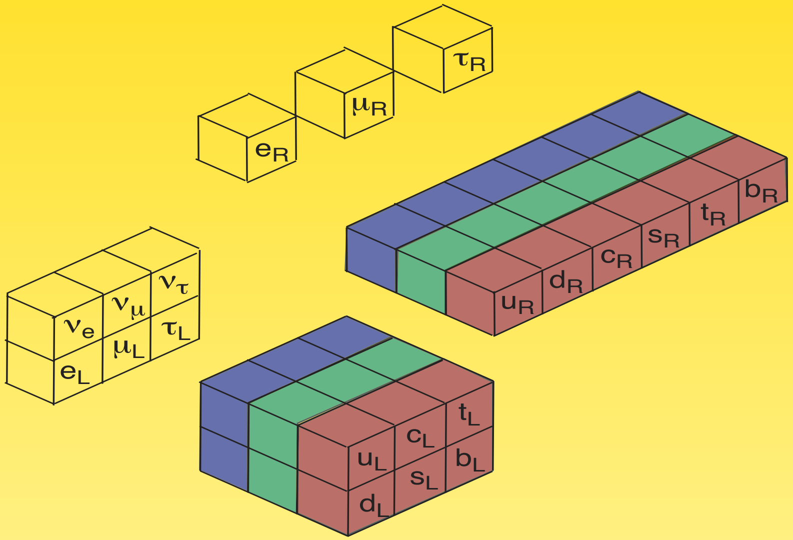
I Basics

I1) Introduction

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	$\#$	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

18 free parameters...

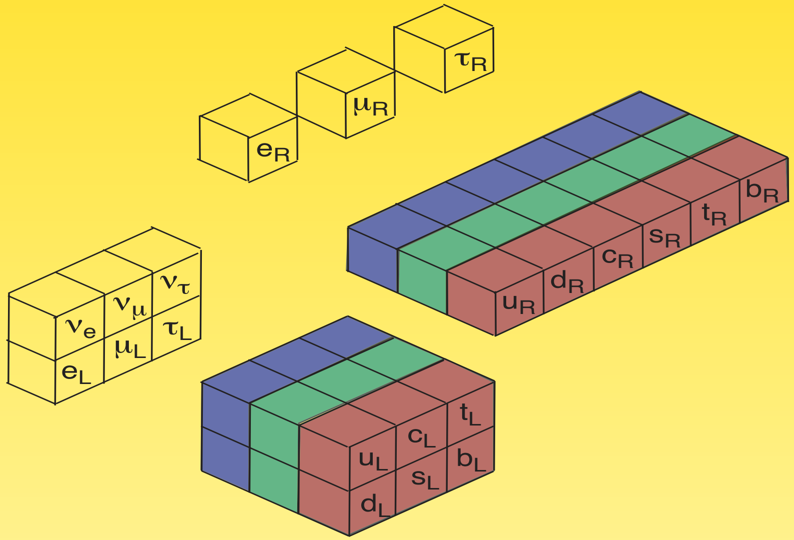
+ Dark Matter

+ Gravitation

+ Dark Energy

+ Baryon Asymmetry

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
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$+$ Neutrino Mass m_ν

Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	$\#$	$\sum$
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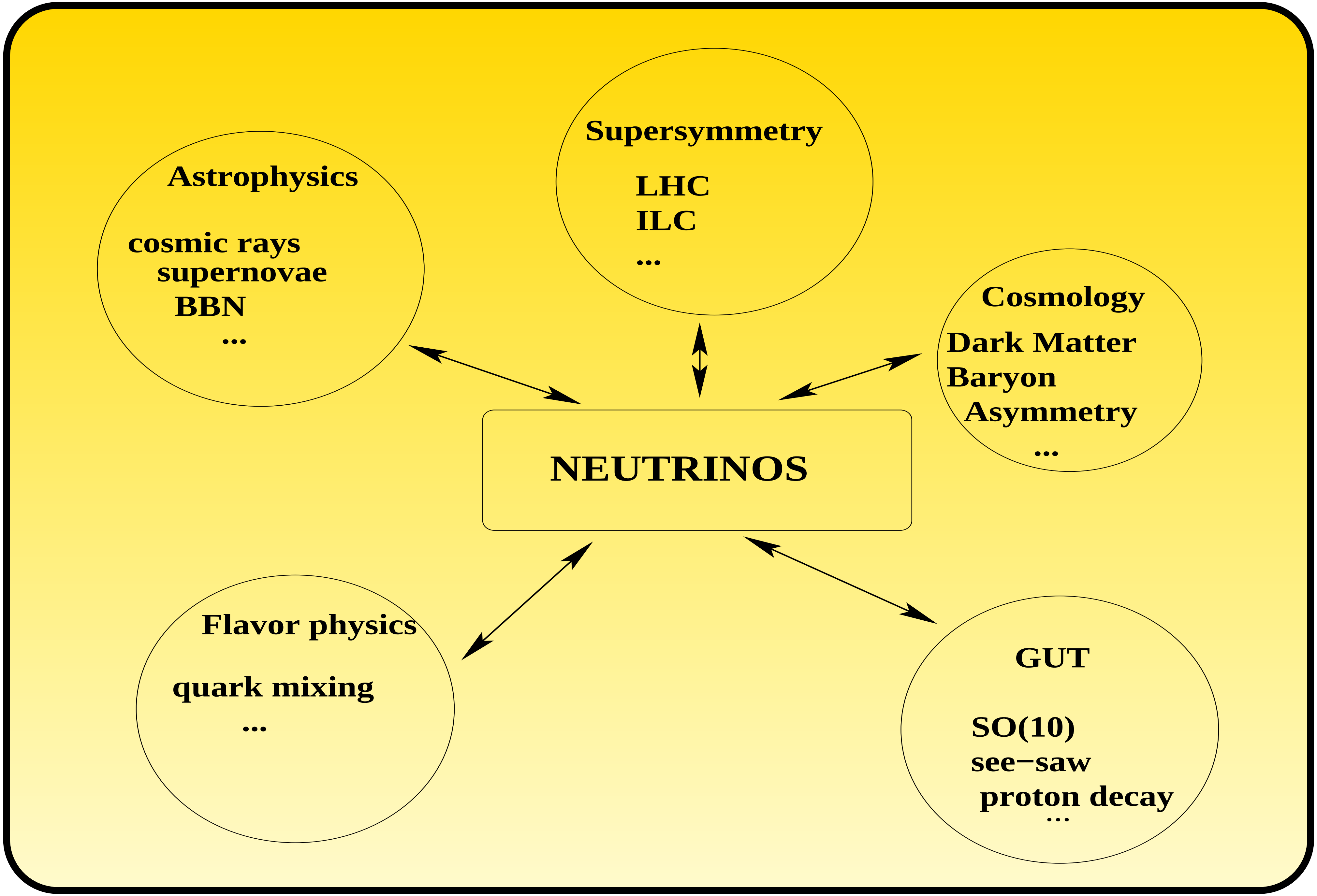
Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	$\#$	$\sum$		Species	$\#$	$\sum$
Quarks	10	10		Quarks	10	10
Leptons	3	13	$\longrightarrow$	Leptons	$12\ (10)$	$22\ (20)$
Charge	3	16		Charge	3	$25\ (23)$
Higgs	2	18		Higgs	2	$27\ (25)$

Two roads towards more understanding: Higgs and Flavor


Supersymmetry
LHC
ILC
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmology
Dark Matter
Baryon
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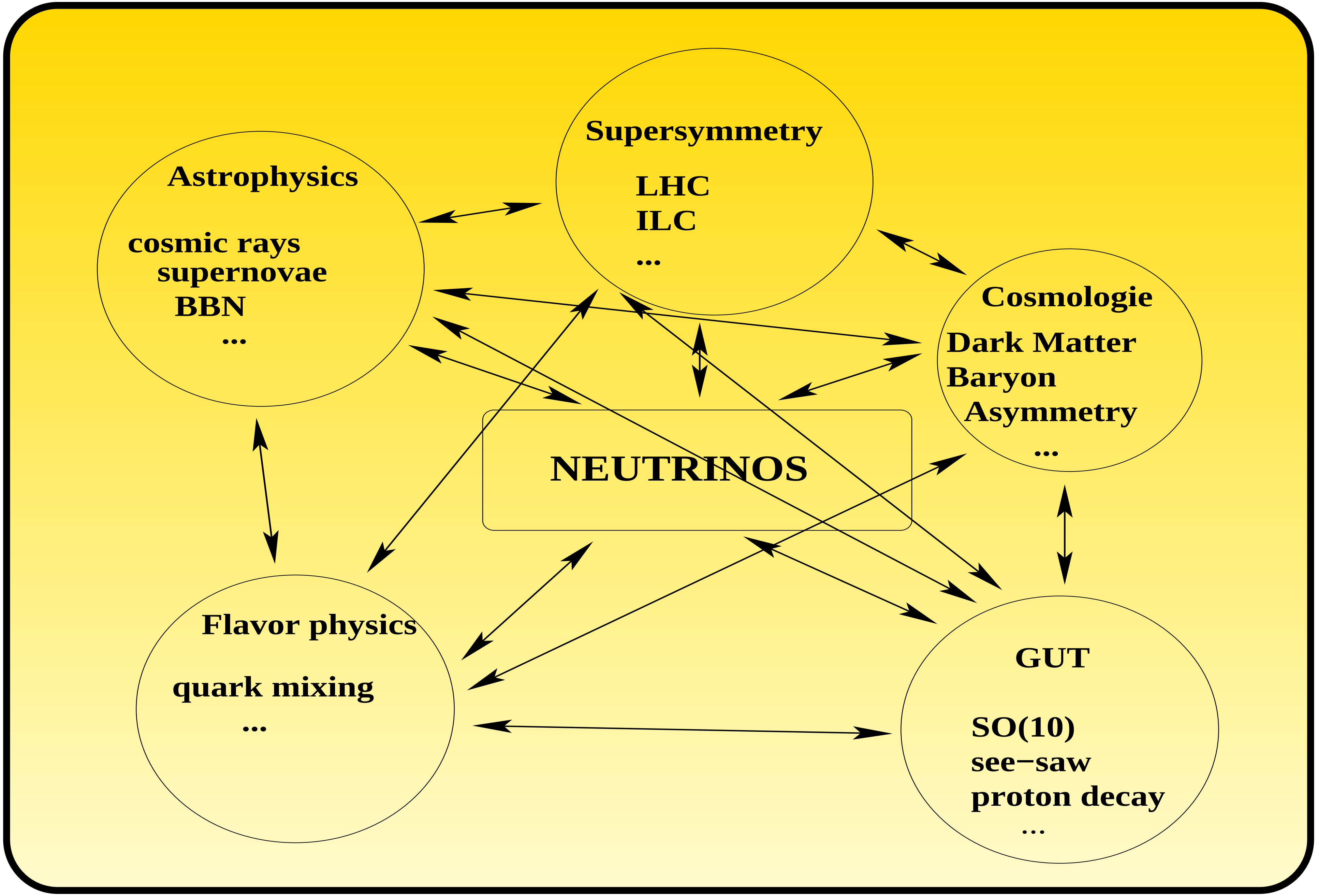
NEUTRINOS
Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
...


Astrophysics
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General Remarks

- Neutrinos interact weakly: can probe things not testable by other means
  - solar interior
  - geo-neutrinos
  - cosmic rays
- Neutrinos have no mass in SM
  - probe scales $m_\nu \propto 1/\Lambda$
  - happens in GUTs
  - connected to new concepts, e.g. Lepton Number Violation

$\Rightarrow$ particle and source physics

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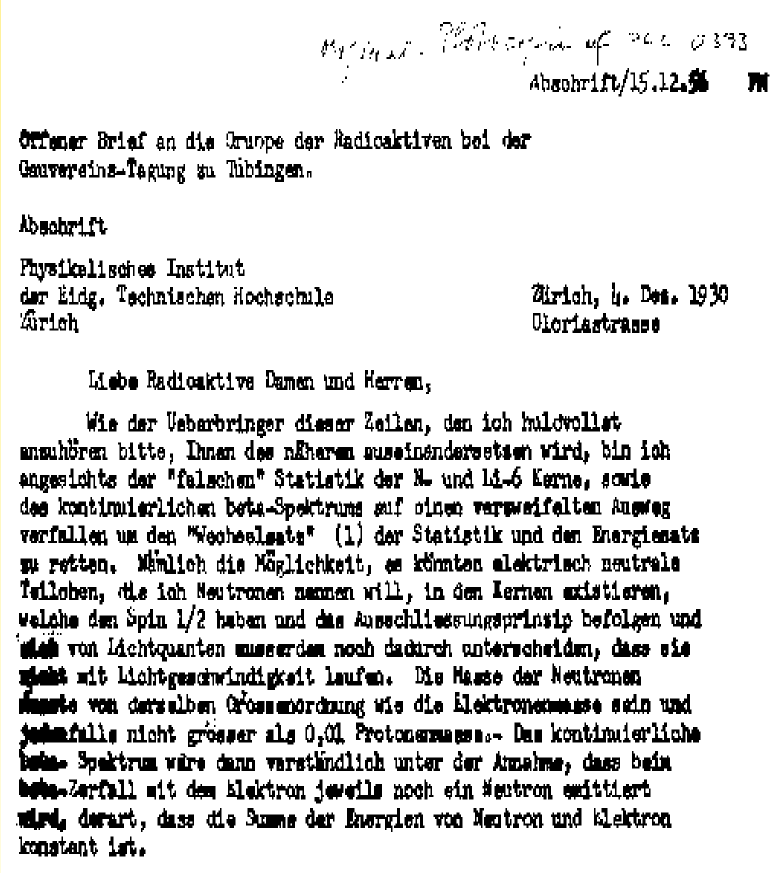
I Basics

I2) History

1926 problem in spectrum of β-decay

1930 Pauli postulates "neutron"



Abschrift/15.12.56

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

1932 Fermi theory of β-decay

1956 discovery of $\bar{\nu}_e$ by Cowan and Reines (NP 1985)

1957 Pontecorvo suggests neutrino oscillations

1958 helicity $h(\nu_e) = -1$ by Goldhaber $\Rightarrow V - A$

1962 discovery of ν_μ by Lederman, Steinberger, Schwartz (NP 1988)

1970 first discovery of solar neutrinos by Ray Davis (NP 2002); solar neutrino problem

1987 discovery of neutrinos from SN 1987A (Koshiba, NP 2002)

1991 $N_\nu = 3$ from invisible Z width

1998 SuperKamiokande shows that atmospheric neutrinos oscillate

2000 discovery of ν_τ

2002 SNO solves solar neutrino problem

2010 the third mixing angle

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I Basics

I1) **Introduction**

I2) **History of the neutrino**

I3) **Fermion mixing, neutrinos and the Standard Model**

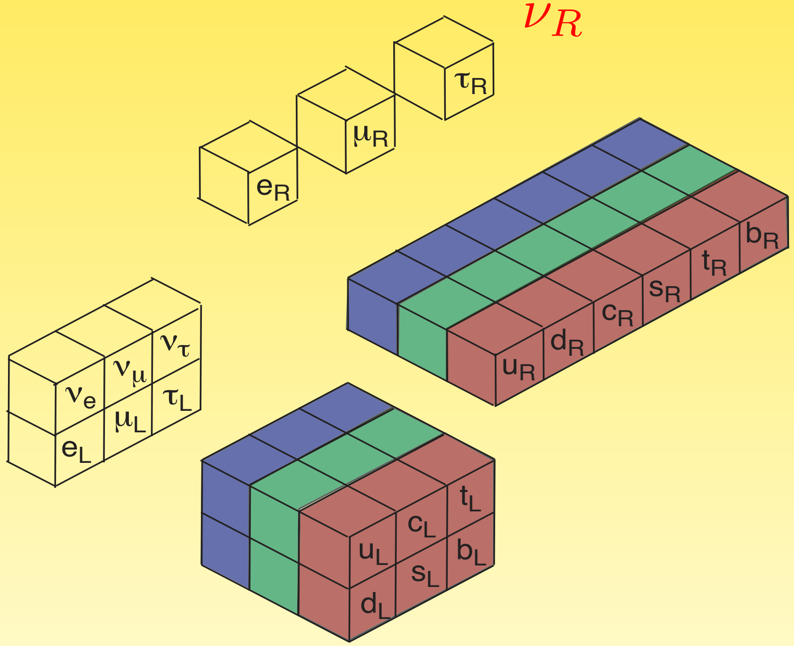
I3) Neutrinos and the Standard Model

$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{\rm em}$ with $Q = I_3 + \frac{1}{2}\,Y$

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \sim (1,2,-1)$$

$$e_R \qquad \sim (1,1,-2)$$

$$\nu_R \qquad \sim (1,1,0)$$ total SINGLET!!



Mass Matrices

3 generations of quarks

$$L_1' = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \ , \quad L_2' = \begin{pmatrix} c' \\ s' \end{pmatrix}_L \ , \quad L_3' = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$$

$$u'_R\,,\ c'_R\,,\ t'_R \equiv u'_{i,R} \quad \text{and} \quad d'_R\,,\ s'_R\,,\ b'_R \equiv d'_{i,R}$$

gives mass term

$$\begin{aligned} -\mathcal{L}_Y &= \sum_{i,j} \overline{L_i'} \left[g_{ij}^{(d)}\,\Phi\, d'_{j,R} + g_{ij}^{(u)}\,\tilde{\Phi}\, u'_{j,R} \right] \\ &\xrightarrow{\text{EWSB}} \sum_{i,j} \frac{v}{\sqrt{2}}\, g_{ij}^{(d)}\, \overline{d'_{i,L}}\, d'_{j,R} + \frac{v}{\sqrt{2}}\, g_{ij}^{(u)}\, \overline{u'_{i,L}}\, u'_{j,R} \\ &= \overline{d'_L}\, M^{(d)}\, d'_R + \overline{u'_L}\, M^{(u)}\, u'_R \end{aligned}$$

arbitrary complex 3×3 matrices in "flavor (interaction, weak) basis"

Diagonalization

$$U_d^\dagger\, M^{(d)}\, V_d = D^{(d)} = \mathrm{diag}(m_d, m_s, m_b)$$
$$U_u^\dagger\, M^{(u)}\, V_u = D^{(u)} = \mathrm{diag}(m_u, m_c, m_t)$$

with unitary matrices $U_{u,d}U_{u,d}^\dagger = U_{u,d}^\dagger U_{u,d} = V_{u,d}V_{u,d}^\dagger = V_{u,d}^\dagger V_{u,d} = \mathbb{1}$

in Lagrangian:

$$-\mathcal{L}_Y = \overline{d'_L}\, M^{(d)}\, d'_R + \overline{u'_L}\, M^{(u)}\, u'_R$$
$$\underbrace{\overline{d'_L}\, U_d}_{\overline{d_L}} \quad \underbrace{U_d^\dagger\, M^{(d)}\, V_d}_{D^{(d)}} \quad \underbrace{V_d^\dagger\, d'_R}_{d_R} \quad + \quad \underbrace{\overline{u'_L}\, U_u}_{\overline{u_L}} \quad \underbrace{U_u^\dagger\, M^{(u)}\, V_u}_{D^{(u)}} \quad \underbrace{V_u^\dagger\, u'_R}_{u_R}$$

physical (mass, propagation) states $u_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$

in interaction terms:

$$-\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u'_L} \gamma^\mu d'_L$$

$$\frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{u'_L} U_u}_{\overline{u_L}} \gamma^\mu \underbrace{U_u^\dagger U_d}_{V} \underbrace{U_d^\dagger d'_L}_{d_L}$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix survives:

$$\boxed{V = U_u^\dagger U_d}$$

Structure in Wolfenstein-parametrization:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with $\lambda = \sin\theta_C = 0.2253 \pm 0.0007,\ A = 0.808^{+0.022}_{-0.015}$,
$\overline{\rho} = (1-\frac{\lambda^2}{2})\,\rho = 0.132^{+0.022}_{-0.014},\ \overline{\eta} = 0.341 \pm 0.013$

Lesson to learn:

$$|V| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

<u>small mixing</u> in the quark sector

related to hierarchy of masses?

$$M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} = U\,D\,U^T \quad \text{with } U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

where $D = \mathrm{diag}(m_1, m_2)$

from 11-entry one gets

$$\tan\theta = \sqrt{\frac{m_1}{m_2}}$$

compare with $\sqrt{m_d/m_s} \simeq 0.22$ and $\tan\theta_C \simeq 0.23$

Number of parameters in V for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase u_i, d_i	$-(2\,N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,(N-2)\,(N-1)$

Masses in the SM:

$$-\mathcal{L}_Y = g_e\,\overline{L}\,\Phi\,e_R + g_\nu\,\overline{L}\,\tilde{\Phi}\,\nu_R + h.c.$$

with

$$L = \left(\begin{array}{c} \nu_e \\ e \end{array}\right)_L \text{ and } \tilde{\Phi} = i\tau_2\,\Phi^* = i\tau_2 \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right)^* = \left(\begin{array}{c} \phi^0 \\ -\phi^+ \end{array}\right)^*$$

after EWSB: $\langle \Phi \rangle \to (0, v/\sqrt{2})^T$ and $\langle \tilde{\Phi} \rangle \to (v/\sqrt{2}, 0)^T$

$$-\mathcal{L}_Y = g_e\,\frac{v}{\sqrt{2}}\,\overline{e_L}\,e_R + g_\nu\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R + h.c. \equiv m_e\,\overline{e_L}\,e_R + m_\nu\,\overline{\nu_L}\,\nu_R + h.c.$$

$\Leftrightarrow$ in a renormalizable, lepton number conserving model with Higgs doublets the absence of ν_R means absence of m_ν

Lepton Masses

$$-\mathcal{L}_Y = \overline{e'_L}\, M^{(\ell)}\, e'_R$$

$$= \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}}\ \underbrace{U_\ell^\dagger\, M^{(\ell)}\, V_\ell}_{D^{(\ell)}}\ \underbrace{V_\ell^\dagger\, e'_R}_{e_R}$$

and in charged current term:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\, W_\mu^+\, \overline{e'_L}\, \gamma^\mu\, \nu'_L$$

$$\frac{g}{\sqrt{2}}\, W_\mu^+\ \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}}\ \gamma^\mu\ \underbrace{U_\ell^\dagger\, U_\nu}_{U}\ \underbrace{U_\nu^\dagger\, \nu'_L}_{\nu_L}$$

Rotation of ν_L is arbitrary in absence of m_ν: choose $U_\nu = U_\ell$

$\Rightarrow$ Pontecorvo-Maki-Nakagawa-Saki (PMNS) matrix

$\boxed{U = \mathbb{1}}$ for massless neutrinos!!

$\Rightarrow$ individual lepton numbers L_e, L_μ, L_τ are conserved

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II Neutrino Oscillations

II1) The PMNS matrix

Neutrinos have mass, so:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\,\overline{\ell_L}\,\gamma^\mu\,U\,\nu_L\,W_\mu^- \quad \text{with } U = U_\ell^\dagger\,U_\nu$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\boxed{\nu_\alpha = U_{\alpha i}^*\,\nu_i}$$

connects flavor states ν_α $(\alpha = e, \mu, \tau)$ to mass states ν_i $(i = 1, 2, 3)$

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ν_i, ℓ_i	$-(2\,N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,(N-2)\,(N-1)$

this assumes $\bar{\nu}\nu$ mass term, what if $\nu^T\nu$?

