Flavor: Theory

What is flavor?

We have doublets and singlets ~~in~~

$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu \\ e \end{pmatrix}$ $\qquad u_R, d_R, e_R$

<u>But</u>: comes in 3 copies, only different in <u>mass</u>

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad + \text{RH singlets}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad + \text{RH singlets}$$

leads to interesting phenomenology

leads to many measurable parameters

<u>But</u>: we don't really know why there are copies, what determines the parameter values, ...

1) How to describe flavor

recall: $\mathcal{L}_Y = -g_b \bar{L} \Phi b_R - g_t \bar{L} \tilde{\Phi} t_R$ $\qquad L = \begin{pmatrix} t \\ b \end{pmatrix}_L$

$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} / \sqrt{2}$

$\tilde{\Phi} = i \tau_2 \Phi^*$

$$\Rightarrow \mathcal{L}_Y \supset -\frac{g_b}{\sqrt{2}} \bar{b}_L b_R - \frac{g_t}{\sqrt{2}} \bar{t}_L t_R$$

$$\boxed{m_{b,t} = \frac{g_{b,t} v}{\sqrt{2}}}$$

Now: 3 generations $L_1' = \begin{pmatrix} u' \\ d' \end{pmatrix}_L$; $L_2' = \begin{pmatrix} c' \\ s' \end{pmatrix}_L$; $L_3' = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$

$u_R', c_R', t_R' \equiv u_{i,R}'$ $\qquad d_R', s_R', b_R' \equiv d_{i,R}'$

"Flavor states"

$\Rightarrow$ most general Yukawa term:

$$\mathcal{L}_Y = -\sum_{i,j} \bar{L}_i' \left[g_{ij}^{(d)} \Phi d_{jR}' + g_{ij}^{(u)} \tilde{\Phi} u_{jR}' \right]$$

$$= -\sum \bar{d}_{iL}' \underbrace{\frac{g_{ij}^{(d)}}{\sqrt{2}} v}_{\equiv M_{ij}^{(d)}} d_{jR}' + \bar{u}_{iL}' \underbrace{\frac{g_{ij}^{(u)}}{\sqrt{2}} v}_{\equiv M_{ij}^{(u)}} u_{jR}'$$

$$\Rightarrow \mathcal{L}_Y \equiv -\left[\overline{d'_L}\, M^{(d)} d'_R + \overline{u'_L}\, M^{(u)} u_R\right]$$

Mass matrices $M^{(u)}, M^{(d)}$

and $d'_{L,R} \equiv \begin{pmatrix} d'_{L,R} \\ s'_{L,R} \\ b'_{L,R} \end{pmatrix}$, $u'_L = \begin{pmatrix} u'_{L,R} \\ c'_{L,R} \\ t'_{L,R} \end{pmatrix}$

Diagonalization:

$$U_d^+ M^{(d)} V_d = D^d = \mathrm{diag}(m_d, m_s, m_b)$$

$$U_u^+ M^{(u)} V_u = D^u = \mathrm{diag}(m_u, m_c, m_t)$$

$$U_u U_u^+ = V_u V_u^+ = \dots = \mathbb{1}$$

$\Rightarrow$ Basis transformation:

$$-\mathcal{L}_Y = \underbrace{\overline{d'_L}\, U_d}_{\equiv \overline{d_L}}\, \underbrace{U_d^+ M^{(d)} V_d}_{D^d}\, \underbrace{V_d^+ d'_R}_{\equiv d_R} + \underbrace{\overline{u'_L}\, U_u}_{\equiv \overline{u_L}}\, \underbrace{U_u^+ M^{(u)} V_u}_{D^u}\, \underbrace{V_u^+ u'_R}_{\equiv u_R}$$

In the new basis the mass states are diagonal

"mass states" e.g. $U_d^+ d'_L = d_L$

"physical", "propagation"

I have to transform in complete Lagrangian:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \, \overline{u'_L} \, \gamma^\mu \, d'_L$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \, \underbrace{\overline{u'_L} \, U_u}_{\overline{u_L}} \, U_u^+ \, \gamma^\mu \, U_d \, \underbrace{U_d^+ \, d'_L}_{d_L}$$

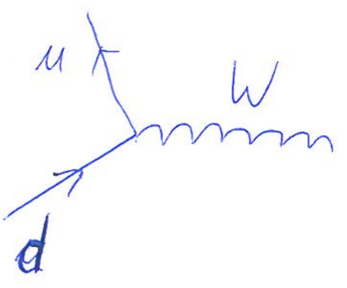
$$\Rightarrow \boxed{V = U_u^+ U_d}$$ survives in charged current

CKM - Matrix

<u>C</u>abibbo
<u>K</u>obayashi
<u>M</u>askawa

(Note: in neutral current $\propto \overline{u'_L} \, u'_L$ $\Rightarrow$ no effect at tree level)

Feynman-rules:



$$i g \gamma_\mu (1-\gamma_5) \frac{1}{2\sqrt{2}} V_{ud}$$

V can be parametrized, but I need to know the number of parameters in V.