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ℓ_α	$-N$	$N(N-1)$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases	extra phases
2	1	1	1
3	3	3	2
4	6	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,N\,(N-1)$	$N-1$

Extra $N-1$ "Majorana phases" because of mass term $\nu^T\,\nu$

(absent for Dirac neutrinos)

Majorana Phases

- connected to Majorana nature, hence to Lepton Number Violation
- I can always write: $U = \tilde{U}\, P$, where all Majorana phases are in $P = \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, \ldots)$:
- 2 families:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

- 3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

Dirac vs. Majorana neutrinos

See next term: Standard Model II (Prof. Lindner)

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II Neutrino Oscillations

II2) Neutrino Oscillations in Vacuum

Neutrino produced with charged lepton α is **flavor state**

$$|\nu(0)\rangle = |\nu_\alpha\rangle = U_{\alpha j}^* \, |\nu_j\rangle$$

evolves with time as

$$|\nu(t)\rangle = U_{\alpha j}^* \, e^{-i\,E_j\,t} \, |\nu_j\rangle$$

amplitude to find state $|\nu_\beta\rangle = U_{\beta i}^* \, |\nu_i\rangle$:

$$\mathcal{A}(\nu_\alpha \to \nu_\beta, t) = \langle \nu_\beta | \nu(t) \rangle = U_{\beta i} \, U_{\alpha j}^* \, e^{-i\,E_j\,t} \quad \underbrace{\langle \nu_i | \nu_j \rangle}_{\delta_{ij}}$$

$$= U_{\alpha i}^* \, U_{\beta i} \, e^{-i\,E_i\,t}$$

Probability:

$$P(\nu_\alpha \to \nu_\beta, t) \equiv P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha \to \nu_\beta, t)|^2$$

$$= \sum_{ij} \underbrace{U^*_{\alpha i}\, U_{\beta i}\, U^*_{\beta j}\, U_{\alpha j}}_{\mathcal{J}^{\alpha\beta}_{ij}} \quad \underbrace{e^{-i\,(E_i - E_j)\,t}}_{e^{-i\Delta_{ij}}}$$

$$= \ldots =$$

$$\boxed{P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{\mathcal{J}^{\alpha\beta}_{ij}\}\, \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{\mathcal{J}^{\alpha\beta}_{ij}\} \sin \Delta_{ij}}$$

with phase

$$\tfrac{1}{2}\Delta_{ij} = \tfrac{1}{2}\,(E_i - E_j)\,t \simeq \tfrac{1}{2}\left(\sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2}\right) L$$

$$\simeq \tfrac{1}{2}\left(p_i\,(1 + \tfrac{m_i^2}{2p_i^2}) - p_j\,(1 + \tfrac{m_j^2}{2p_j^2})\right) L \simeq \tfrac{m_i^2 - m_j^2}{2E}\, L$$

$$\boxed{\frac{1}{2}\Delta_{ij} = \frac{m_i^2 - m_j^2}{4\,E}\, L \simeq 1.27 \left(\frac{\Delta m^2_{ij}}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{km}}\right) \left(\frac{\mathrm{GeV}}{E}\right)}$$

$$\boxed{P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} \mathrm{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\}\,\sin^2\frac{\Delta_{ij}}{2} + 2\sum_{j>i}\mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\}\sin\Delta_{ij}}$$

- $\alpha = \beta$: survival probability
- $\alpha \neq \beta$: transition probability
- requires $U \neq \mathbb{1}$ and $\Delta m_{ij}^2 \neq 0$
- $\sum\limits_{\alpha} P_{\alpha\beta} = 1 \leftrightarrow$ conservation of probability
- $\mathcal{J}_{ij}^{\alpha\beta}$ invariant under $U_{\alpha j} \to e^{i\phi_\alpha}\, U_{\alpha j}\, e^{i\phi_j}$

  $\Rightarrow$ Majorana phases drop out!

CP Violation

In oscillation probabilities: $U \to U^*$ for anti-neutrinos

Define asymmetries:

$$\Delta_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$
$$= 4 \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

- 2 families: U is real and $\mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} = 0 \ \forall \alpha, \beta, i, j$
- 3 families:

$$\Delta_{e\mu} = -\Delta_{e\tau} = \Delta_{\mu\tau} = \left(\sin \frac{\Delta m^2_{21}}{2\,E} L + \sin \frac{\Delta m^2_{32}}{2\,E} L + \sin \frac{\Delta m^2_{13}}{2\,E} L\right) J_{\mathrm{CP}}$$

where $J_{\mathrm{CP}} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\}$

$$= \tfrac{1}{8}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin 2\theta_{13}\, \cos\theta_{13}\, \sin\delta$$

vanishes for one $\Delta m^2_{ij} = 0$ or one $\theta_{ij} = 0$ or $\delta = 0, \pi$

- CP violation in survival probabilities vanishes:

$$P(\nu_\alpha \to \nu_\alpha) - P(\overline{\nu}_\alpha \to \overline{\nu}_\alpha) \propto \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\alpha}\} = \sum_{j>i} \mathrm{Im}\{U_{\alpha i}^* \, U_{\alpha i} \, U_{\alpha j}^* \, U_{\alpha j}\} = 0$$

- Recall that $U = U_\ell^\dagger \, U_\nu$

  If charged lepton masses diagonal, then m_ν is diagonalized by PMNS matrix:

$$m_\nu = U \, \mathrm{diag}(m_1, m_2, m_3) \, U^T$$

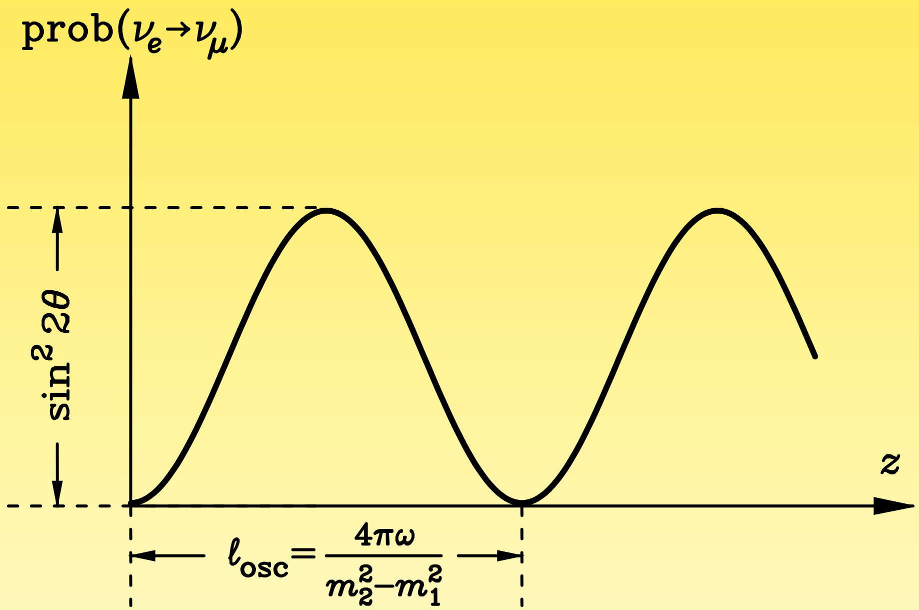
  Define $h = m_\nu \, m_\nu^\dagger$ and find that

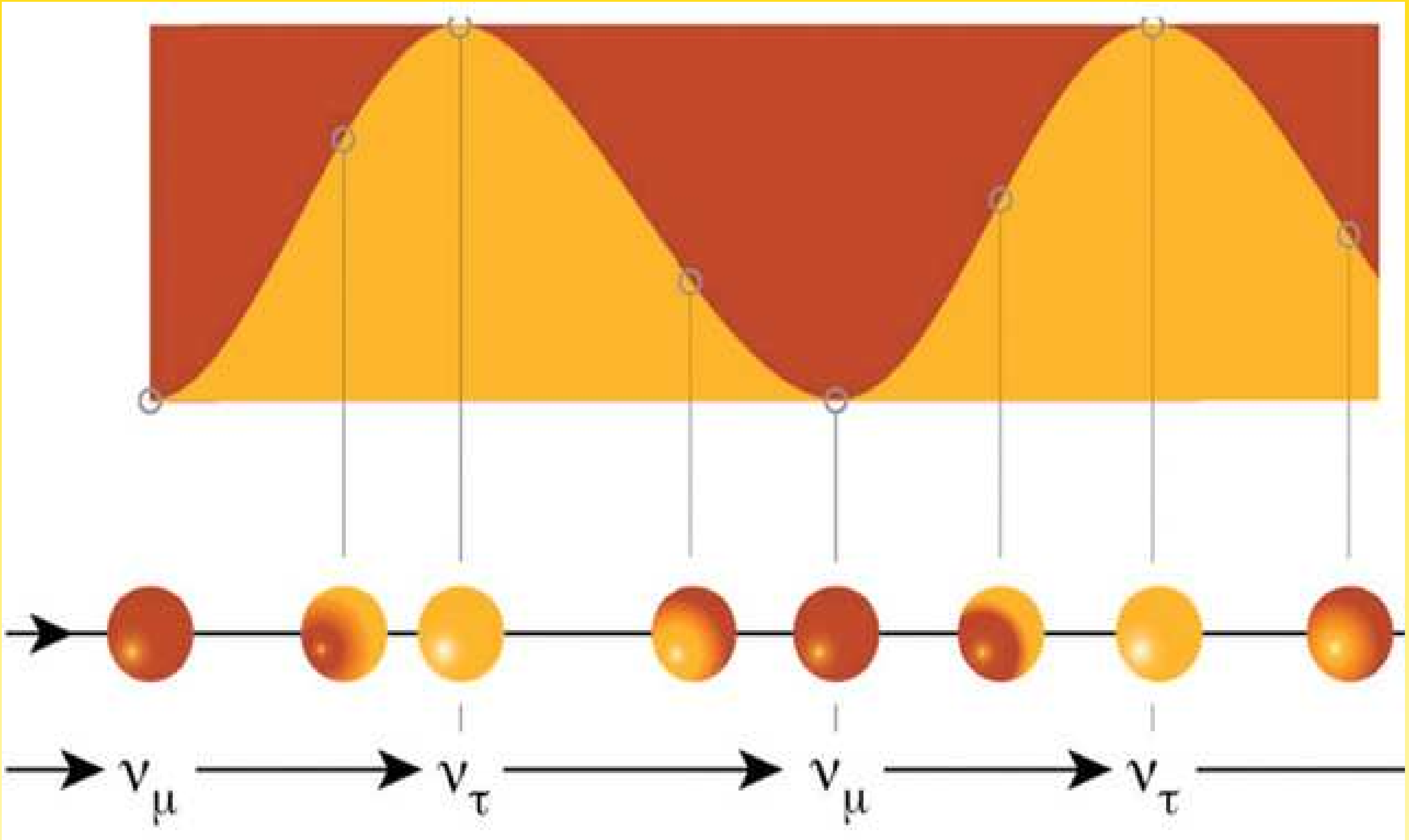
$$\mathrm{Im}\,\{h_{12} \, h_{23} \, h_{31}\} = \Delta m_{21}^2 \, \Delta m_{31}^2 \, \Delta m_{32}^2 \, J_{\mathrm{CP}}$$

Two Flavor Case

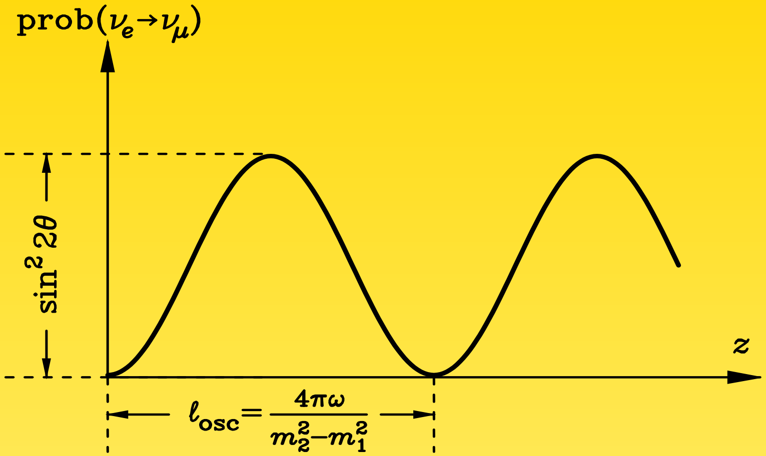
$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Rightarrow \mathcal{J}_{12}^{\alpha\alpha} = |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = \frac{1}{4}\sin^2 2\theta$$

and transition probability is $\boxed{P_{\alpha\beta} = \sin^2 2\theta \, \sin^2 \frac{\Delta m_{21}^2}{4\,E} L}$





ν_μ
ν_τ
ν_μ
ν_τ

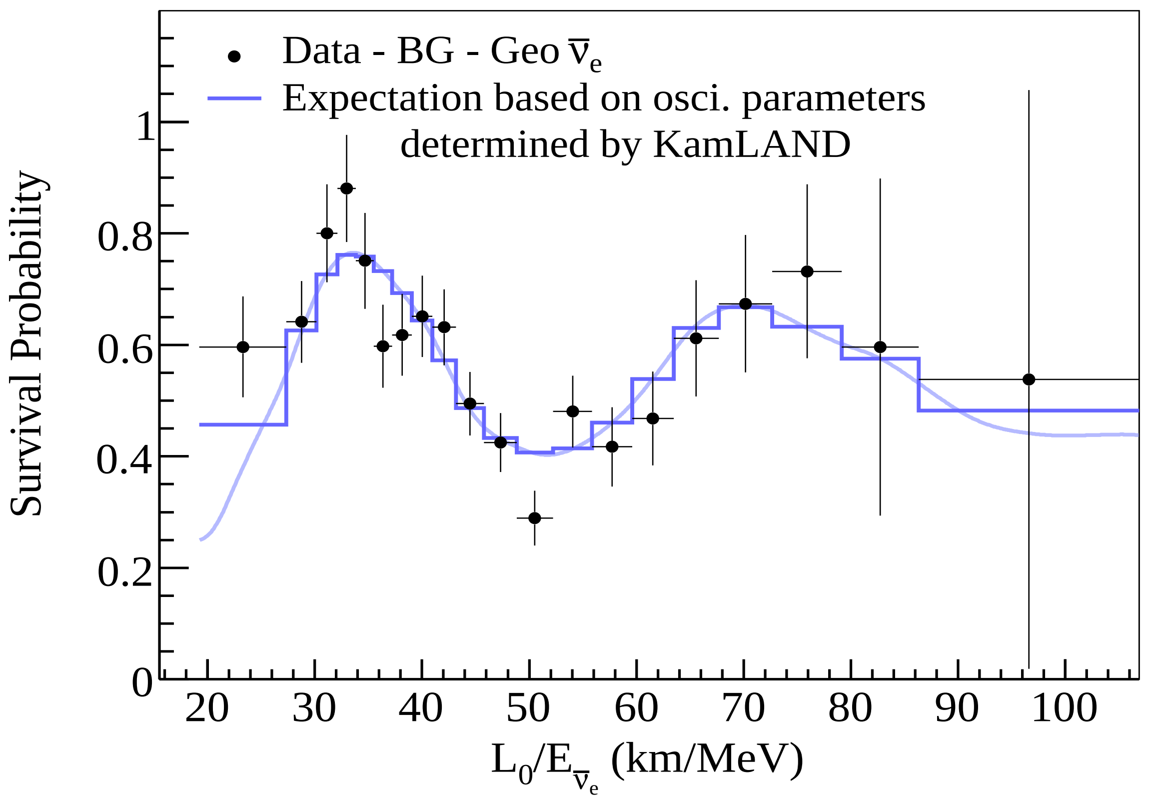


- amplitude $\sin^2 2\theta$
- maximal mixing for $\theta = \pi/4 \Rightarrow \nu_\alpha = \sqrt{\frac{1}{2}}\,(\nu_1 + \nu_2)$
- oscillation length $L_{\text{osc}} = 4\pi\, E/\Delta m^2_{21} = 2.48\,\frac{E}{\text{GeV}}\,\frac{\text{eV}^2}{\Delta m^2_{21}}$ km

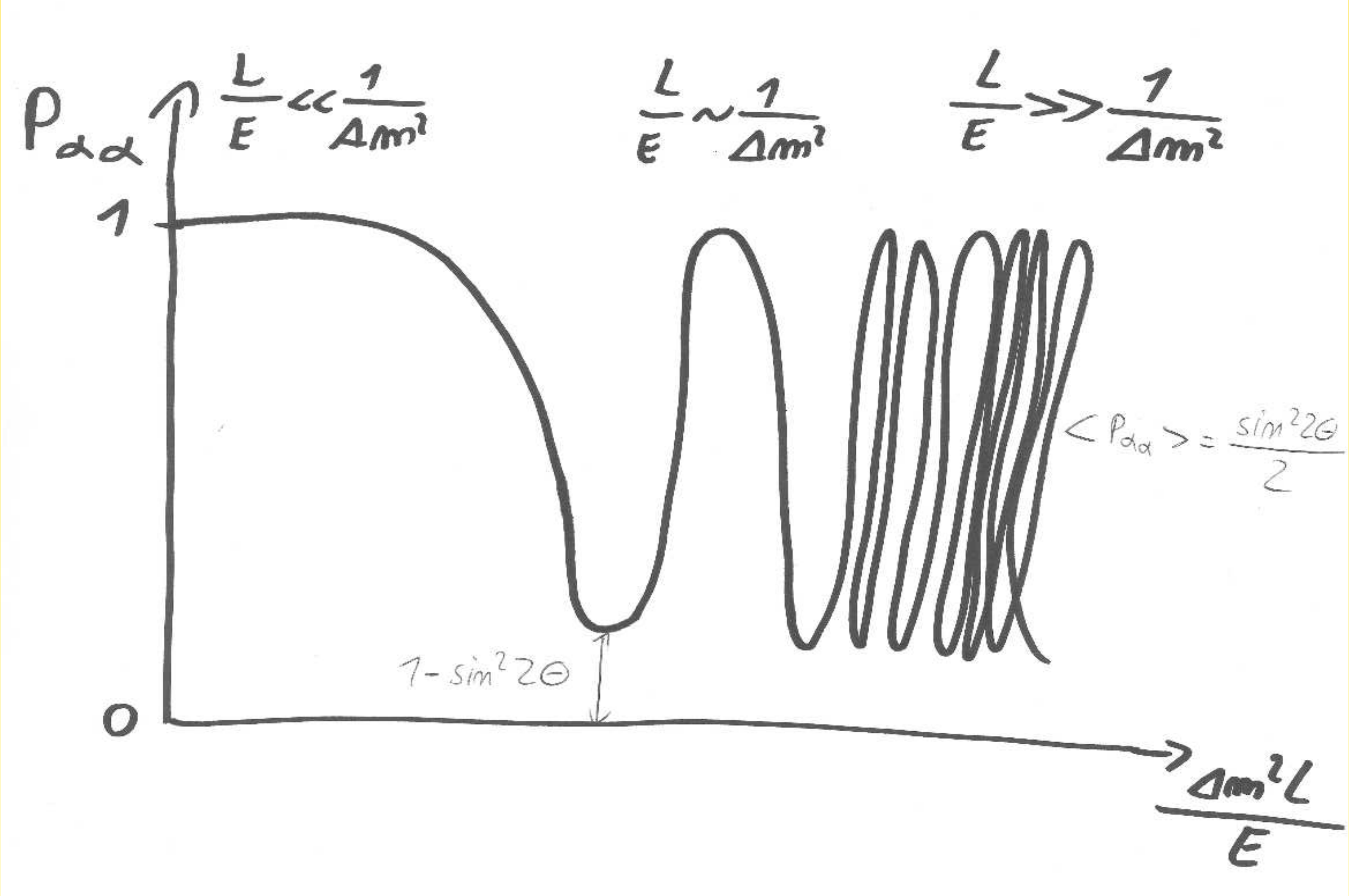
  $\Rightarrow P_{\alpha\beta} = \sin^2 2\theta\, \sin^2 \pi \frac{L}{L_{\text{osc}}}$

  is distance between two maxima (minima)

  e.g.: $E =$ GeV and $\Delta m^2 = 10^{-3}$ eV^2: $L_{\text{osc}} \simeq 10^3$ km



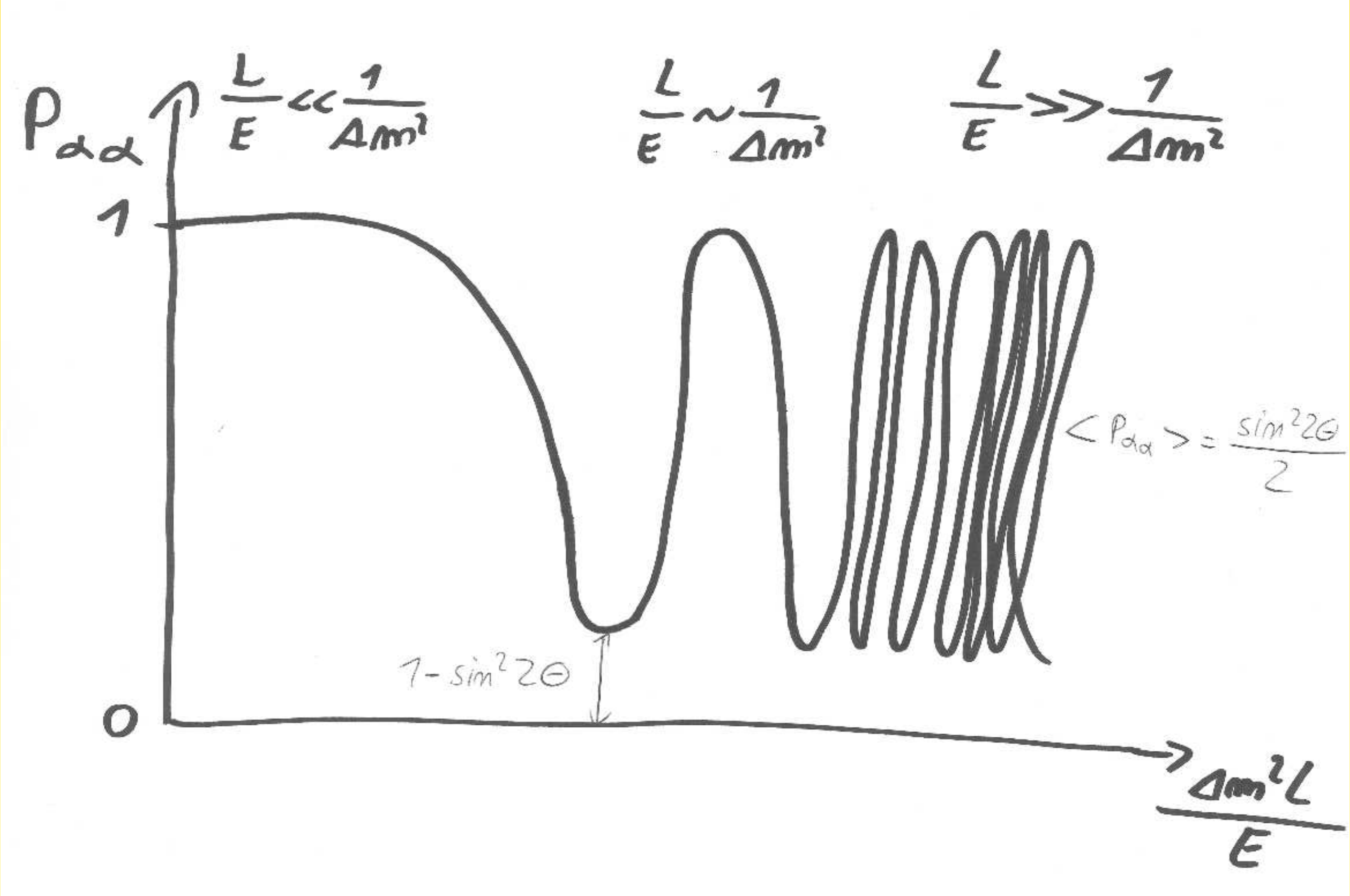
Data - BG - Geo $\overline{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\overline{\nu}_e}$ (km/MeV)



$L \gg L_{\mathrm{osc}}$: fast oscillations $\langle \sin^2 \pi L/L_{\mathrm{osc}} \rangle = \frac{1}{2}$

and $P_{\alpha\alpha} = 1 - 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$

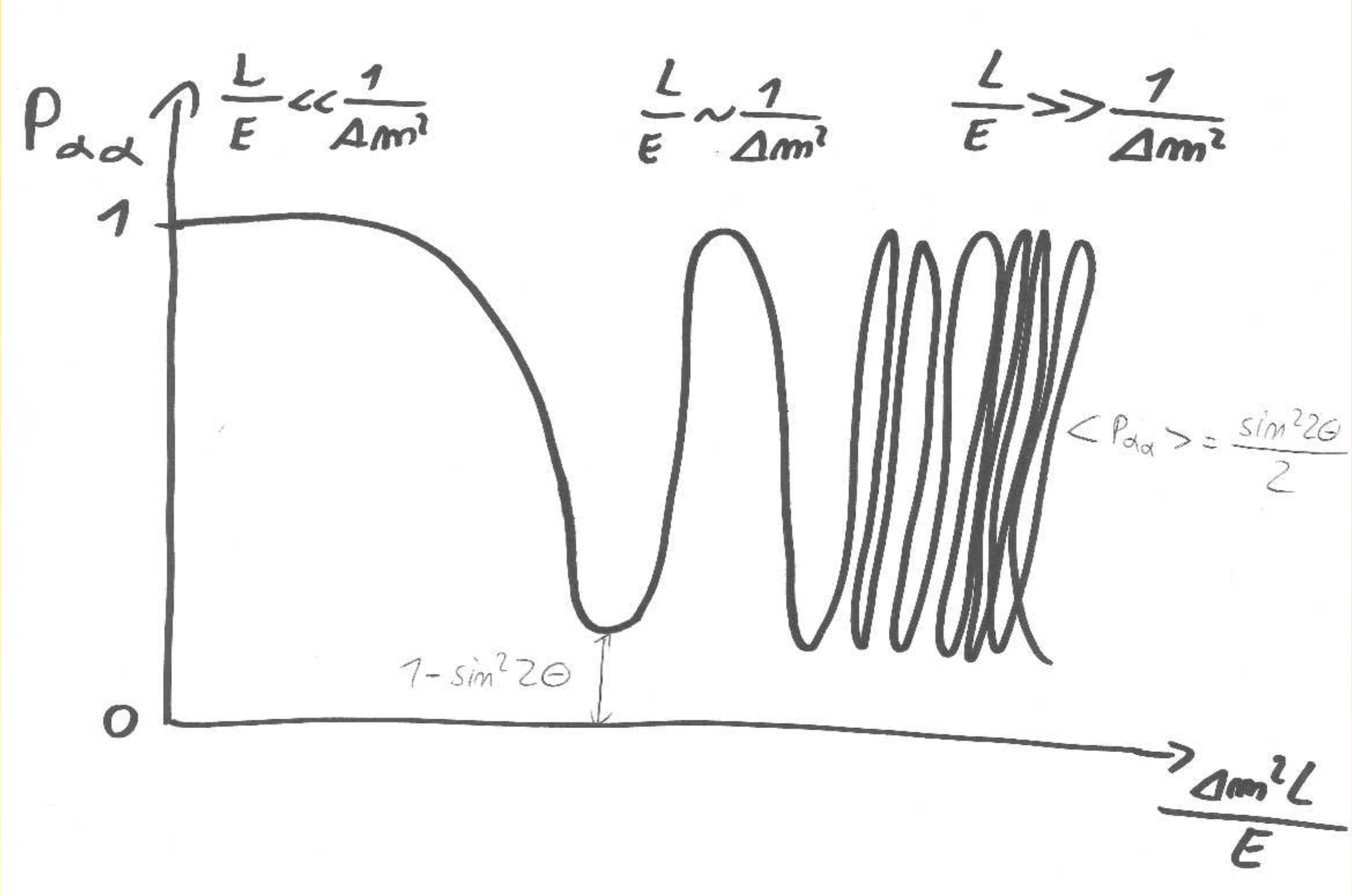
sensitivity to mixing



$L \gg L_{\rm osc}$: fast oscillations $\langle \sin^2 \pi L/L_{\rm osc} \rangle = \frac{1}{2}$

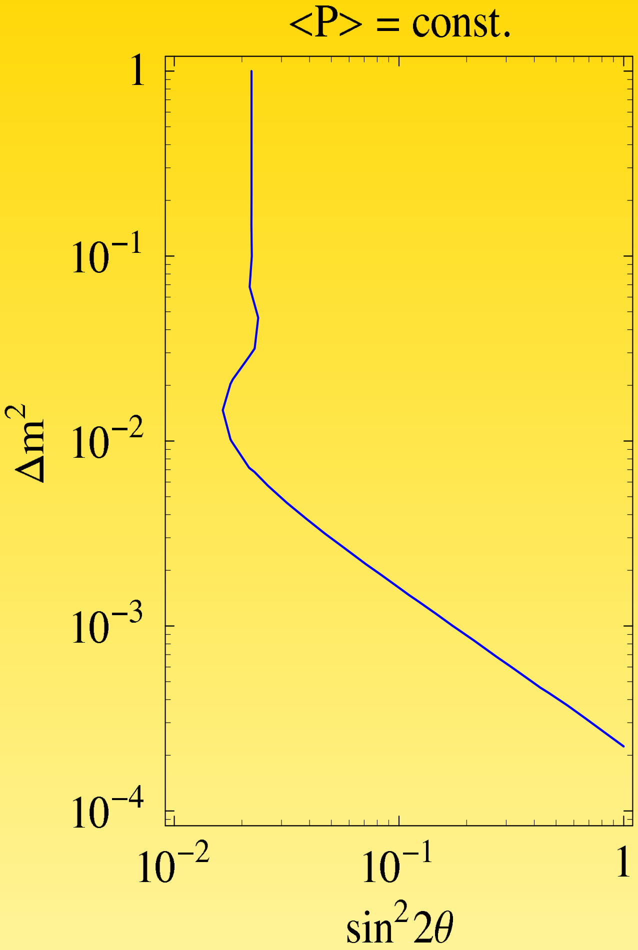
and $P_{\alpha\beta} = 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2\,|U_{\beta 1}|^2 + |U_{\alpha 2}|^2\,|U_{\beta 2}|^2 = \frac{1}{2}\sin^2 2\theta$

sensitivity to mixing



$L \ll L_{\rm osc}$: hardly oscillations and $P_{\alpha\beta} = \sin^2 2\theta\,(\Delta m^2 L/(4E))^2$

sensitivity to product $\sin^2 2\theta\,\Delta m^2$



large Δm^2: sensitivity to mixing

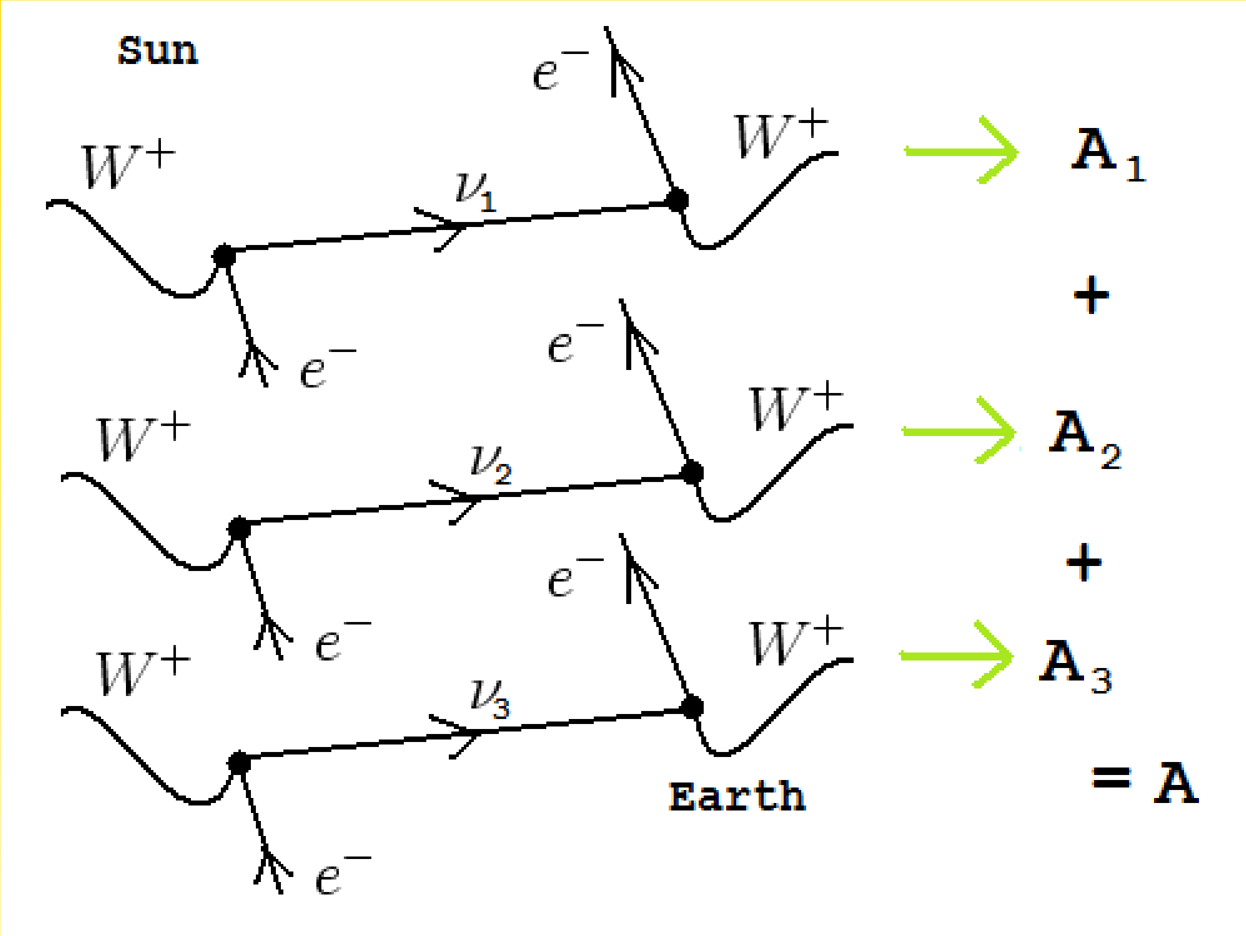
small Δm^2: sensitivity to $\sin^2 2\theta\, \Delta m^2$

maximal sensitivity when $\Delta m^2 L/E \simeq 2\pi$

Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\min}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e, \ \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^2$	$10 \ldots 10^4$	10^{-6}
Sun	ν_e	$10^{-3} \ldots 10^{-2}$	10^8	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^2	10^{-5}
Accelerator LBL	$\overset{(-)}{\nu}_e, \ \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^2	10^{-1}
Accelerator SBL	$\overset{(-)}{\nu}_e, \ \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	1	1

Quantum Mechanics



Can't distinguish the individual m_i: coherent sum of amplitudes and *interference*

Quantum Mechanics

Textbook calculation is completely wrong!!

- $E_i - E_j$ is not Lorentz invariant
- massive particles with different p_i and same E violates energy and/or momentum conservation
- definite p: in space this is e^{ipx}, thus no localization

Quantum Mechanics

consider E_j and $p_j = \sqrt{E_j^2 - m_j^2}$:

$$p_j \simeq E + m_j^2 \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0} \equiv E - \xi \frac{m_j^2}{2E} , \quad \text{with } \xi = -2E \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0}$$

$$E_j \simeq p_j + m_j^2 \left.\frac{\partial E_j}{\partial m_j^2}\right|_{m_j=0} = p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E}\,(1-\xi)$$

in pion decay $\pi \to \mu\nu$:

$$E_j = \frac{m_\pi^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^2}{2m_\pi^2}$$

thus,

$$\xi = \frac{1}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8 \ \text{ in } E_i - E_j \simeq (1-\xi)\frac{\Delta m_{ij}^2}{2E}$$

wave packet with size $\sigma_x (\gtrsim 1/\sigma_p)$ and group velocity $v_i = \partial E_i / \partial p_i = p_i / E_i$:

$$\psi_i \propto \exp\left\{ -i(E_i\, t - p_i\, x) - \frac{(x - v_i\, t)^2}{4\sigma_x^2} \right\}$$

1) wave packet separation should be smaller than σ_x!

$$L\,\Delta v < \sigma_x \Rightarrow \frac{L}{L_{\rm osc}} < \frac{p}{\sigma_p}$$

(loss of *coherence*: interference impossible)

2) m_ν^2 should NOT be known too precisely!

if known too well: $\Delta m^2 \gg \delta m_\nu^2 = \dfrac{\partial m_\nu^2}{\partial p_\nu}\, \delta p_\nu \Rightarrow \delta x_\nu \gg \dfrac{2\, p_\nu}{\Delta m^2} = \dfrac{L_{\rm osc}}{2\pi}$

(I know which state ν_i is exchanged, *localization*)

In both cases: $P_{\alpha\alpha} = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$ (same as for $L \gg L_{\rm osc}$)

Quantum Mechanics

total amplitude for $\alpha \to \beta$ should be given by

$$A \propto \sum_j \int \frac{d^3p}{2E_j} \mathcal{A}^*_{\beta j}\, \mathcal{A}_{\alpha j} \exp\{-i(E_j t - px)\}$$

with production and detection amplitudes

$$\mathcal{A}_{\alpha j}\, \mathcal{A}^*_{\beta j} \propto \exp\left\{-\frac{(p-\tilde{p}_j)^2}{4\sigma_p^2}\right\}$$

we expand around $\tilde{p}_j$:

$$E_j(p) \simeq E_j(\tilde{p}_j) + \left.\frac{\partial E_j(p)}{\partial p}\right|_{p=\tilde{p}_j} (p-\tilde{p}_j) = \tilde{E}_j + v_j\,(p-\tilde{p}_j)$$

and perform the integral over p:

$$A \propto \sum_j \exp\left\{-i(\tilde{E}_j t - \tilde{p}_j x) - \frac{(x-v_j t)^2}{4\sigma_x^2}\right\}$$

the probability is the integral of $|A|^2$ over t:

$$P = \int dt\, |A|^2 \propto \exp\left\{ -i \left[(\tilde{E}_j - \tilde{E}_k) \frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k) \right] x \right\}$$
$$\times \exp\left\{ -\frac{(v_j - v_k)^2 x^2}{4\sigma_x^2 (v_j^2 + v_k^2)} - \frac{(\tilde{E}_j - \tilde{E}_k)^2}{4\sigma_p^2 (v_j^2 + v_k^2)} \right\}$$

now express average momenta, energy and velocity as

$$\tilde{p}_j \simeq E - \xi \frac{m_j^2}{2E}$$

$$\tilde{E}_j \simeq E + (1 - \xi) \frac{m_j^2}{2E} \,, \qquad v_j = \frac{\tilde{p}_j}{\tilde{E}_j} \simeq 1 - \frac{m_j^2}{2E^2}$$

this we insert in first exponential of P:

$$\left[(\tilde{E}_j - \tilde{E}_k) \frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k) \right] = \frac{\Delta m_{jk}^2 L}{2E}$$

the second exponential (damping term) can also be rewritten and the final probability is

$$P \propto \exp\left\{ -i\frac{\Delta m^2_{ij}}{2E}L - \left(\frac{L}{L^{\rm coh}_{jk}}\right)^2 - 2\pi^2(1-\xi)^2\left(\frac{\sigma_x}{L^{\rm osc}_{jk}}\right)^2 \right\}$$

with

$$L^{\rm coh}_{jk} = \frac{4\sqrt{2}E^2}{|\Delta m^2_{jk}|}\sigma_x \text{ and } L^{\rm osc}_{jk} = \frac{4\pi E}{|\Delta m^2_{jk}|}$$

expressing the two conditions (coherence and localization) for oscillation discussed before

Contents

II Neutrino Oscillations

III3) Results and their interpretation – what have we learned?

- Main results as by-products:
  - check solar fusion in Sun $\rightarrow$ solar neutrino problem
  - look for nucleon decay $\rightarrow$ atmospheric neutrino oscillations
- almost all current data described by 2-flavor formalism
- future goal: confirm genuine 3-flavor effects:
  - third mixing angle (check!)
  - mass ordering
  - CP violation
- have entered precision era

Interpretation in 3 Neutrino Framework

assume $\Delta m^2_{21} \ll \Delta m^2_{31} \simeq \Delta m^2_{32}$ and small θ_{13}:

- atmospheric and accelerator neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31}}{4\,E} L$$

- solar and KamLAND neutrinos: $\Delta m^2_{31} L/E \gg 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21}}{4\,E} L$$

- short baseline reactor neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4\,E} L$$

Solar Neutrinos

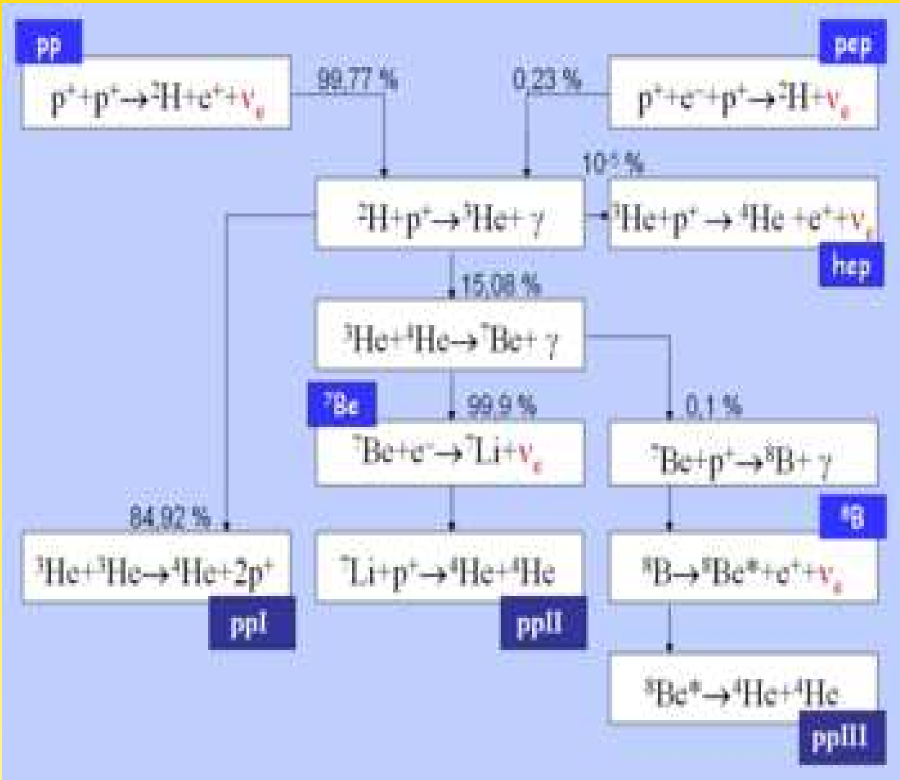
98% of energy production in fusion of net reaction

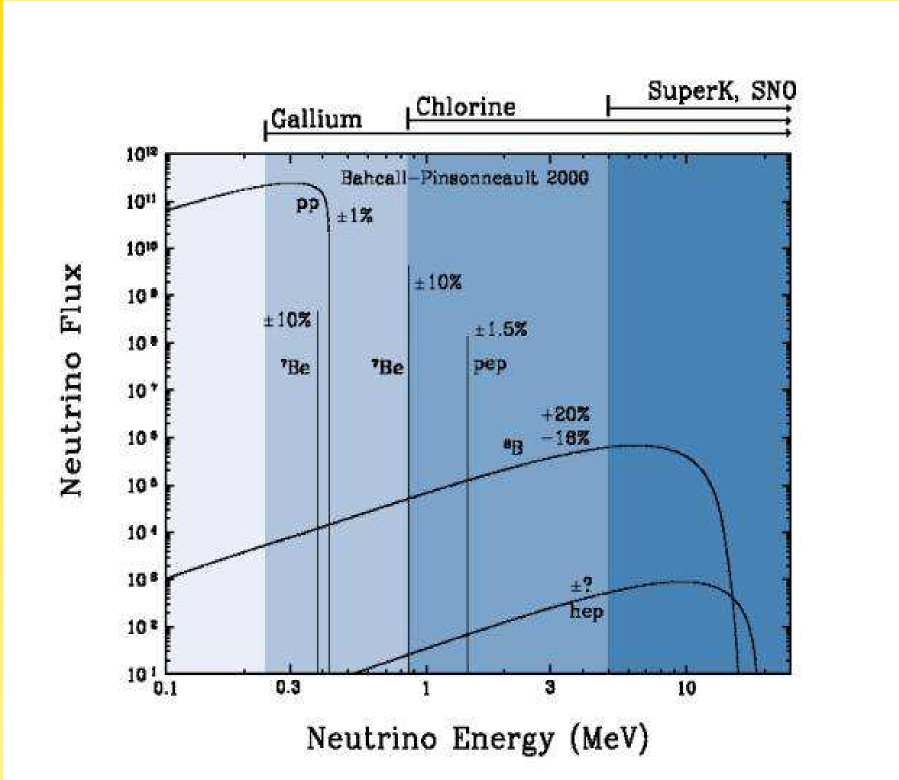
$$4\,p + 2\,e^- \rightarrow {}^4\mathrm{He}^{++} + 2\,\nu_e + 26.73\ \mathrm{MeV}$$

26 MeV of the energy go in photons, i.e., 13 MeV per ν_e;

get neutrino flux from solar constant

$$S = 8.5 \times 10^{11}\ \mathrm{MeV\,cm^{-2}\,s^{-1}} \Rightarrow \Phi_\nu = \frac{S}{13\ \mathrm{MeV}} = 6.5 \times 10^{10}\ \mathrm{cm^{-2}\,s^{-1}}$$





Solar Standard Model (SSM) predicts 5 sources of neutrinos from pp-chain

Bahcall *et al.*

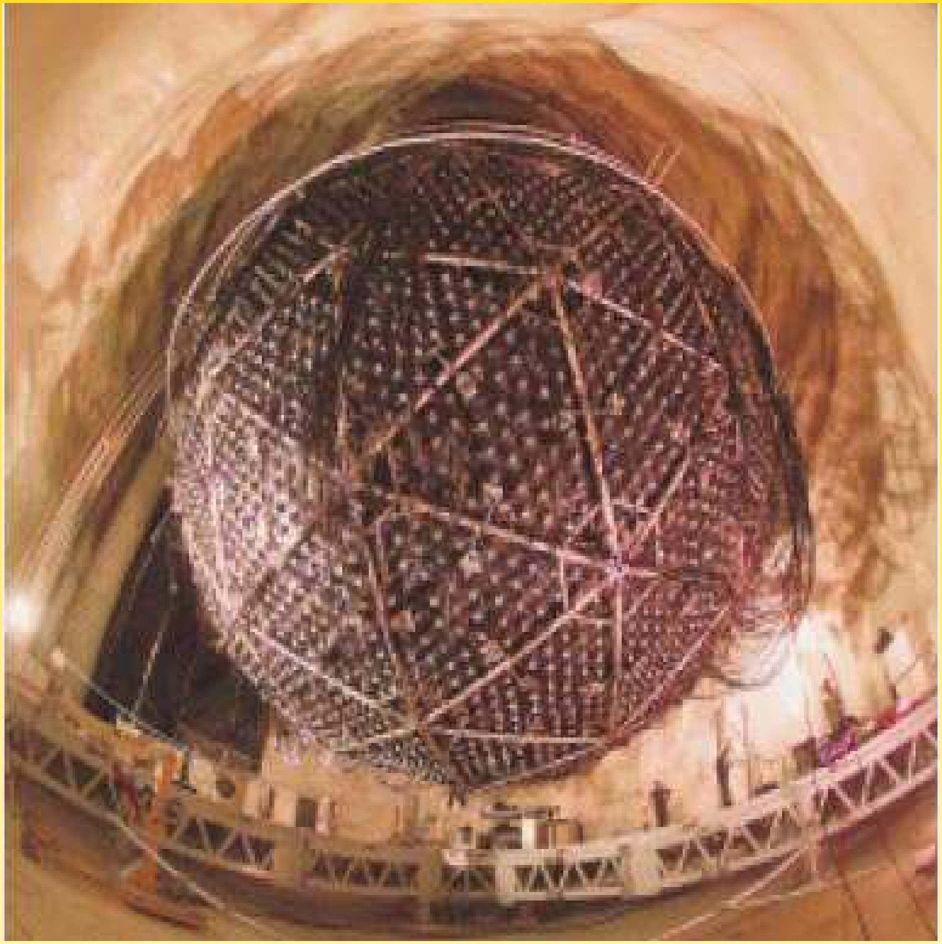
Different experiments sensitive to different energy, hence different neutrinos

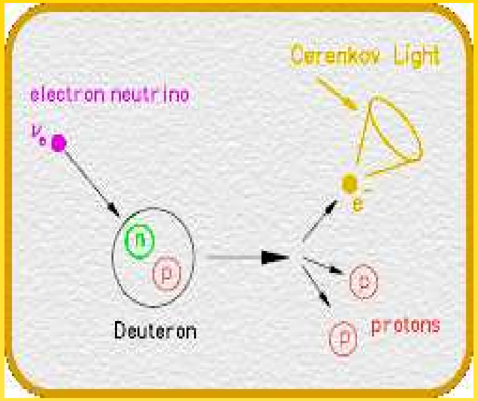
- Homestake: $\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$
- Gallex, GNO, SAGE: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$
- (Super)Kamiokande: $\nu_e + e^- \to \nu_e + e^-$

All find less neutrinos than predicted by SSM, deficit is energy dependent:

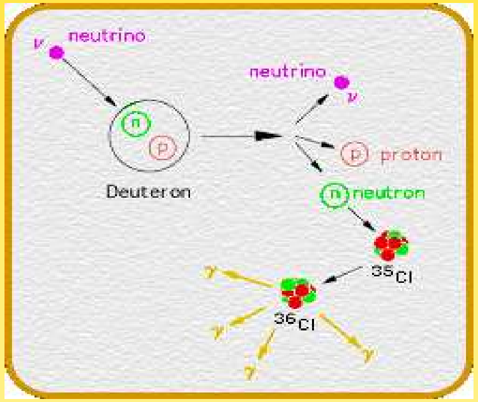
"solar neutrino problem"

Breakthrough came with SNO experiment, using heavy water

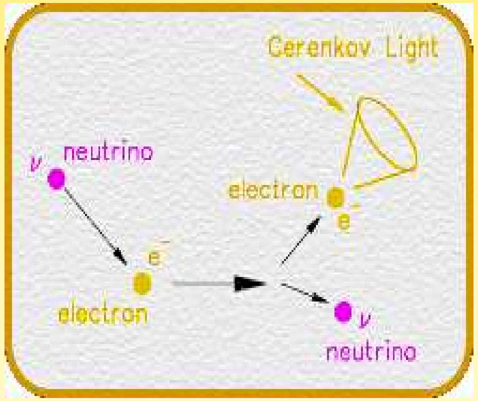




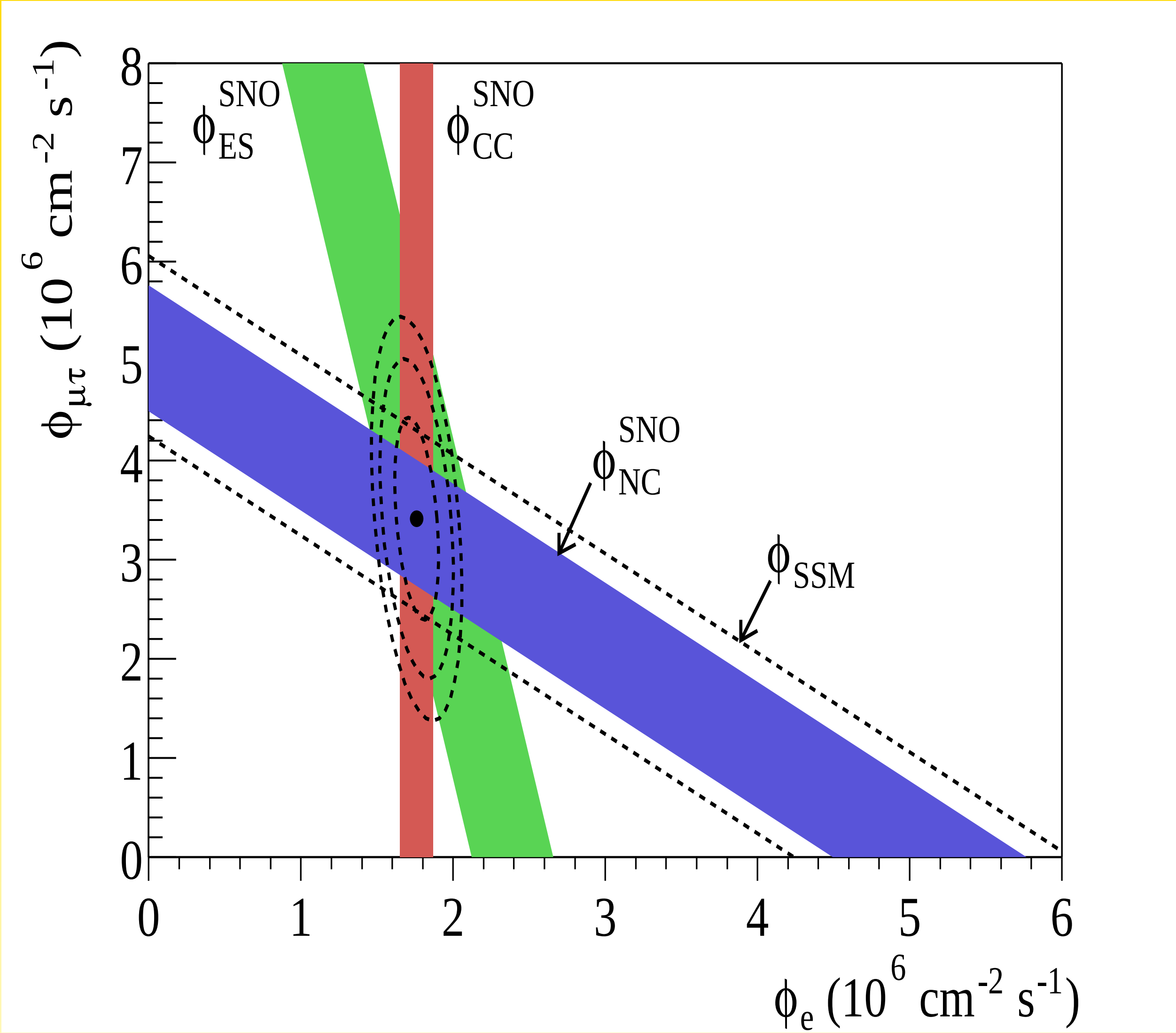
charged current: $\Phi(\nu_e)$



neutral current: $\Phi(\nu_e) + \Phi(\nu_{\mu\tau})$



elastic scattering: $\Phi(\nu_e) + 0.15\,\Phi(\nu_{\mu\tau})$


ϕ_{ES}^{SNO}
ϕ_{CC}^{SNO}
ϕ_{NC}^{SNO}
ϕ_{SSM}
$\phi_{\mu\tau}$ (10^6 cm^{-2} s^{-1})
ϕ_e (10^6 cm^{-2} s^{-1})
0
1
2
3
4
5
6
7
8

Results of fits give

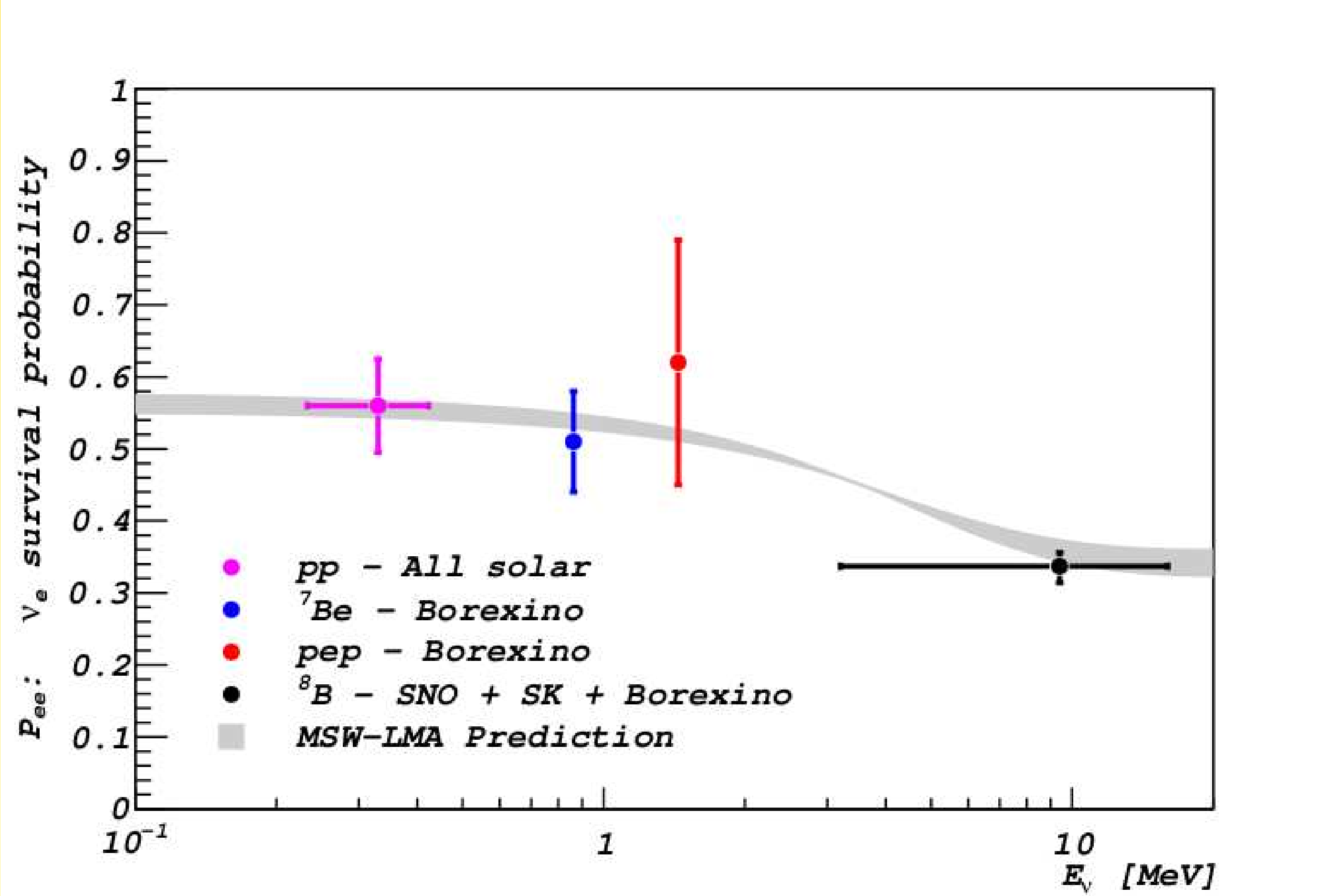
$$\boxed{\sin^2\theta_{12} \simeq 0.30}$$

$$\boxed{\Delta m^2_{21} \equiv \Delta m^2_{\odot} \simeq 8 \times 10^{-5}\ \mathrm{eV}^2}$$

only works with matter effects and resonance in Sun

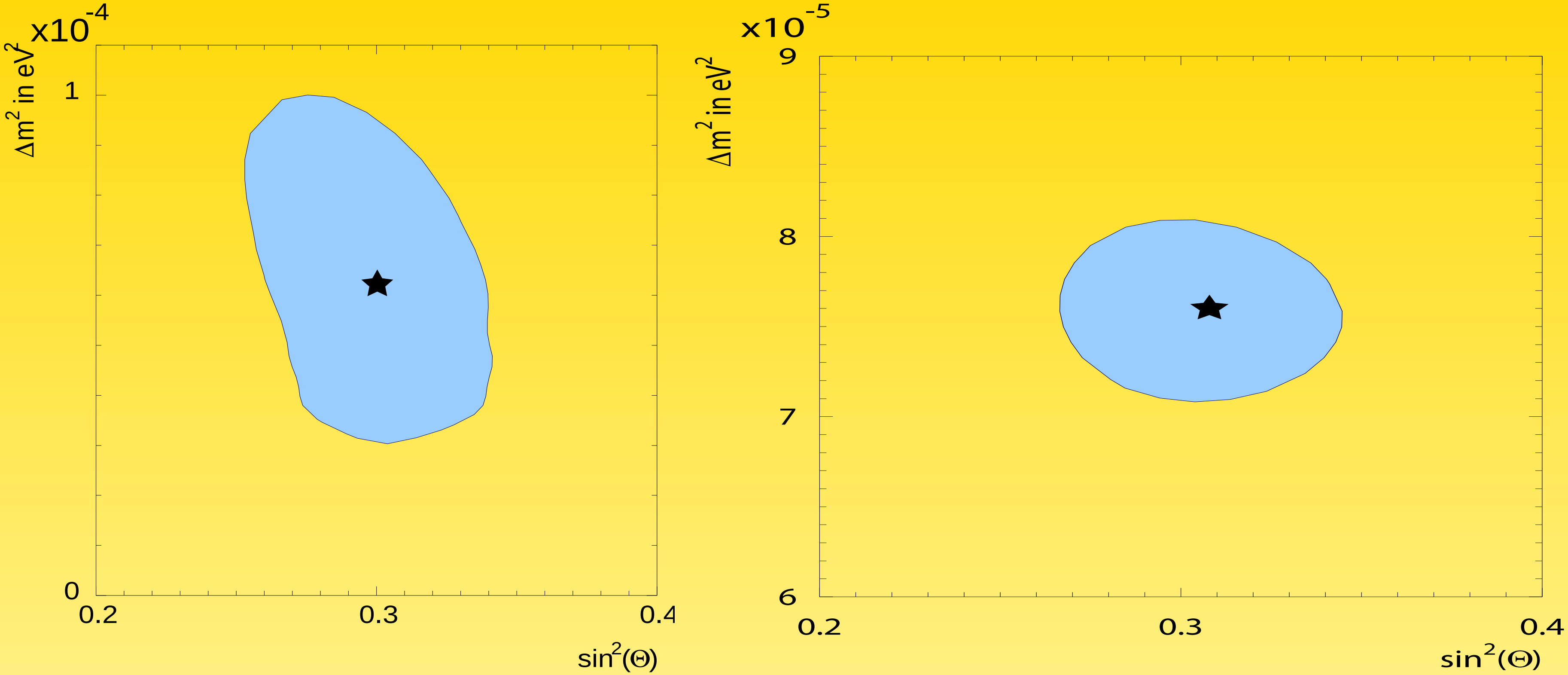
$$\Rightarrow \Delta m^2_{\odot}\, \cos 2\theta_{12} = (m_2^2 - m_1^2)\,(\cos^2\theta_{12} - \sin^2\theta_{12}) > 0$$

choosing $\cos 2\theta_{12} > 0$ fixes $\Delta m^2_{\odot} > 0$



low E: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{12} \simeq \frac{5}{9}$

large E: $P_{ee} = \sin^2\theta_{12} = \frac{1}{3}$

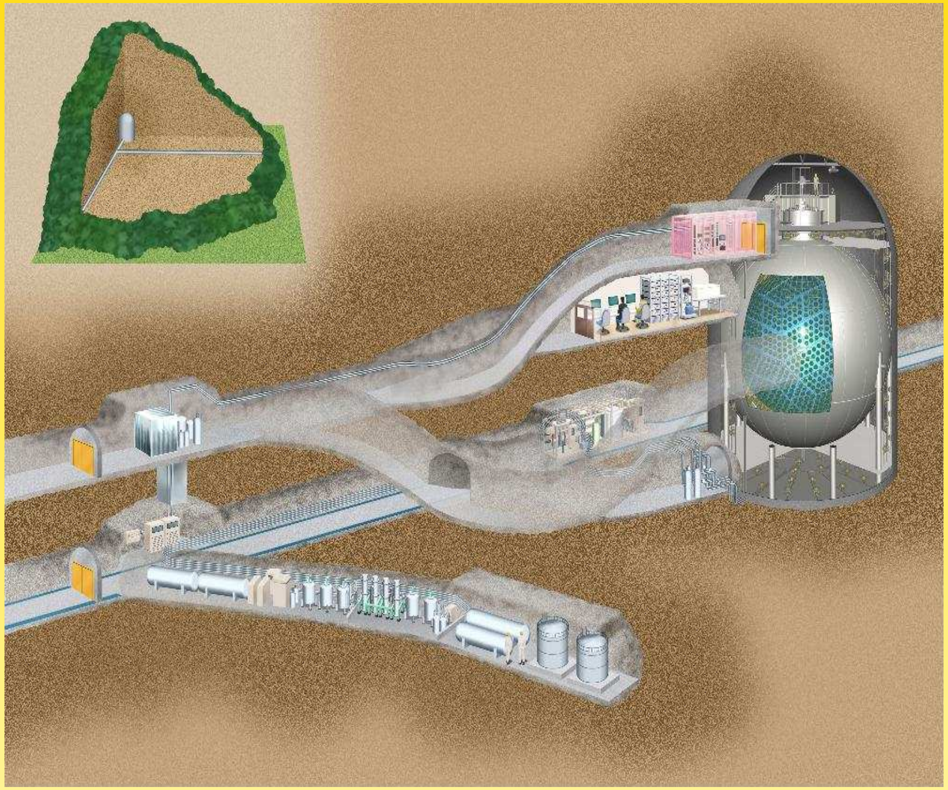


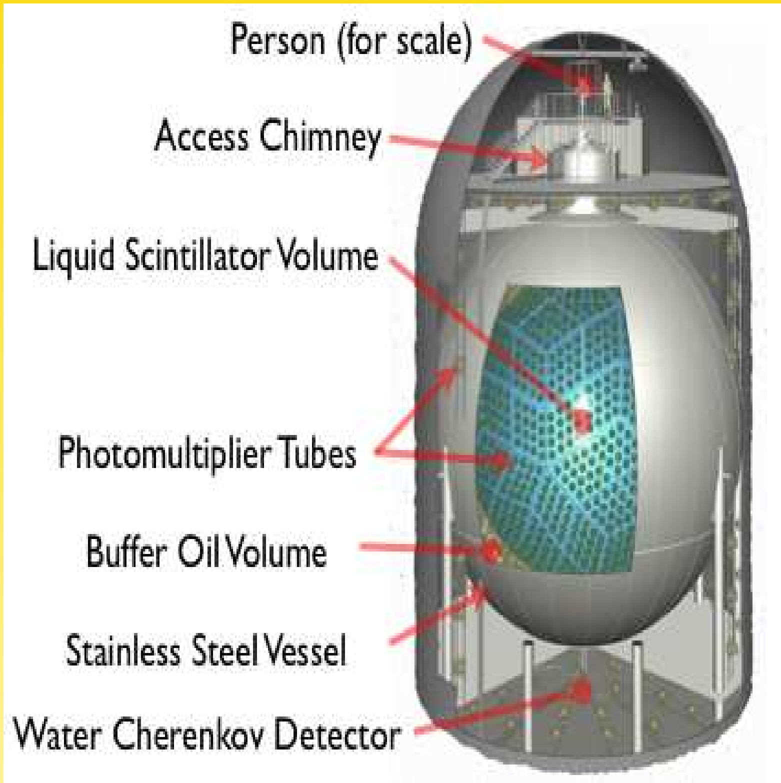
KamLAND: reactor neutrinos

$n \to p + e^- + \overline{\nu_e}$ with $E \simeq \text{few MeV}$

If $L \simeq 100$ km:

$$\frac{\Delta m^2_\odot}{E} L \sim 1 \Rightarrow \text{ solar } \nu \text{ parameters!!}$$

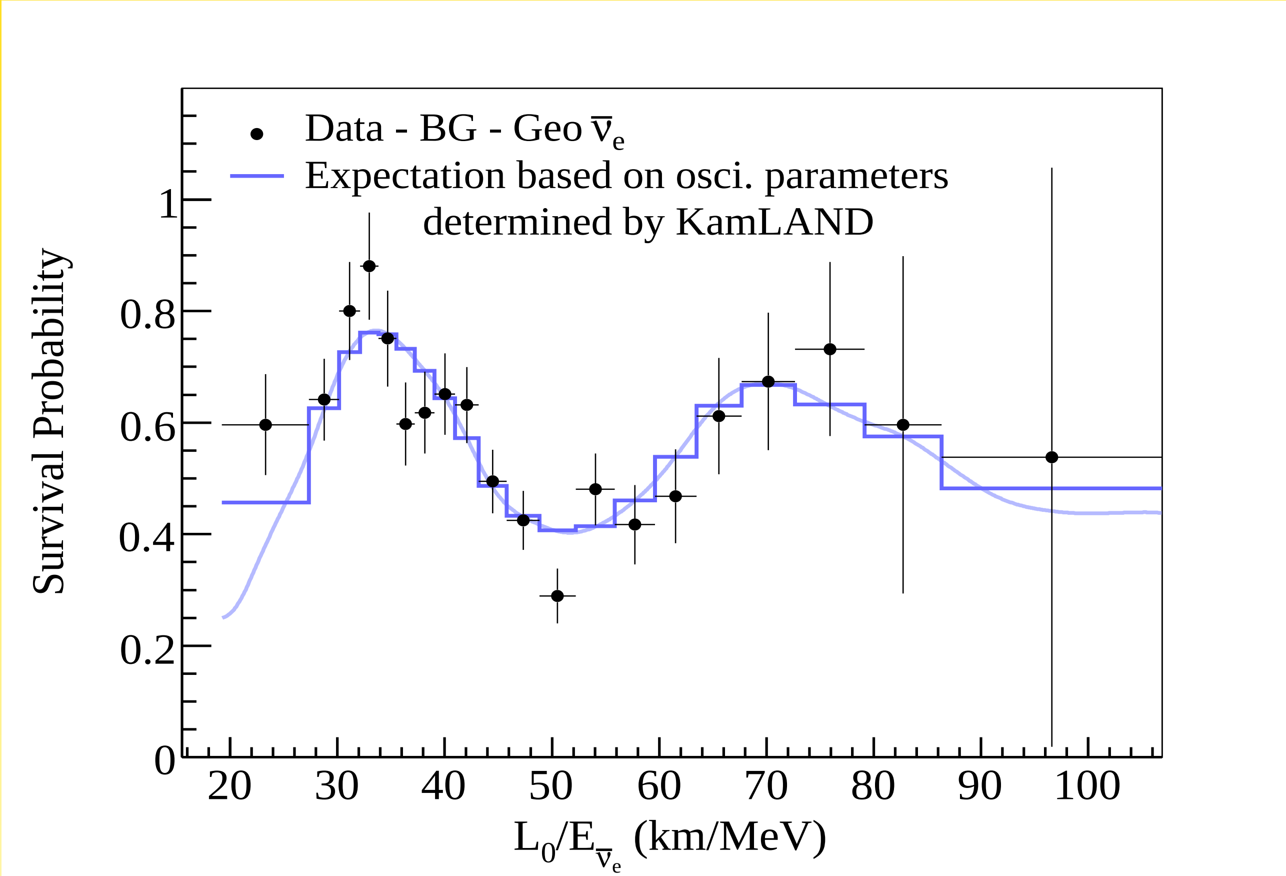




$\overline{\nu_e} + p \to n + e^+$ with $E_\nu \simeq E_{\rm prompt} + E_n^{\rm recoil} + 0.8$ MeV

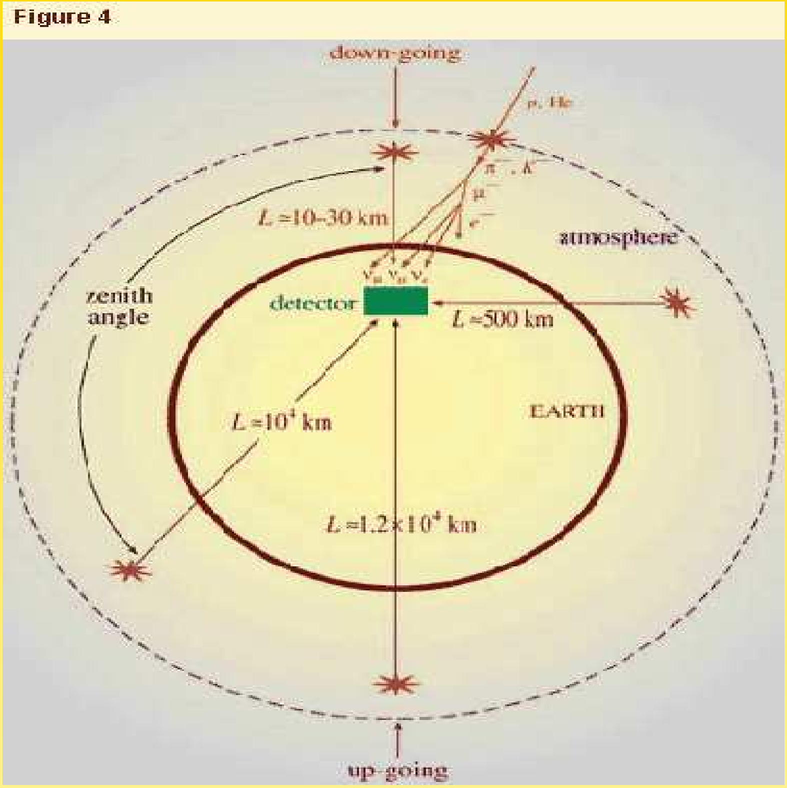
200 μs later: $n + p \to d + \gamma$

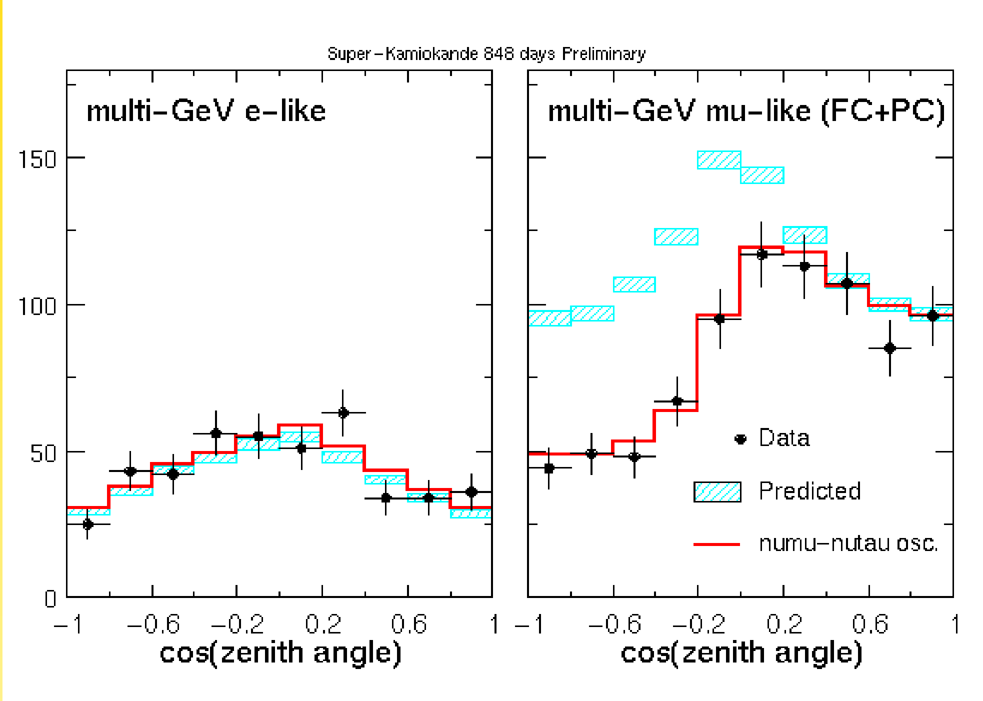
Neutrinos do oscillate



KamLAND

Atmospheric Neutrinos



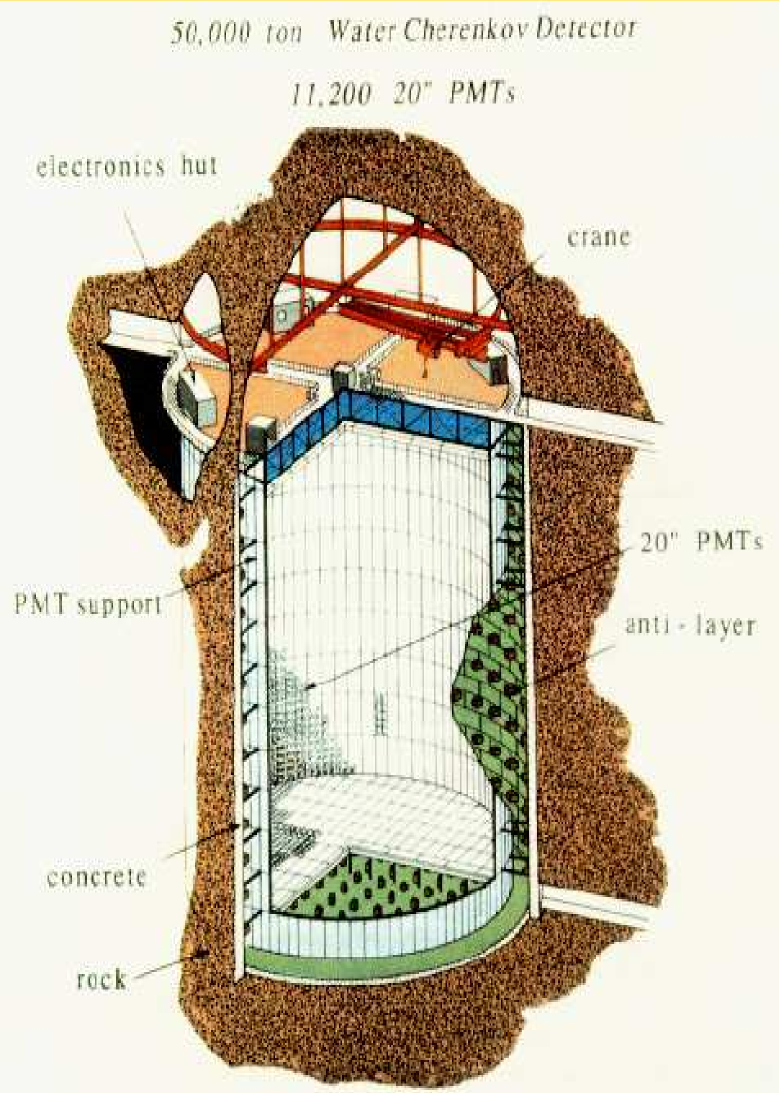


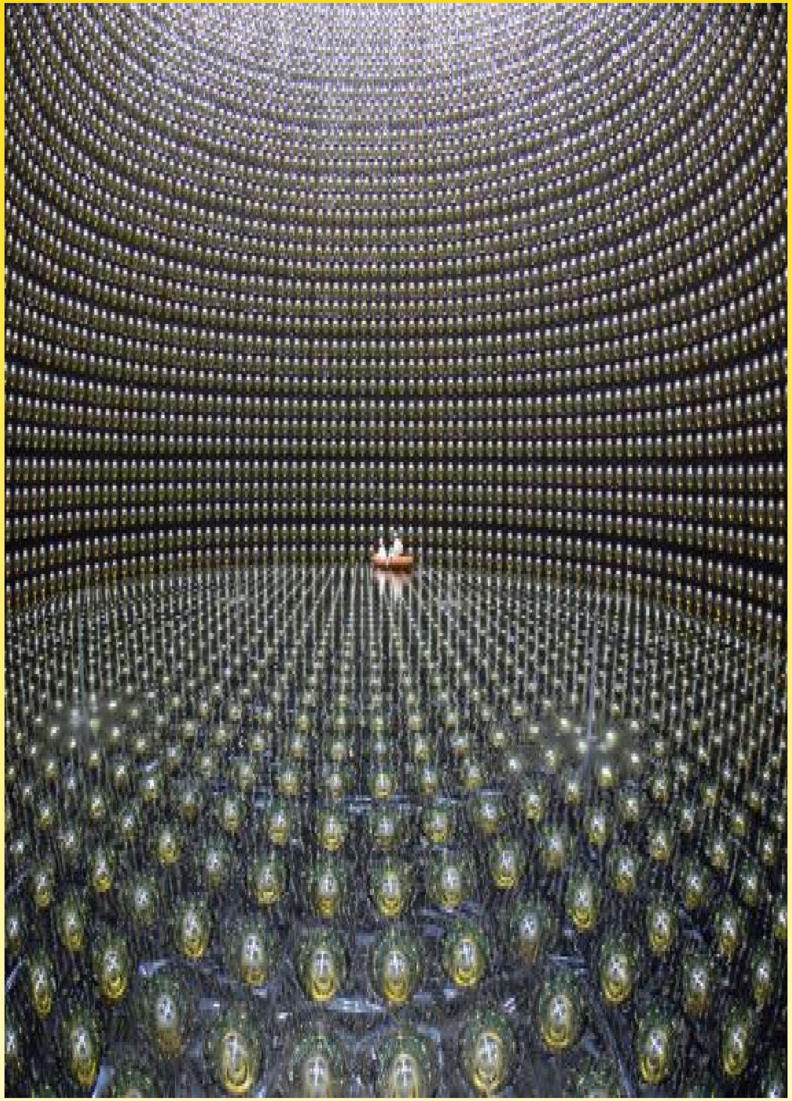
zenith angle $\cos\theta = 1$ $\quad L \simeq 500$ km

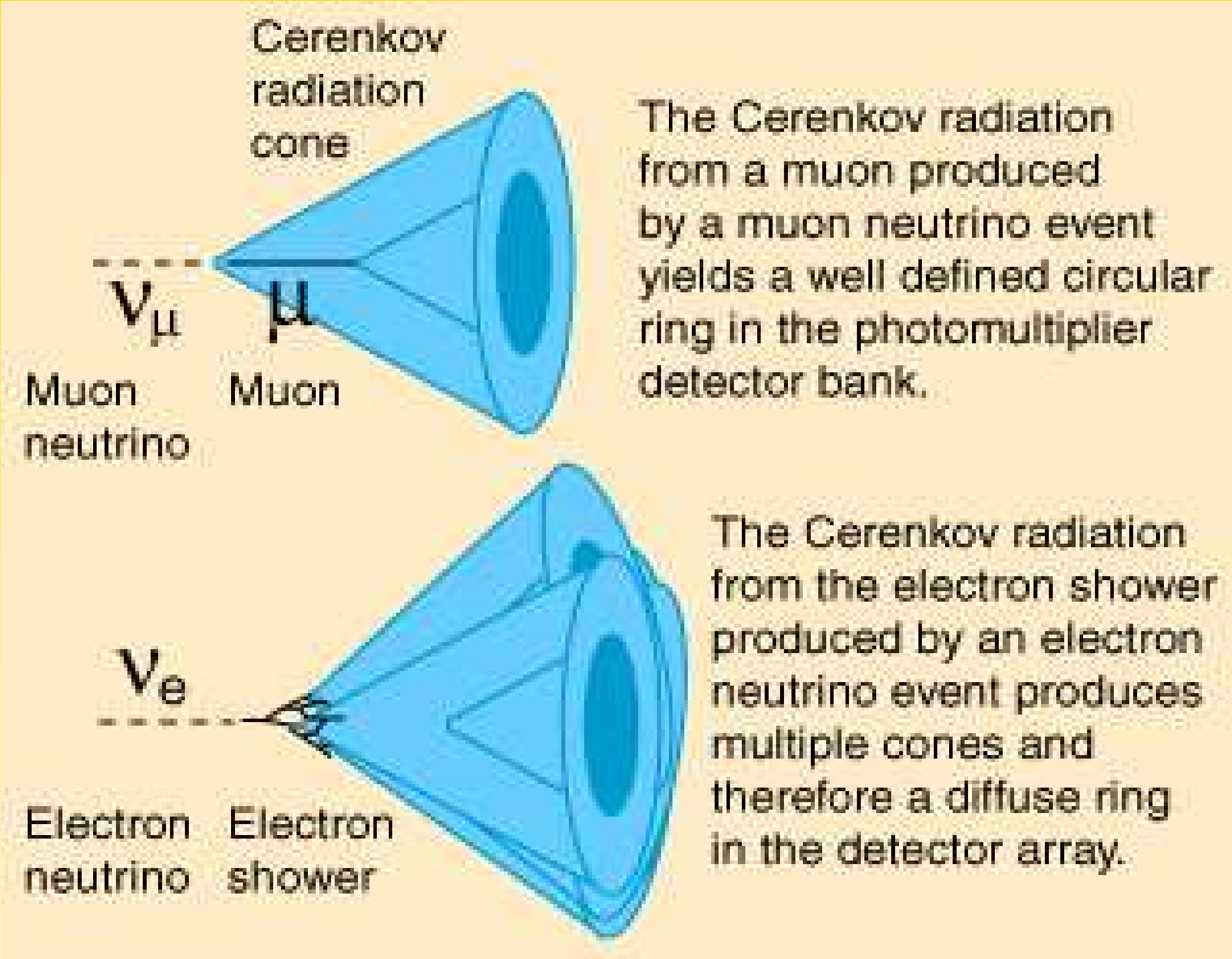
zenith angle $\cos\theta = 0$ $\quad L \simeq 10$ km $\quad$ down-going

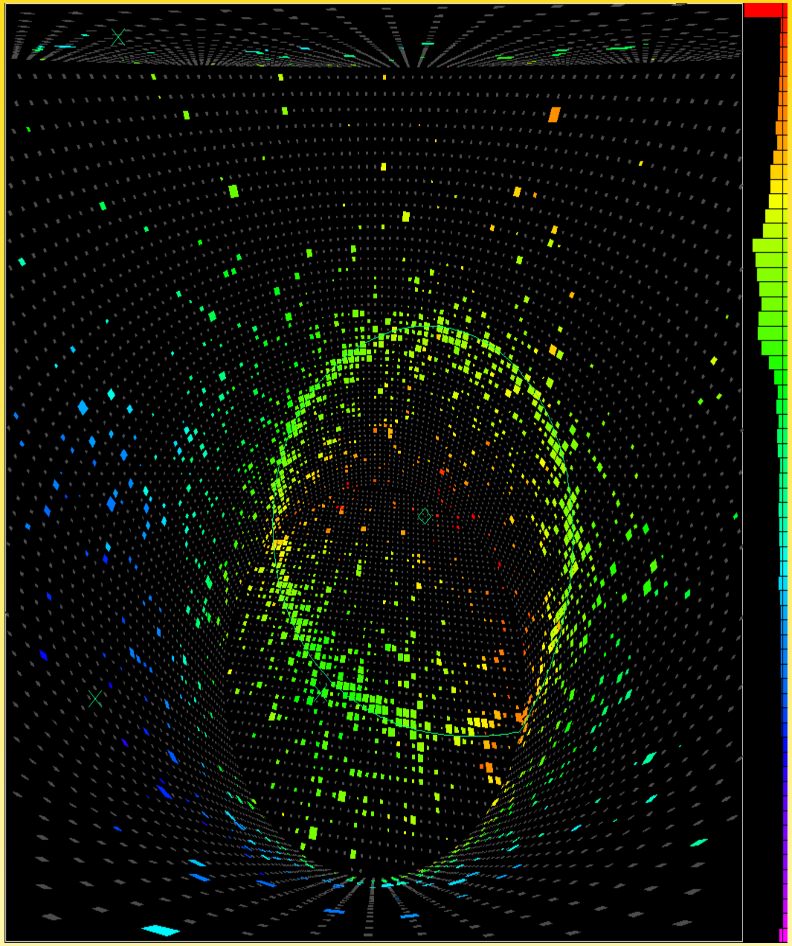
zenith angle $\cos\theta = -1$ $\quad L \simeq 10^4$ km $\quad$ up-going

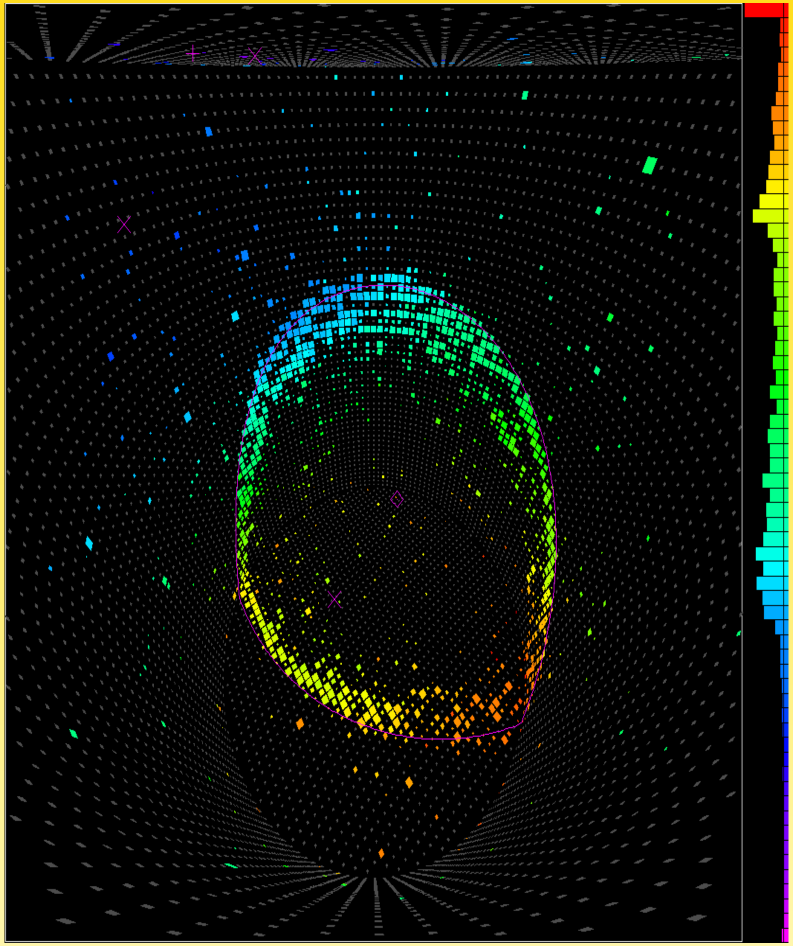
SuperKamiokande



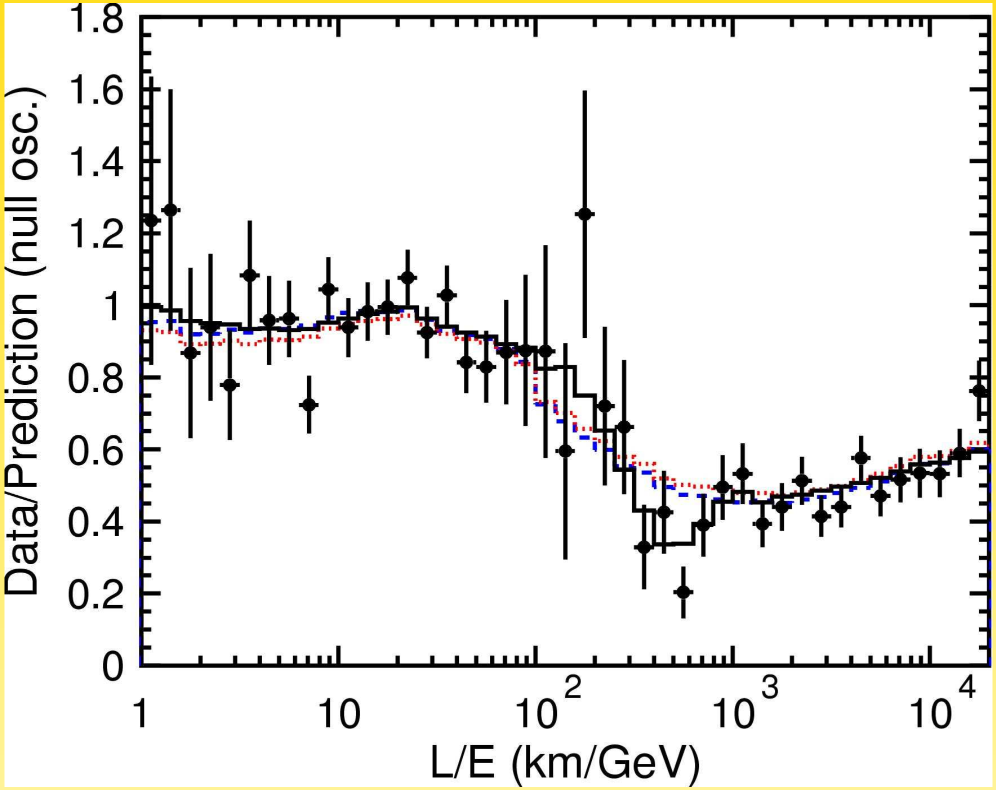



Cerenkov
radiation
cone
ν_μ
μ
Muon
neutrino
Muon
The Cerenkov radiation
from a muon produced
by a muon neutrino event
yields a well defined circular
ring in the photomultiplier
detector bank.
ν_e
Electron
neutrino
Electron
shower
The Cerenkov radiation
from the electron shower
produced by an electron
neutrino event produces
multiple cones and
therefore a diffuse ring
in the detector array.





Atmospheric Neutrinos



Dip at $L/E \simeq 500\ \mathrm{km/GeV} \Rightarrow$ Oscillatory Behavior!!

(3.8 σ evidence for ν_τ appearence)

Testing Atmospheric Neutrinos with Accelerators: K2K, MINOS, T2K, OPERA, NoνA

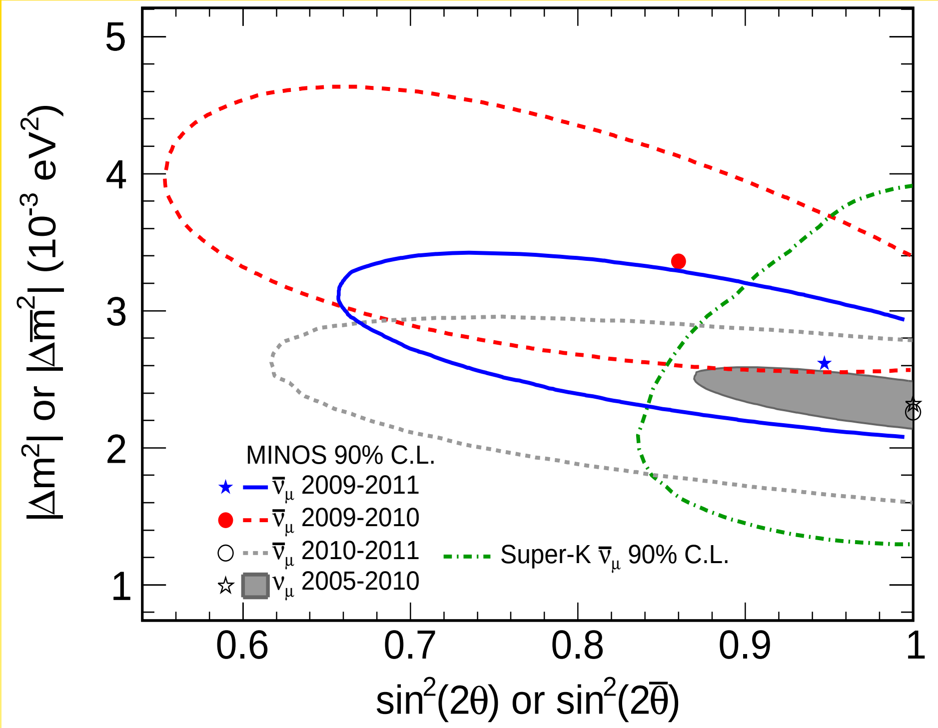
Proton beam

$$p + X \to \pi^{\pm}\,,\; K^{\pm} \to \pi^{\pm} \to \overset{(-)}{\nu}_{\mu} \quad \text{with } E \simeq \mathrm{GeV}$$

If $L \simeq 100$ km:

$$\frac{\Delta m^2_{\mathrm{A}}}{E}\, L \sim 1 \Rightarrow \text{ atmospheric } \nu \text{ parameters!!}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23}\, \sin^2 \frac{\Delta m^2_{31}}{4E} L$$



Results of fits give

$\boxed{\sin^2\theta_{23} \simeq 0.50}$ maximal mixing?!

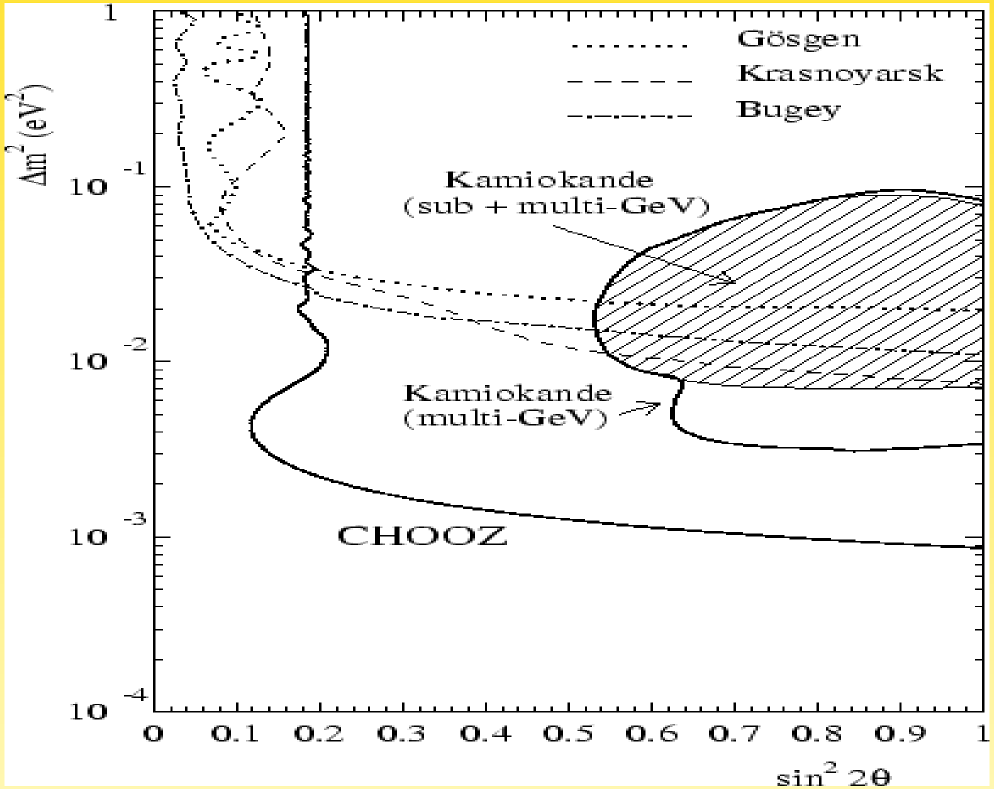
$$\boxed{|\Delta m^2_{31}| \equiv \Delta m^2_{\rm A} \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2} \simeq 30\,\Delta m^2_\odot$$

NEW TREND (2012): less-than-maximal θ_{23}

The third mixing: Short-Baseline Reactor Neutrinos

$E_\nu \simeq \mathrm{MeV}$ and $L \simeq 0.1$ km:

$\frac{\Delta m^2_{\mathrm{A}}}{E} L \sim 1 \Rightarrow$ atmospheric ν parameters!!



$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{\mathrm{A}}}{4E} L$$

3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

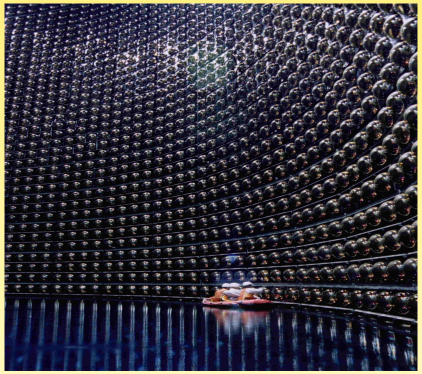
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

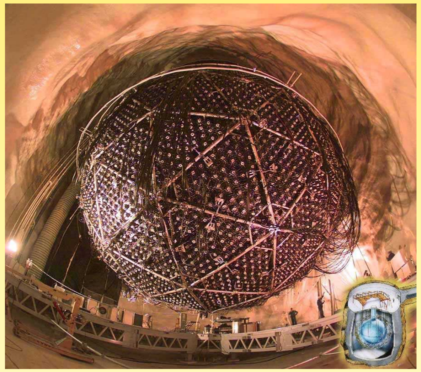
$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

$$
U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} =
$$

$$
\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}}
\underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}}
\underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}
$$





$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2\theta_{23} = \tfrac{1}{2}) \qquad (\sin^2\theta_{13} = 0) \qquad (\sin^2\theta_{12} = \tfrac{1}{3})$$

$$\Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\odot}$$

Tri-bimaximal Mixing

approximation to PMNS matrix:

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$ Harrison, Perkins, Scott (2002)

with mass matrix

$$(m_\nu)_{\rm TBM} = U_{\rm TBM}^* \, m_\nu^{\rm diag} \, U_{\rm TBM}^\dagger = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$A = \frac{1}{3}\left(2\, m_1 + m_2\, e^{-2i\alpha}\right) \,, \quad B = \frac{1}{3}\left(m_2\, e^{-2i\alpha} - m_1\right) \,, \quad D = m_3\, e^{-2i\beta}$$

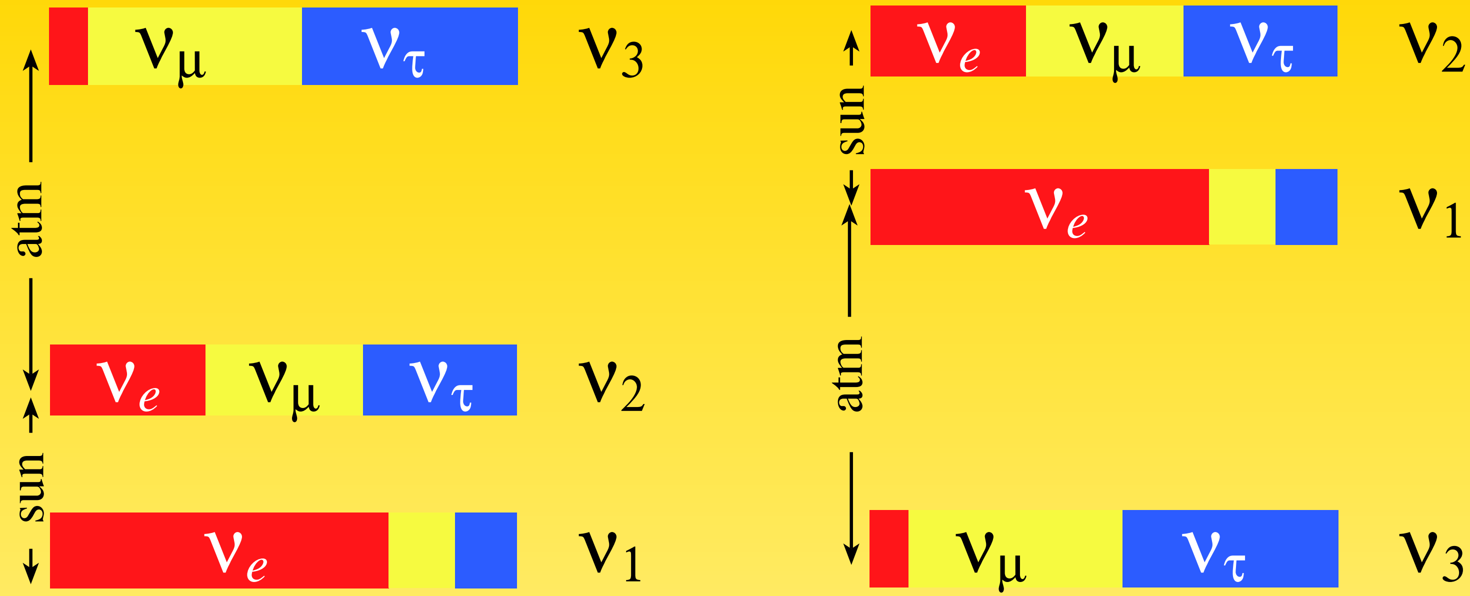
$\Rightarrow$ Flavor symmetries...

Tri-bimaximal Mixing

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

`Harrison, Perkins, Scott (2002)`

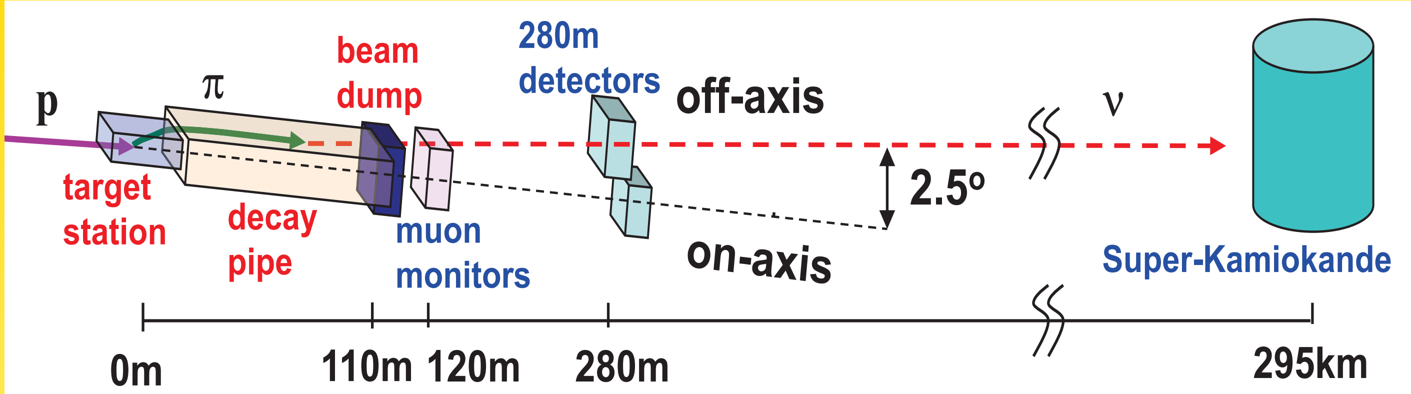
This was still okay till end of 2010...



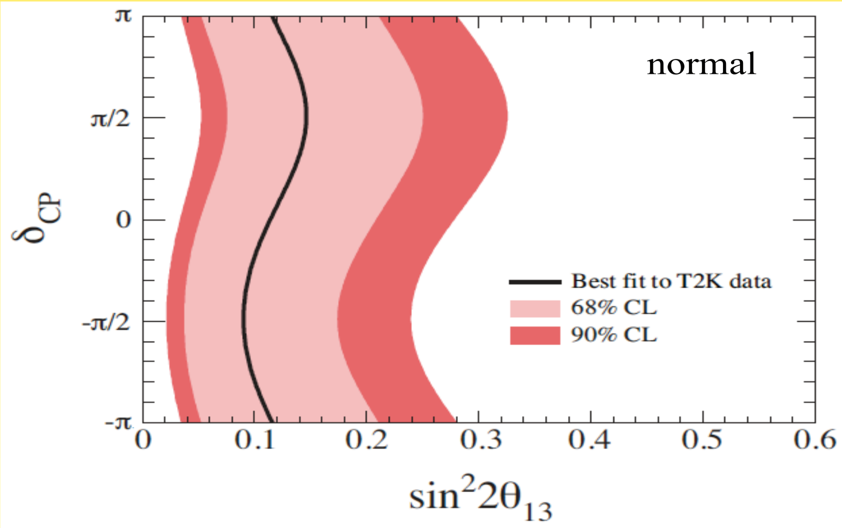
$$|U|^2 \simeq \begin{pmatrix} 0.779\ldots0.848 & 0.510\ldots0.604 & 0.122\ldots0.190 \\ 0.183\ldots0.568 & 0.385\ldots0.728 & 0.613\ldots0.794 \\ 0.200\ldots0.576 & 0.408\ldots0.742 & 0.589\ldots0.775 \end{pmatrix}$$

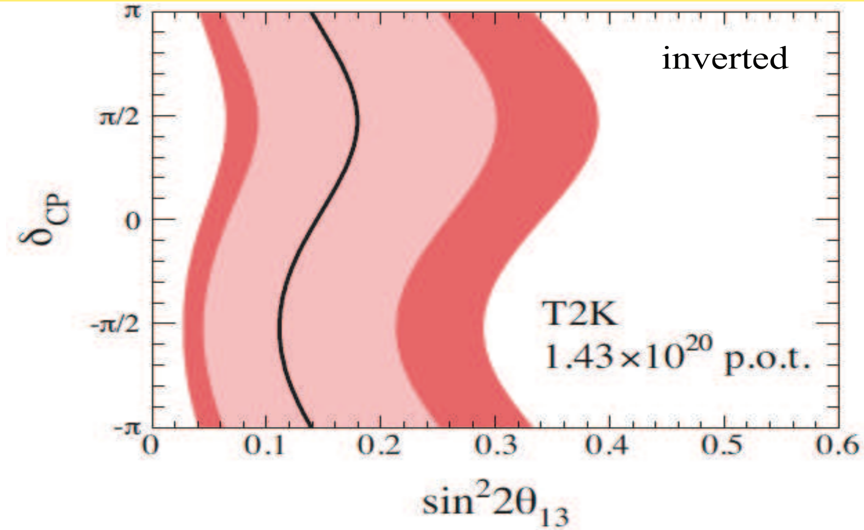
- normal ordering: $\Delta m^2_{31} > 0$
- inverted ordering: $\Delta m^2_{31} < 0$

T2K: 2.5σ



$$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\frac{\Delta m_A^2}{4E}L$$

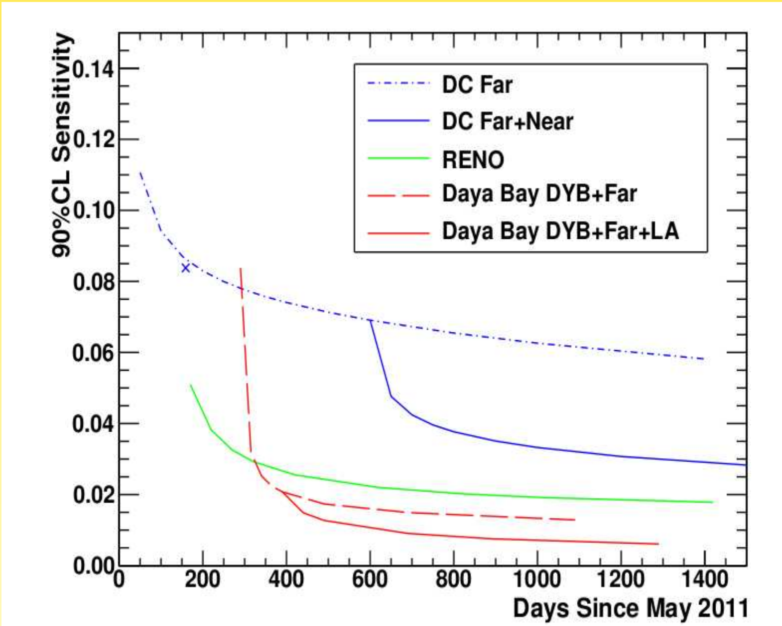




More data

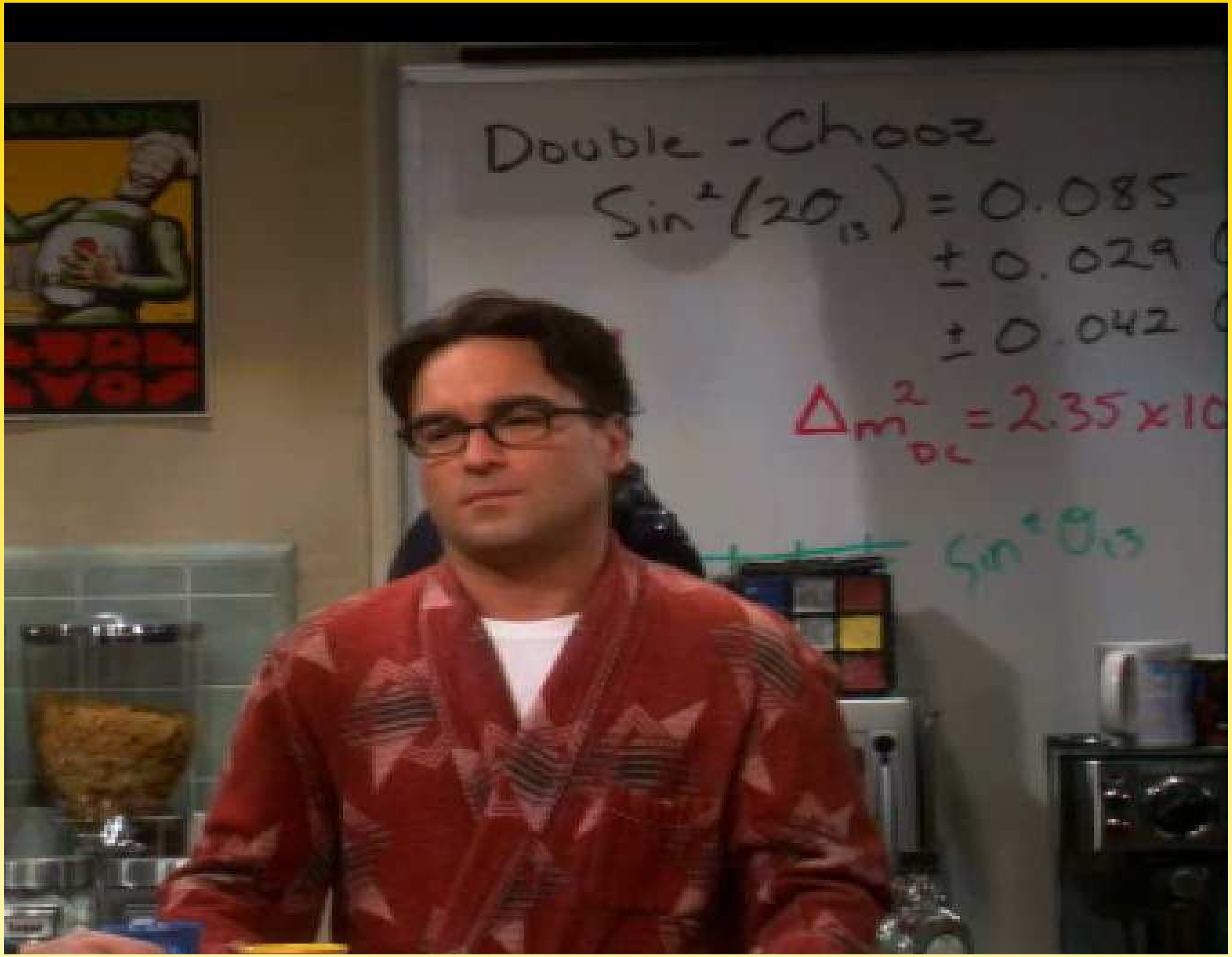
- MINOS: 1.7σ
- Double Chooz: $0.017 < \sin^2 2\theta_{13} < 0.16$ at 90 % C.L.

$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{\rm A}}{4E} L$$



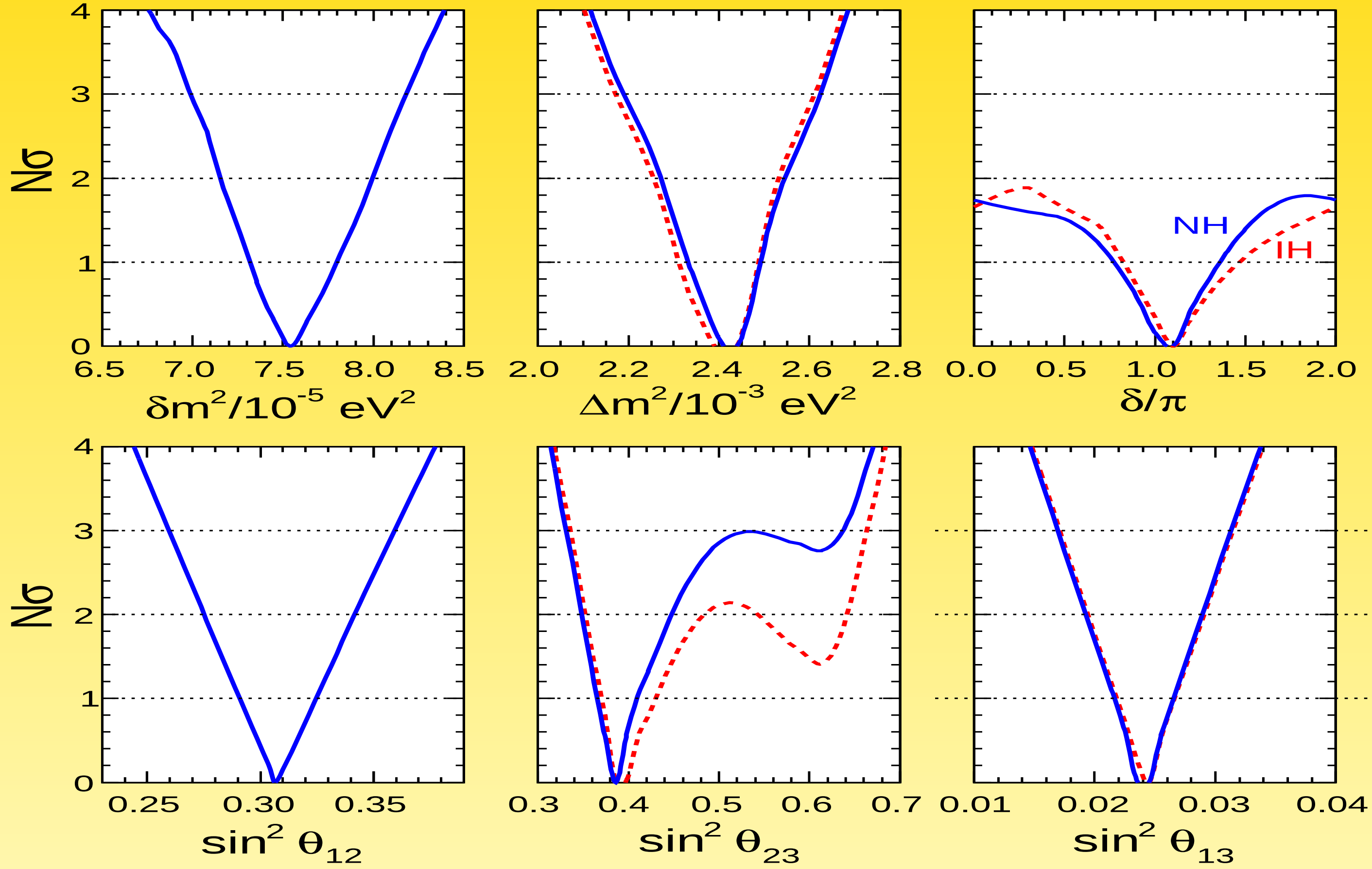
θ_{13}: status

Double Chooz:	$\sin^2 2\theta_{13} = 0.086 \pm 0.051$	$\neq 0$ at 1.9σ (3.1)
Daya Bay:	$\sin^2 2\theta_{13} = 0.092 \pm 0.017$	$\neq 0$ at 5.2σ (> 7)
RENO:	$\sin^2 2\theta_{13} = 0.113 \pm 0.023$	$\neq 0$ at 4.9σ

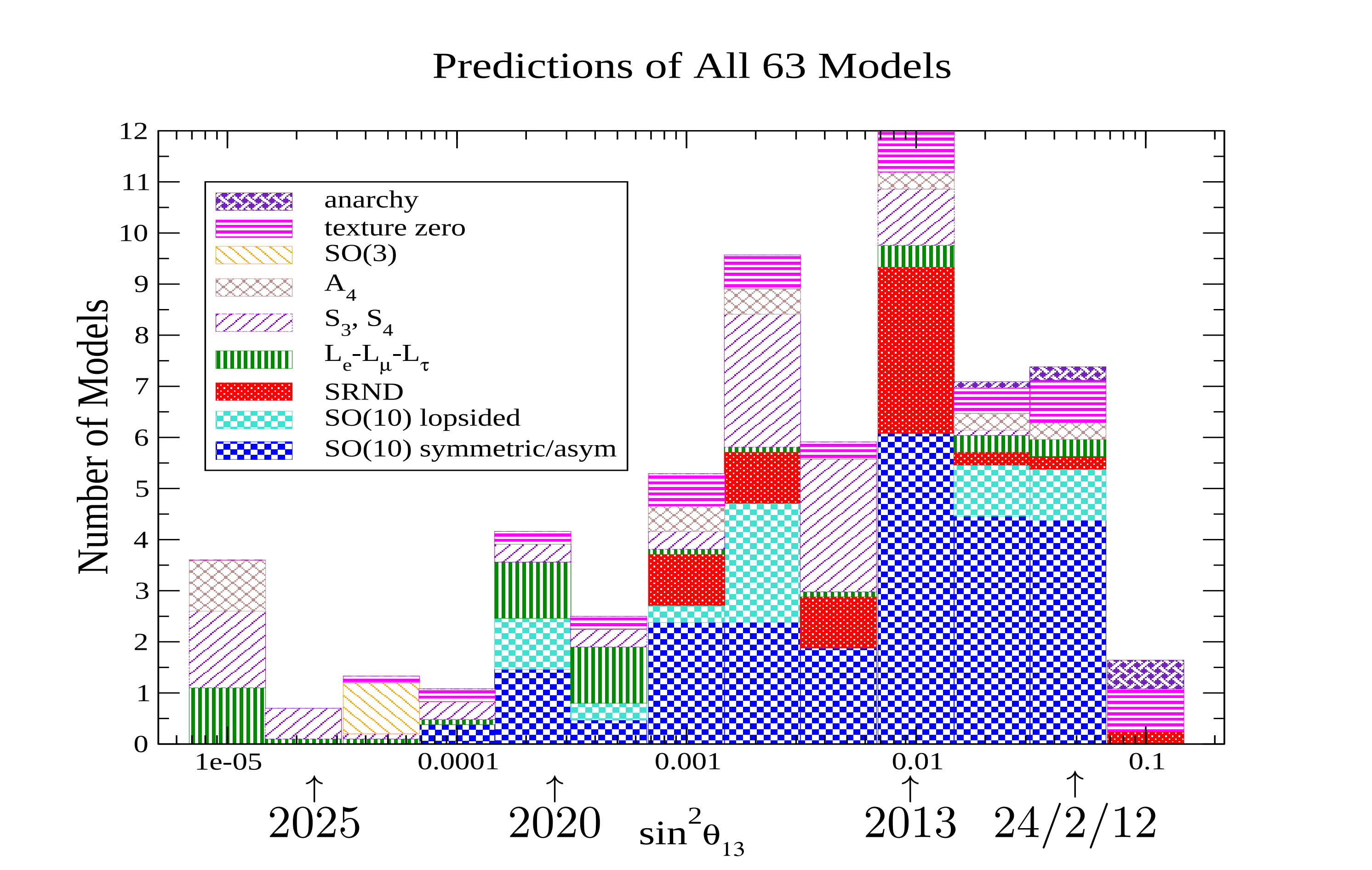


at least, Double Chooz are the only ones who made it to Big Bang Theory...

Synopsis of global 3ν oscillation analysis



What's that good for?



Albright, Chen

CKM vs. PMNS

$$|V_{\mathrm{CKM}}| \simeq \begin{pmatrix} 0.97419 & 0.2257 & 0.00359 \\ 0.2256 & 0.97334 & 0.0415 \\ 0.00874 & 0.0407 & 0.999133 \end{pmatrix}$$

$$|U_{\mathrm{PMNS}}| \simeq \begin{pmatrix} 0.82 & 0.58 & 0 \\ 0.64 & 0.58 & 0.71 \\ 0.64 & 0.58 & 0.71 \end{pmatrix}$$

Contents

II Neutrino Oscillations

II4) Prospects – what do we want to know?

9 physical parameters in m_ν

- θ_{12} and $m_2^2 - m_1^2$ (or $\theta_\odot$ and $\Delta m_\odot^2$)
- θ_{23} and $|m_3^2 - m_2^2|$ (or θ_A and Δm_A^2)
- θ_{13} (or $|U_{e3}|$)
- m_1, m_2, m_3
- sgn$(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β (or α_1 and α_2, or ϕ_1 and ϕ_2, or$\ldots$)

The future: open issues for neutrinos oscillations

Look for *three flavor effects:*

- precision measurements
  - how maximal is θ_{23} ? how small/large is U_{e3} ?
- sign of Δm^2_{32} ?

$$\tan 2\theta_m = f(\mathrm{sgn}(\Delta m^2))$$

- is there CP violation?
- Problems:
  - two *small* parameters: $\Delta m^2_{\odot}/\Delta m^2_{\mathrm{A}} \simeq 1/30$ and $|U_{e3}| \lesssim 0.2$
  - 8-fold degeneracy for fixed L/E and $\nu_e \to \nu_\mu$ channels

Degeneracies

Expand *3 flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_\mathrm{A}$ and $|U_{e3}|$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2\ \sin^2 2\theta_{12}\ \cos^2\theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+\sin\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23} \sin\Delta\ \frac{\sin \hat{A}\Delta\ \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+\cos\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23}\ \cos\Delta \frac{\sin \hat{A}\Delta\ \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

with $\hat{A} = 2\sqrt{2}\, G_F\, n_e\, E / \Delta m^2_\mathrm{A}$ and $\Delta = \frac{\Delta m^2_\mathrm{A}}{4\, E}\, L$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13}-δ degeneracy
- δ-sgn(Δm^2_A) degeneracy

Solutions: more channels, different L/E, high precision,...

Degeneracies

Expand *3 flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_{\rm A}$ and $|U_{e3}|$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \ \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2\ \sin^2 2\theta_{12}\ \cos^2\theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+\sin\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23} \sin\Delta\ \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+\cos\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23}\ \cos\Delta \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

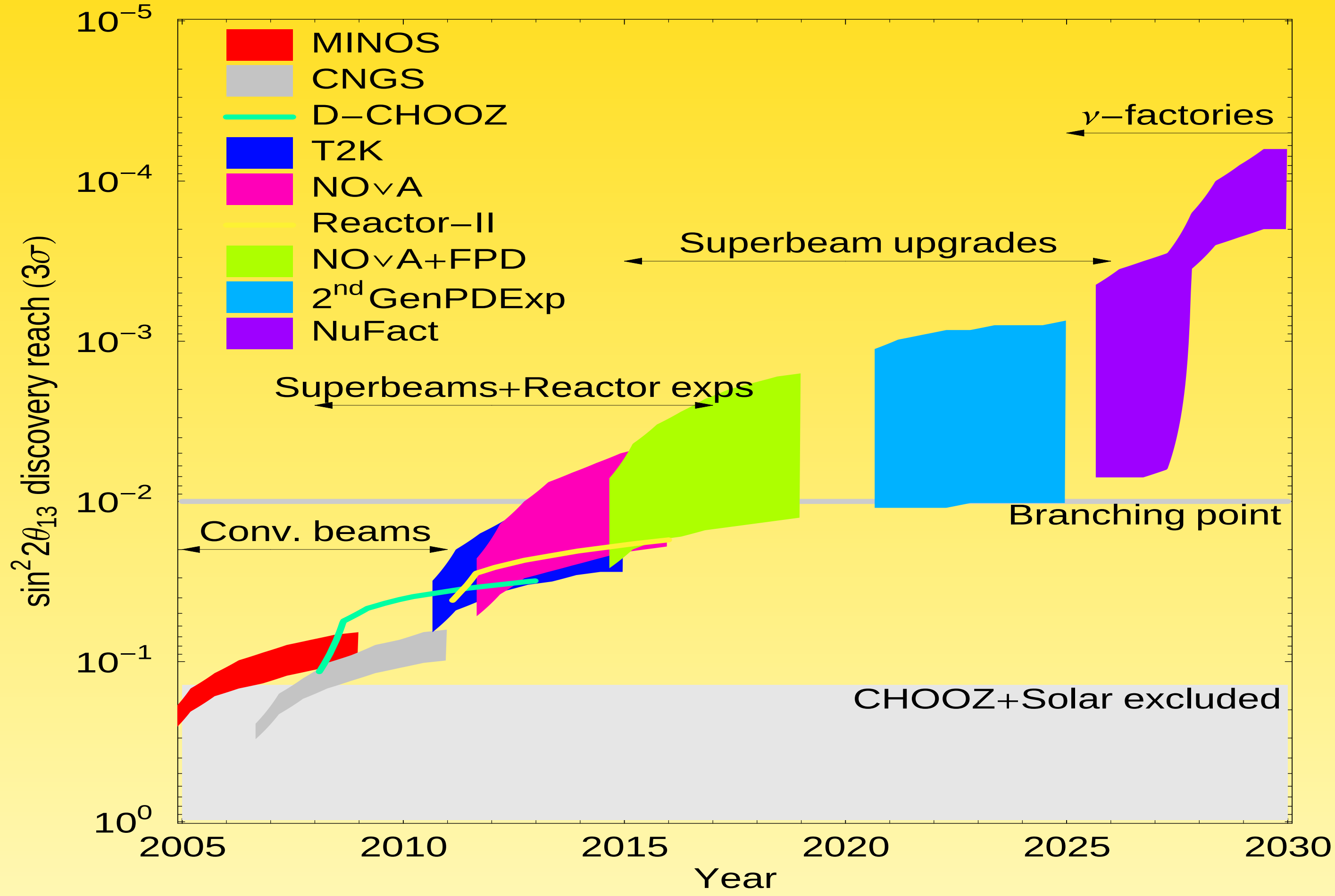
with $\hat{A} = 2\sqrt{2}\, G_F\, n_e\, E / \Delta m^2_{\rm A}$ and $\Delta = \frac{\Delta m^2_{\rm A}}{4\,E}\, L$

If $\hat{A}\,\Delta = \pi$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \ \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2}$$

This is the "magic baseline" of $L = \frac{\sqrt{2}\pi}{G_F\, n_e} \simeq 7500$ km

Typical time scale

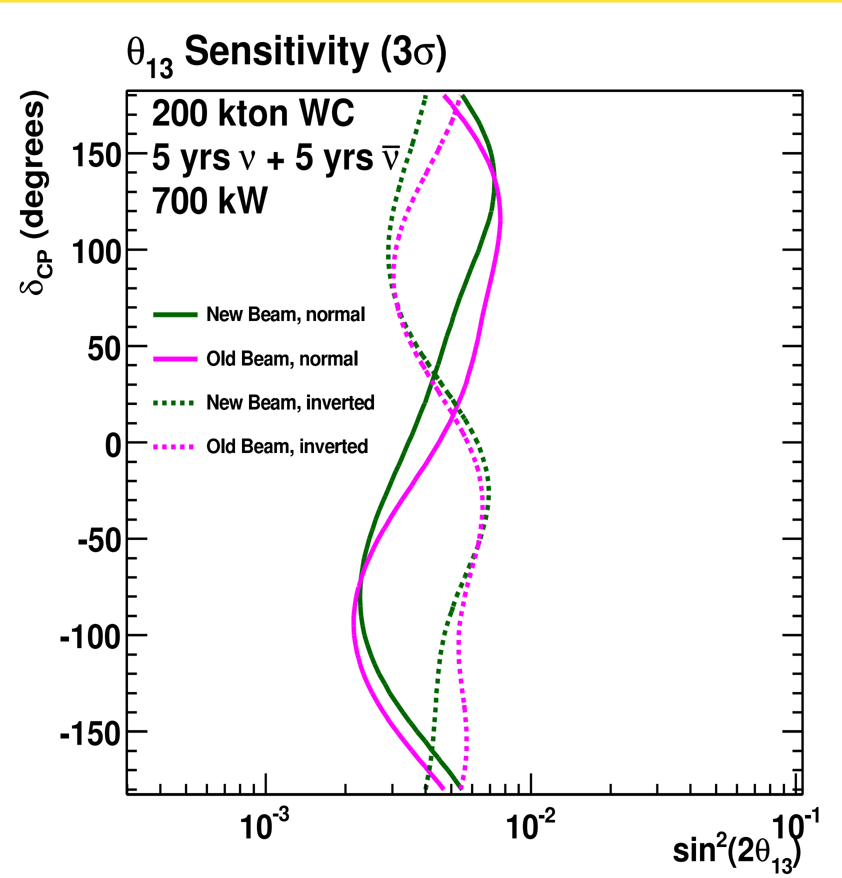


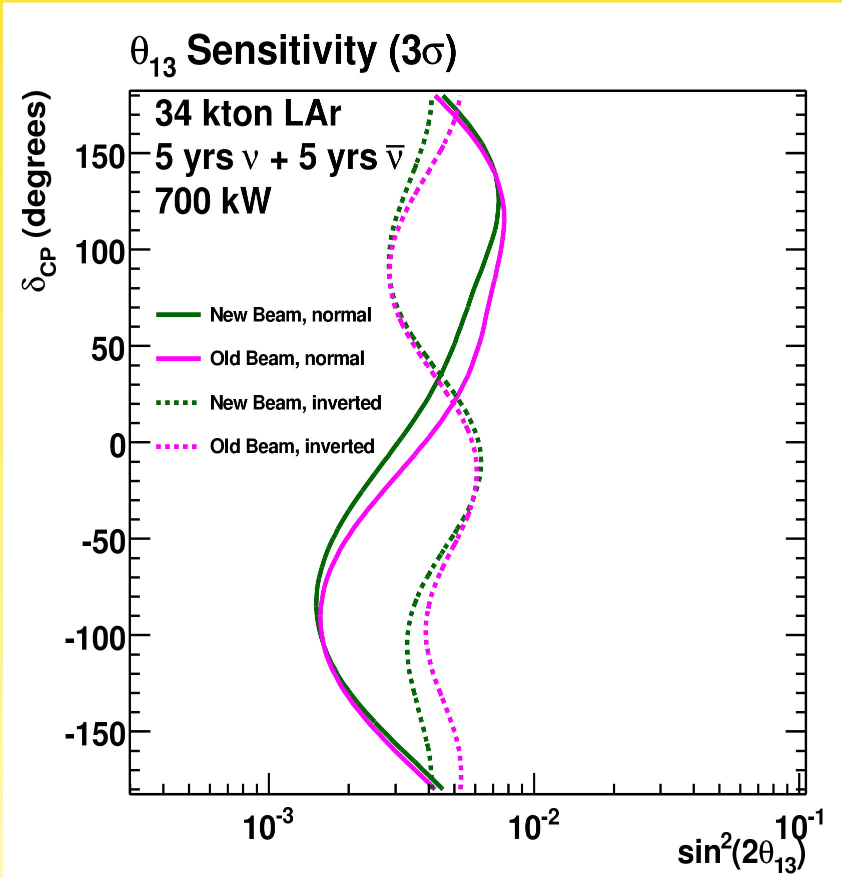
Future experiments

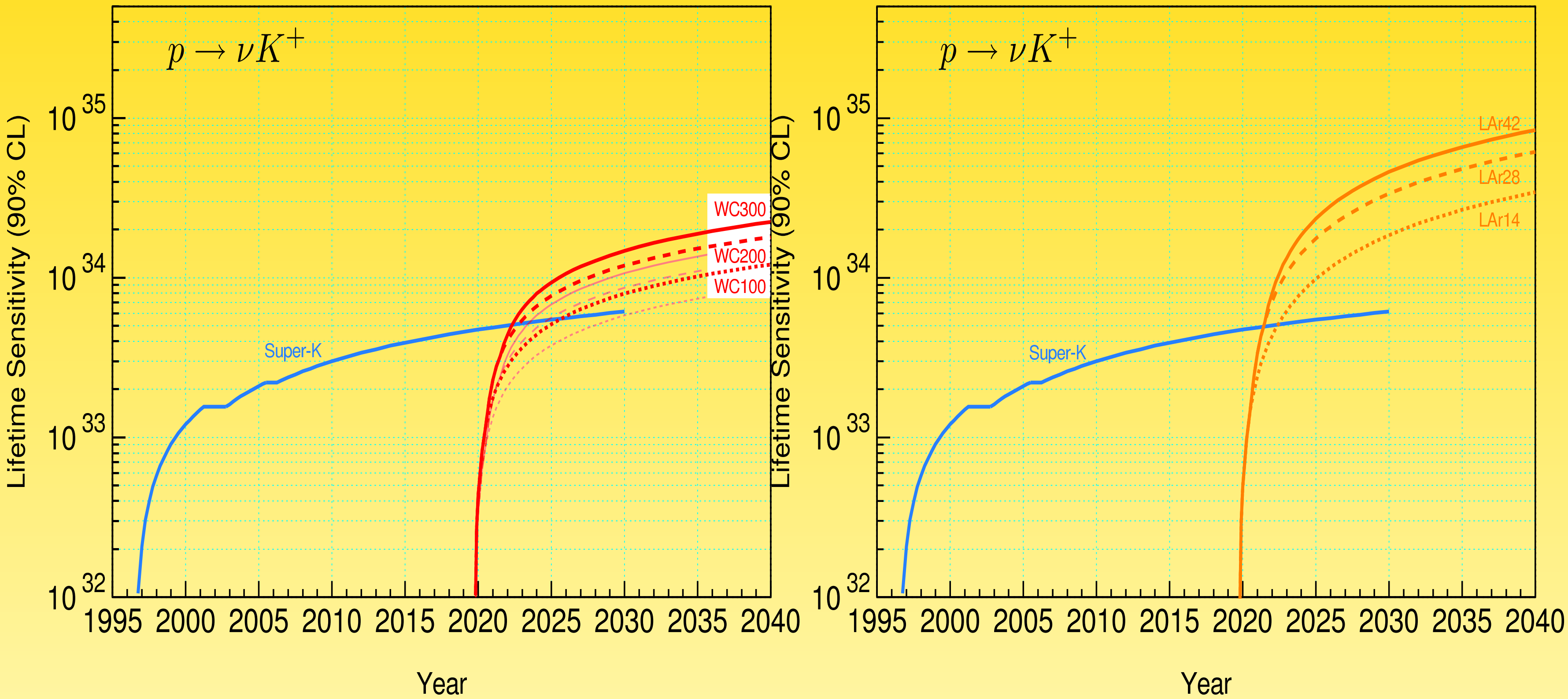
- what detector?
  - Water Cerenkov?
  - liquid scintillator?
  - liquid argon?
- Neutrino Physics
  - oscillations (hierarchy, CP, precision)
  - non-standard physics (NSIs, unitarity violation, steriles, extra forces,. . .)
- other physics
  - SN (burst and relic)
  - geo-neutrinos
  - p-decay

Example LBNE

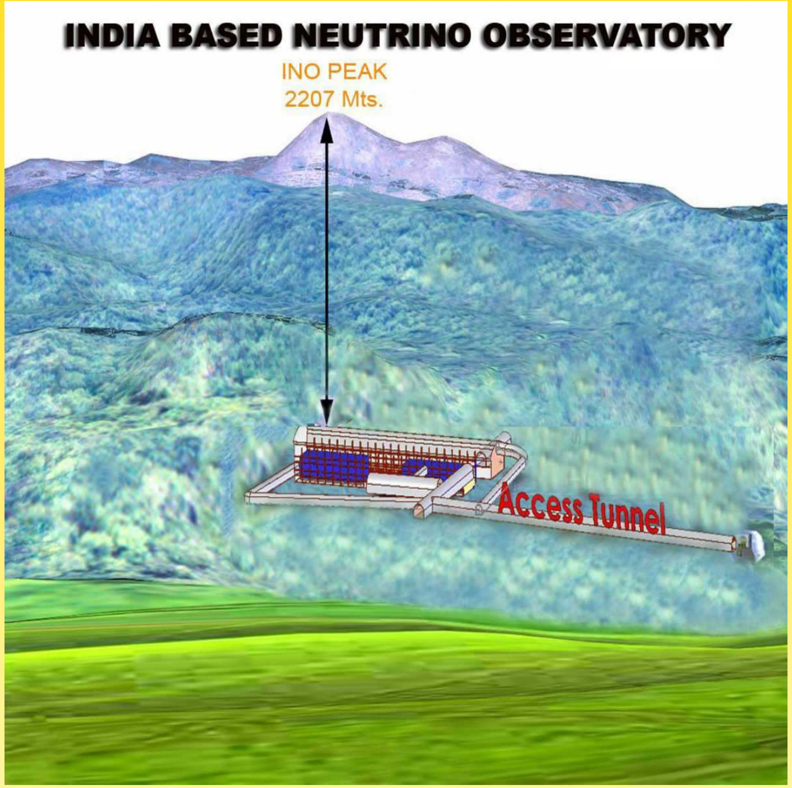
FNAL $\to$ Homestake, $L = 1300$ km

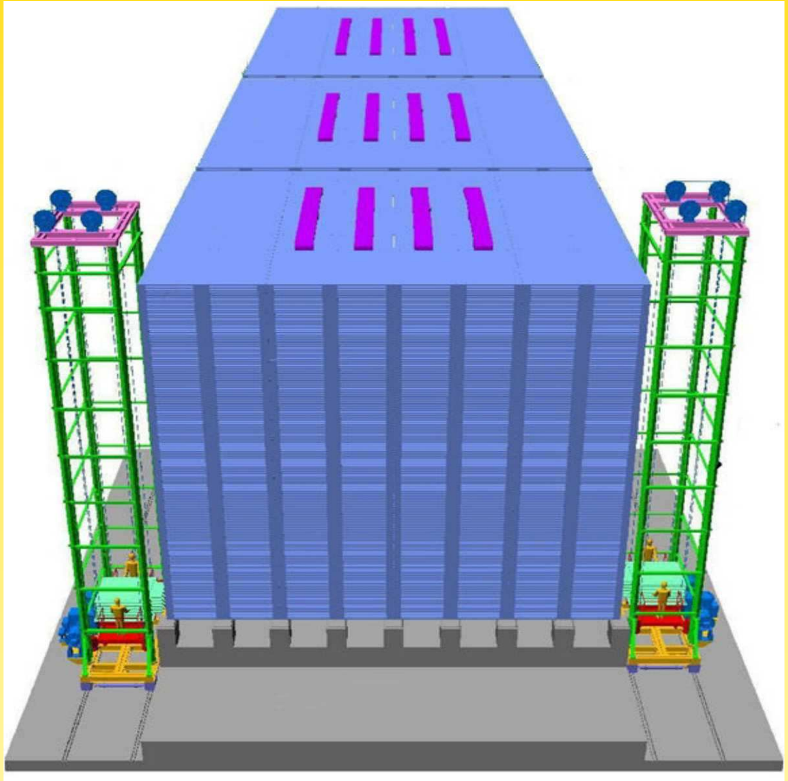





$p \to \nu K^+$
Lifetime Sensitivity (90% CL)
10^{35}
10^{34}
10^{33}
10^{32}
WC300
WC200
WC100
Super-K
1995 2000 2005 2010 2015 2020 2025 2030 2035 2040
Year
$p \to \nu K^+$
Lifetime Sensitivity (90% CL)
LAr42
LAr28
LAr14
Super-K
Year

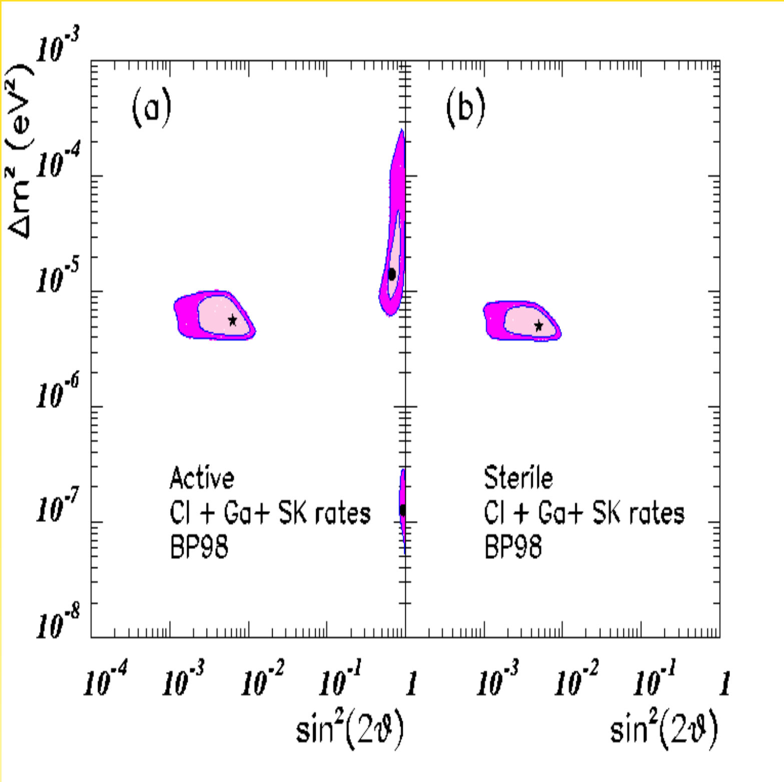
Example ICAL at INO



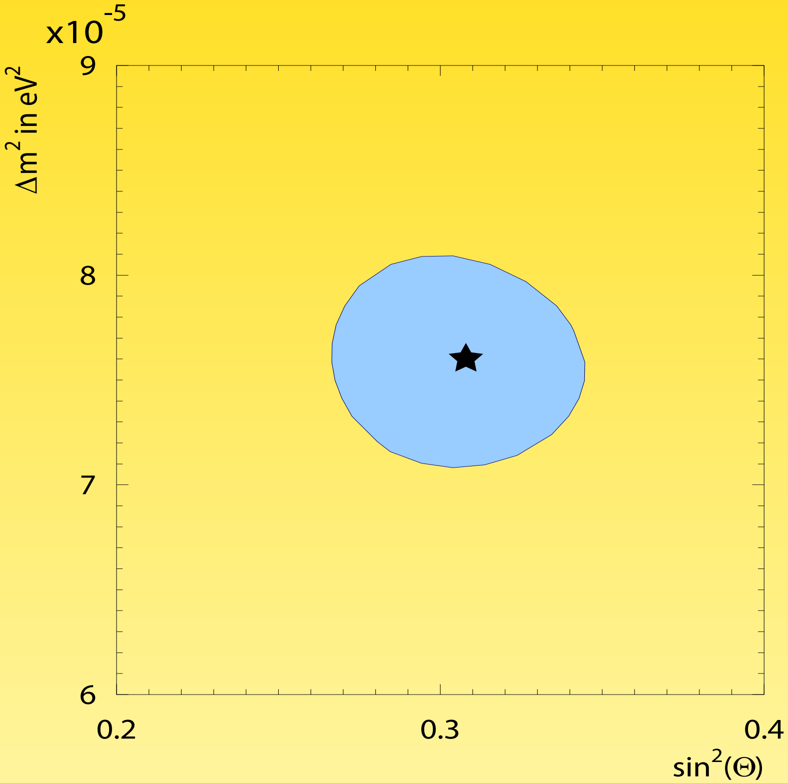


7300 km from CERN, 6600 km from JHF at Tokai

Precision era!

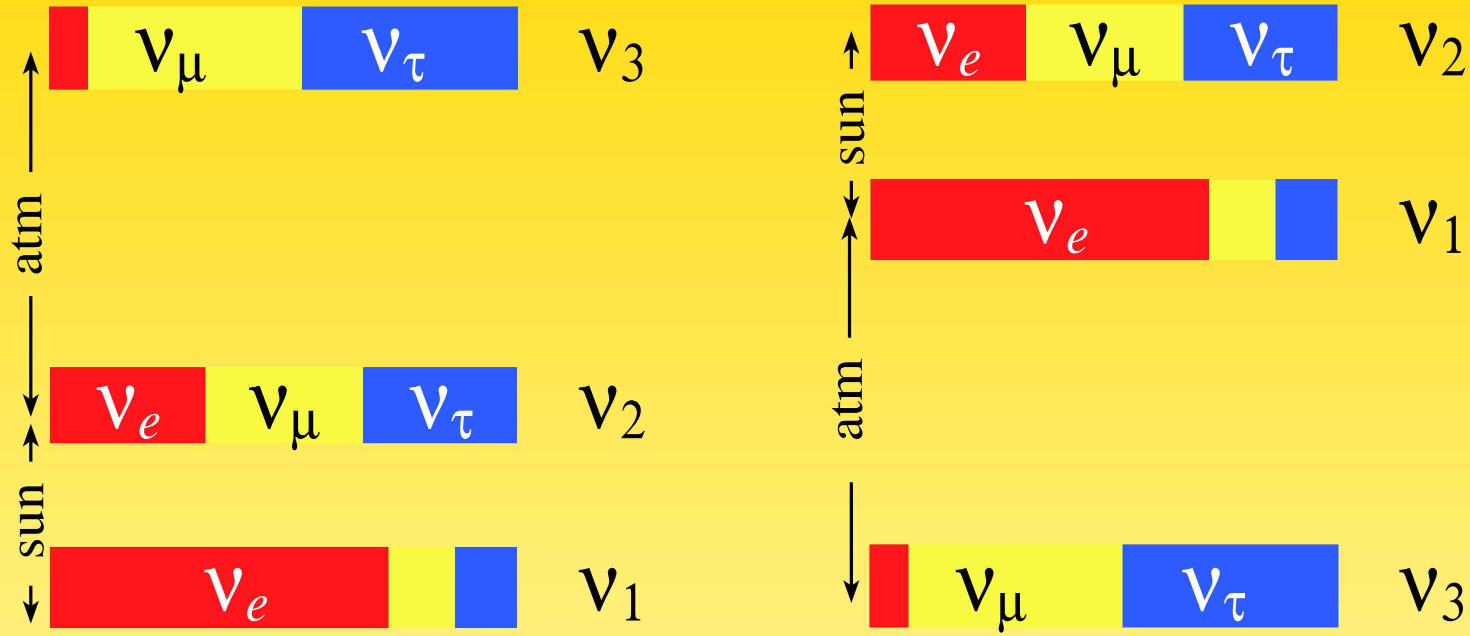


1998



today

Limits on neutrino mass(es)



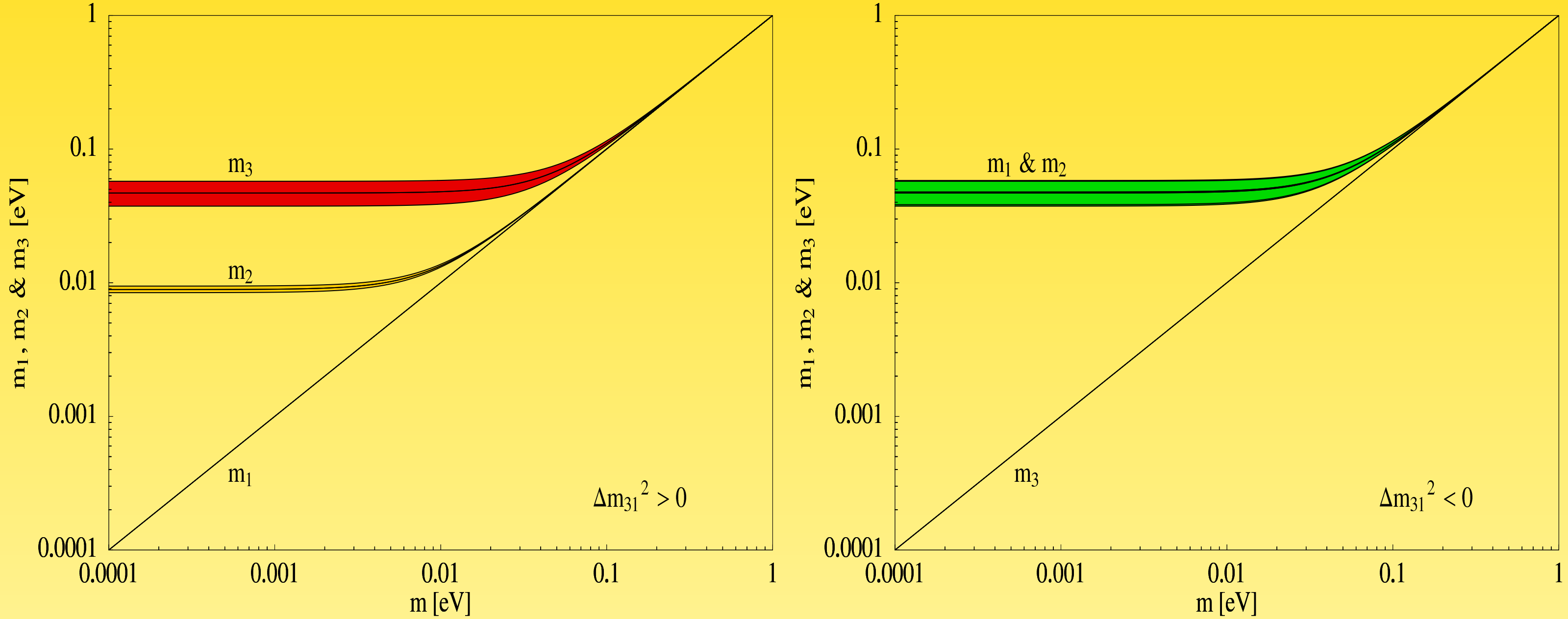
$\boxed{\Delta m^2_{21} = 8 \times 10^{-5}\ \mathrm{eV}^2}$ and $\boxed{|\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = 2 \times 10^{-3}\ \mathrm{eV}^2}$

- normal ordering: $\Delta m^2_{31} > 0$
- inverted ordering: $\Delta m^2_{31} < 0$

We don't know the zero point!

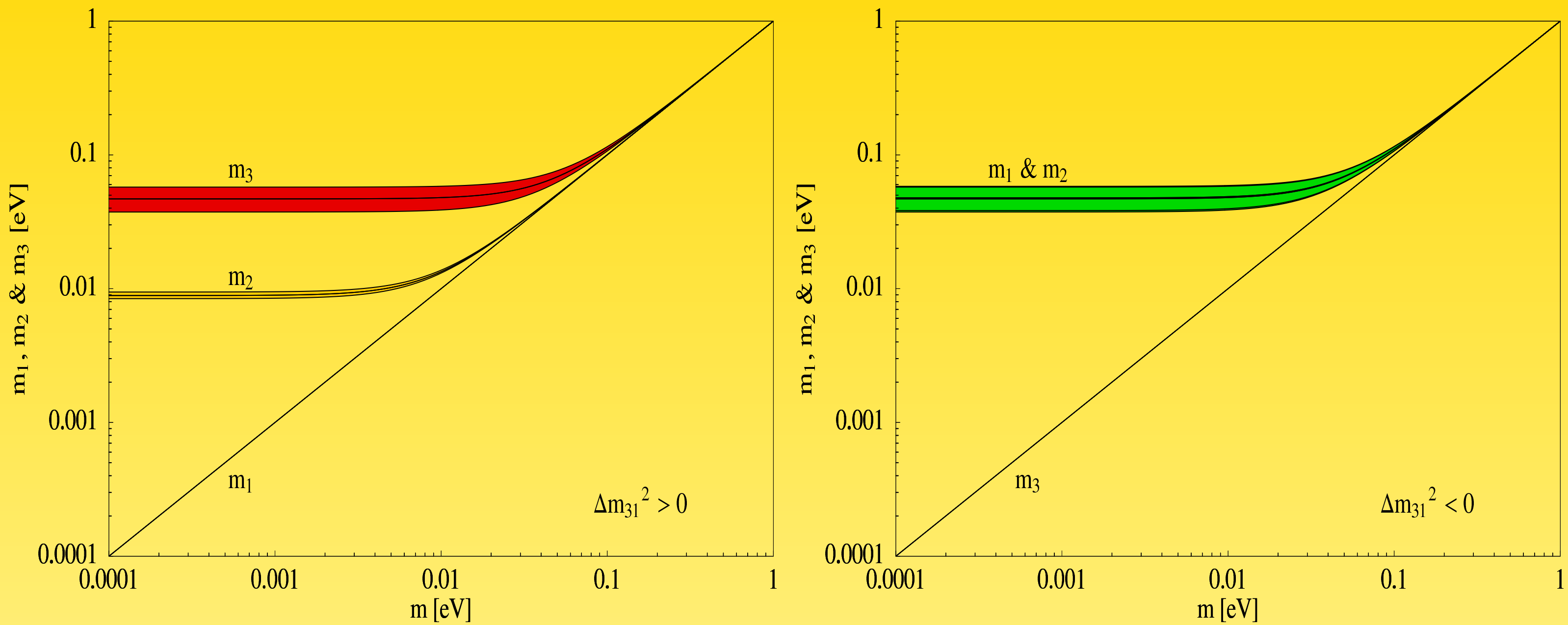
Neutrino masses

- neutrino masses $\leftrightarrow$ scale of their origin
- neutrino mass ordering $\leftrightarrow$ form of m_ν



- $m_3^2 \simeq \Delta m_{\rm A}^2 \gg m_2^2 \simeq \Delta m_\odot^2 \gg m_1^2$: normal hierarchy (NH)
- $m_2^2 \simeq |\Delta m_{\rm A}^2| \simeq m_1^2 \gg m_3^2$: inverted hierarchy (IH)
- $m_3^2 \simeq m_2^2 \simeq m_1^2 \equiv m_0^2 \gg \Delta m_{\rm A}^2$: quasi-degeneracy (QD)

Neutrino masses



Neutrino mass hierarchy is moderate!

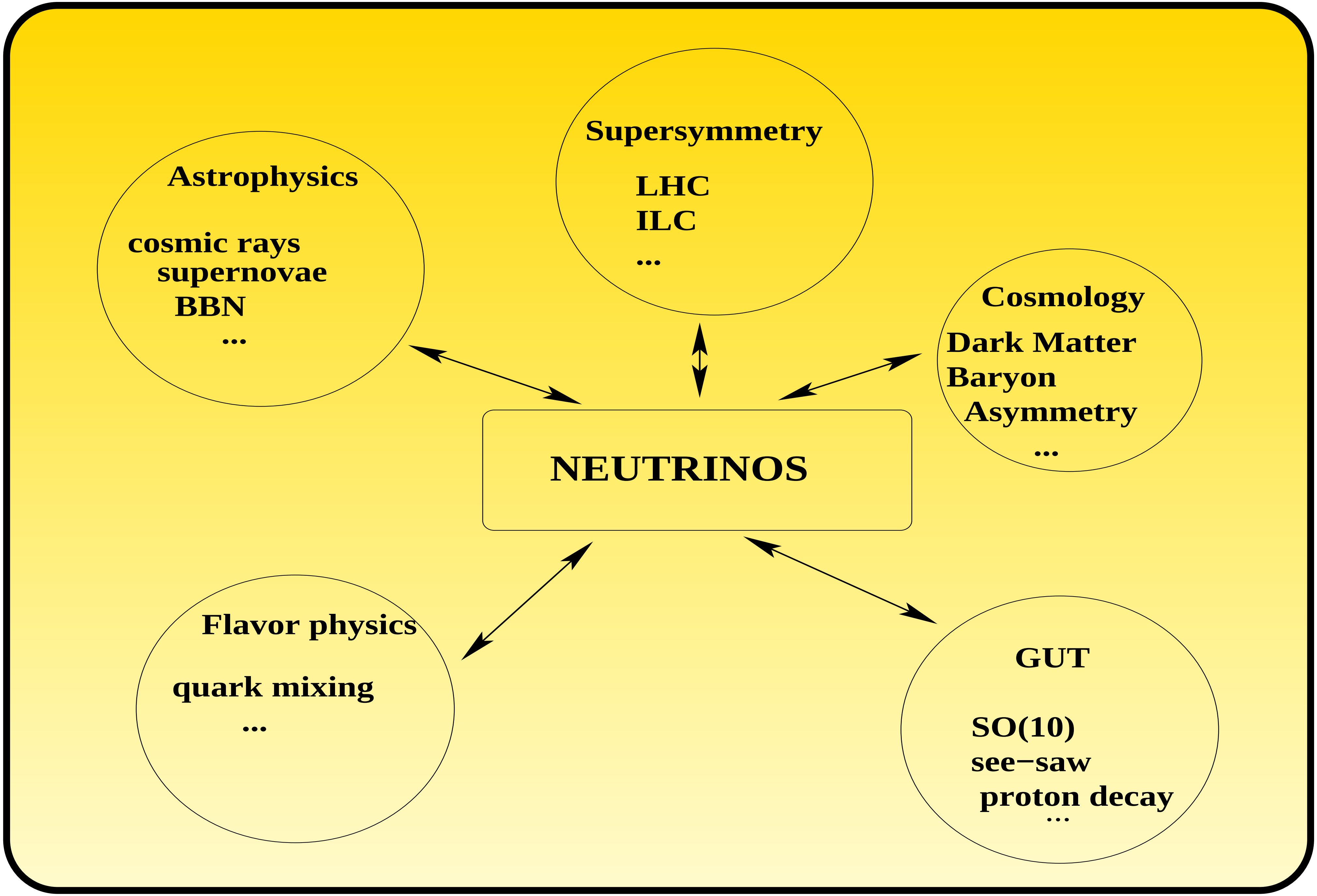
$$\mathrm{NH}: \qquad \frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_\mathrm{A}^2}} \simeq \frac{1}{5}$$

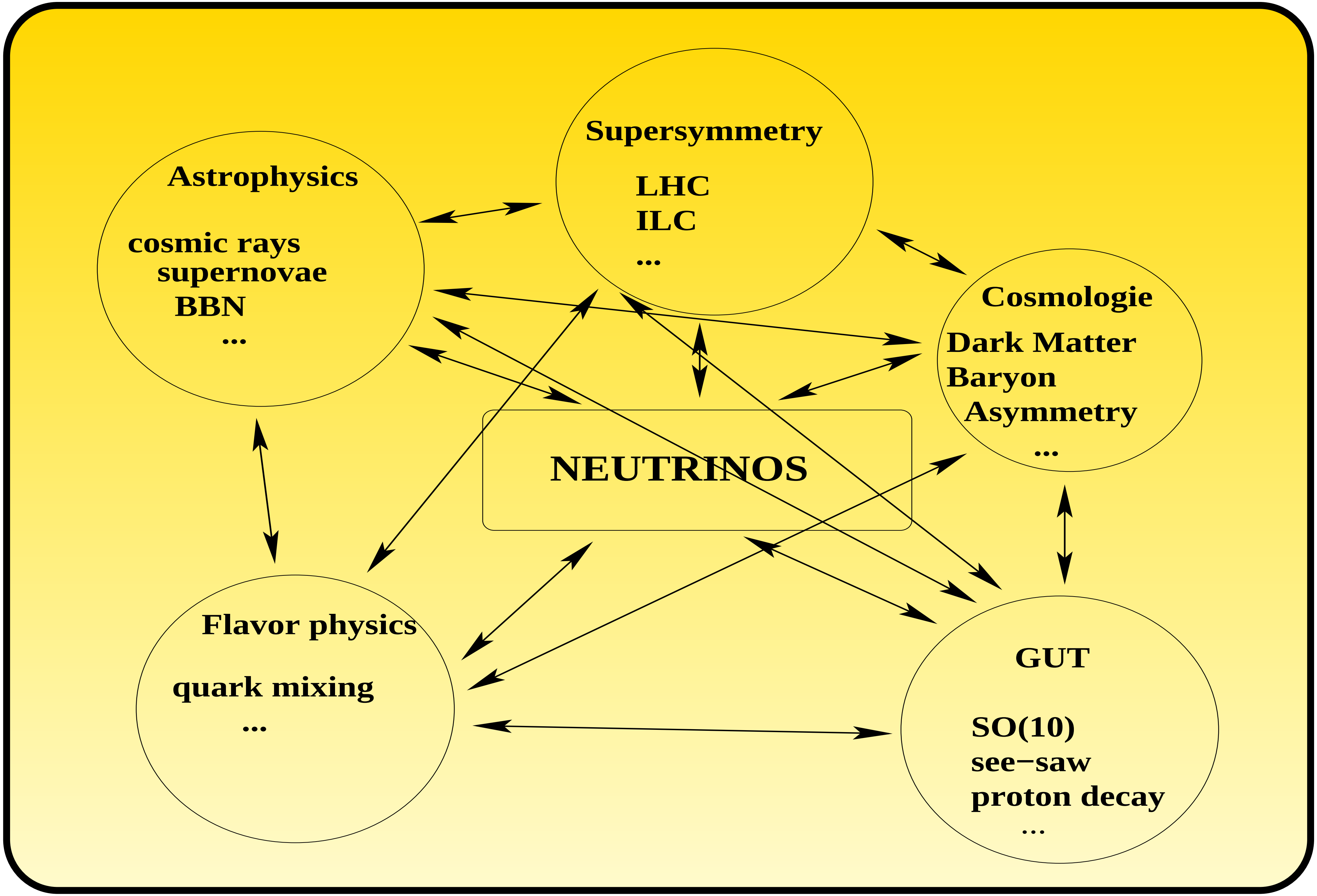
$$\mathrm{IH}: \qquad \frac{m_1}{m_2} \gtrsim 1 - \frac{1}{2}\,\frac{\Delta m_\odot^2}{\Delta m_\mathrm{A}^2} \simeq 0.98$$

Summary

Neutrinos are massive: $\Rightarrow$ 3 Tasks

- determine parameters
- explain why neutrinos are so light
- explain why leptons mix so weirdly


Supersymmetry
LHC
ILC
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmology
Dark Matter
Baryon
Asymmetry
...
NEUTRINOS
Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
...


Supersymmetry
LHC
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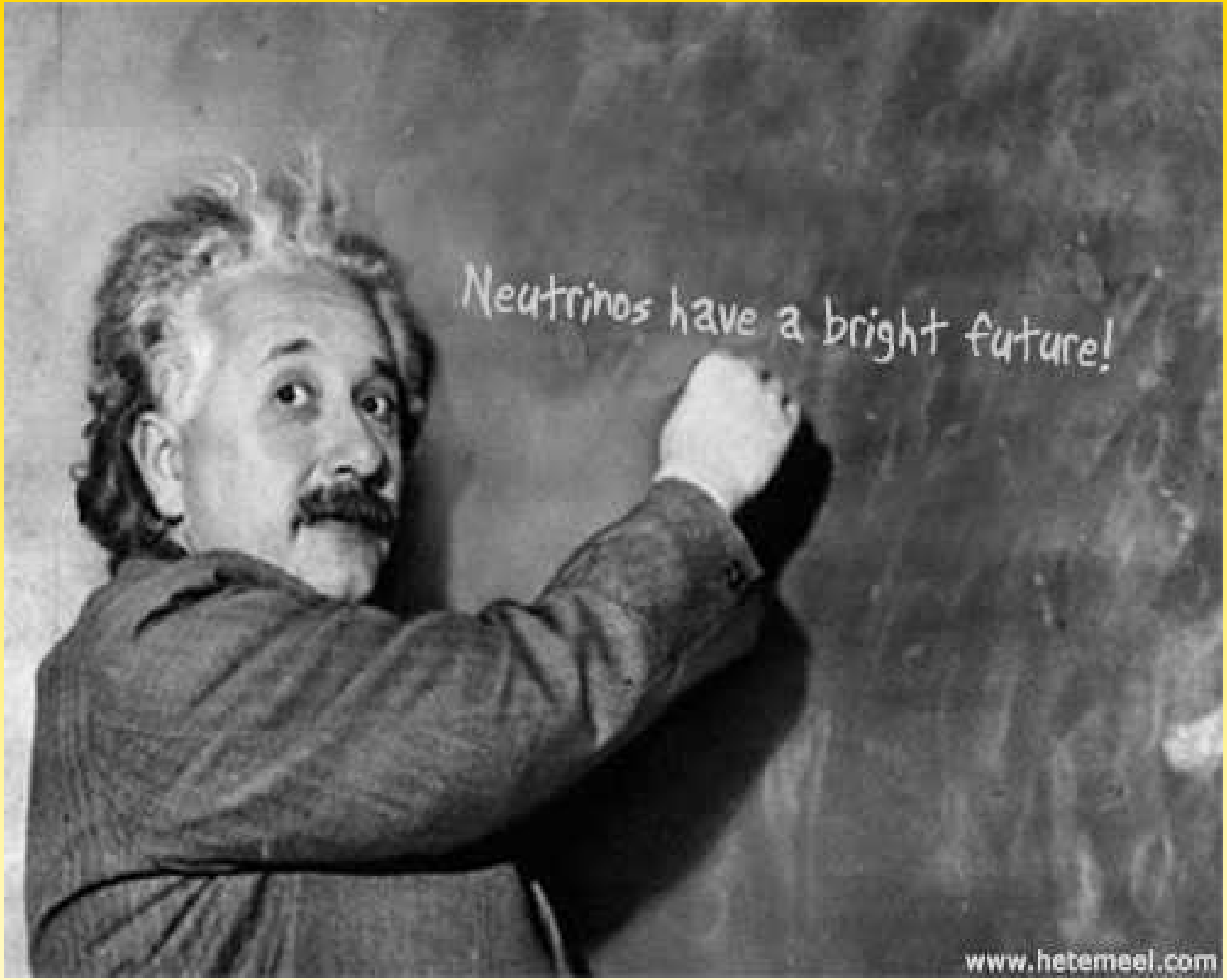
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmologie
Dark Matter
Baryon
Asymmetry
...
NEUTRINOS
Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
...

Experiment	Status	Name	Start
WČ (3 kton)	finished	Kamiokande	1983
WČ (50 kton)	running	SuperKamiokande	1996
WČ (1000 kton)	proposed	HyperK, MEMPHYS	2015?
li. Ar	in discussion	GLACIER	2015?
li. Szintillator	in discussion	LENA	2015?
Monopoles and CR	finished	MaCRO	1994
Solar B	finished	SNO	2001
Solar Be	in construction	Borexino	2006?
Solar pp	running	Gallex, SAGE	1991
Solar pp	proposed	LENS	
Reactor	finished	CHOOZ	1997
Reactor	running	KamLAND	2002
Reactor	proposed	Double-CHOOZ, DayaBay	2009
Long baseline	finished	K2K	1999
Long baseline	in construction	CNGS	2006
Long baseline	in construction	NuMI	2004
Long baseline	proposed	Noνa	2011
Long baseline	funded	T2K	2009
Long baseline	proposed	Super-beam	2010?
Long baseline	in discussion	ν-Fabrik	2020?
Long baseline	in discussion	β-beam	2020?
Cosm. Rays	running	Auger	2006
ν-telescope	funded	ANITA	2007
ν-telescope	in construction	IceCube	2009?
ν-telescope	proposed	KM3NeT	2012?
β-decay at 2 eV	finished	Mainz, Troitsk	1993
β-decay at 0.2 eV	in construction	KATRIN	2008
$0\nu\beta\beta$ at 1 eV	finished	HM	1990
$0\nu\beta\beta$ at 0.1 eV	running	Cuoricino, NEMO3	2003
$0\nu\beta\beta$ at 0.1 eV	in construction	GERDA	2008
$0\nu\beta\beta$ at 0.01 eV	proposed		2012?
t-Scale DM search	proposed		2012?
ν-couplings	finished	NuTeV	1996
$e\bar{e}$-collider (103 GeV)	finished	LEP	1989
$e\bar{e}$-collider (0.5 TeV)	proposed	ILC	2020?
pp-collider (7 TeV)	in construction	LHC	2008
Satellite	running	WMAP	2003
Satellite	in construction	Planck	2007
Satellite	in construction	GLAST	2008
Gravitational waves	running	LIGO + VIRGO	2002


Neutrinos have a bright future!
www.hetemeel.com