N families (3 probably...)

	#	$\#_{tot}$
complex $N \times N$	$2N^2$	$2N^2$
$VV^\dagger = 1\!\!1$	$-N^2$	N^2
rephase u_i, d_i (one unphysical total phase)	$-(2N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}N(N-1)$ Euler angles

$\Rightarrow$ the rest must be phases

families	angles	phases
2	1	0
3	3	1
4	6	3
$\vdots$	$\vdots$	$\vdots$
N	$\frac{1}{2}N(N-1)$	$\frac{1}{2}(N-2)(N-1)$

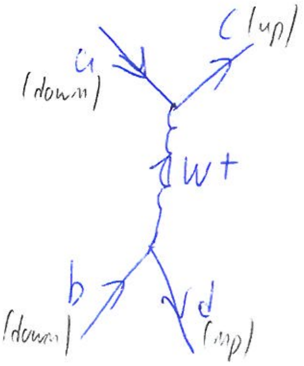
$\Rightarrow$ V complex for $N \geq 3$

for anti-quarks: $V \rightarrow V^*$ and processes can have different rates $\Rightarrow$ **CP violation**

(Nobel prize for KM in 2008)

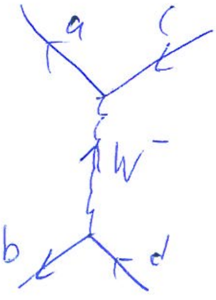
Look at it in more detail: (is actually more subtle than)

$ab \rightarrow cd$ (or $\bar c \bar d \rightarrow \bar a \bar b$)



$$\mathcal{M} \propto V_{ca} V^*_{db} \, \bar u_c \gamma_\mu (1-\gamma_5) u_a \, \bar u_d \gamma^\mu (1-\gamma_5) u_b$$

for anti-particles: $\bar a \bar b \rightarrow \bar c \bar d$ (or $cd \rightarrow ab$)



$$\mathcal{M}' \propto V^*_{ca} V_{db} \, \bar u_a \gamma^\mu (1-\gamma_5) u_c \, \bar u_b \gamma_\mu (1-\gamma_5) u_d$$

$$= \mathcal{M}^\dagger$$

e.g.: $(\bar u_c \gamma_\mu (1-\gamma_5) u_a)^\dagger = u_a^\dagger \gamma_0 \gamma_0 \left[\gamma_\mu^\dagger - (\gamma_\mu \gamma_5)^\dagger\right] \gamma_0 u_c = \bar u_a (\gamma_\mu - \gamma_\mu \gamma_5) u_c$

recall: weak interaction seems to be CP invariant: $\nu_L \Leftrightarrow \overline{\nu_R}$

to check if CP-transformed amplitude is identical to original amplitude:

CP-transformed spinors

$$C: \quad u^c = C\,\bar{u}^T \qquad ; \; C = i\gamma_2\gamma_0$$

$$P: \quad u^P = P\,u \qquad ; \; P = \gamma_0$$

$$CP: \quad P\,u^c$$

<u>e.g.</u>

$$\overline{u_c^c}\,\gamma_\mu(1-\gamma_5)\,u_a^c$$

$$= -u_c^T\,C^{-1}\gamma_\mu(1-\gamma_5)\,C\,\bar{u}_a^T$$

$$= u_c^T\left[\gamma_\mu(1+\gamma_5)\right]^T\bar{u}_a^T$$

$$= -\bar{u}_a\,\gamma_\mu(1+\gamma_5)\,u_c$$

now P: $\quad P^{-1}\gamma_\mu(1+\gamma_5)P = \gamma_\mu^\dagger(1-\gamma_5)$

$\Rightarrow$ CP-transformed ~~amont~~ amplitude

$$\mathcal{M}_{CP} = V_{ca}V_{db}^*\quad \bar{u}_a\,\gamma_\mu^\dagger(1-\gamma_5)\,u_c\quad \bar{u}_b\,\gamma_\mu^\dagger(1-\gamma_5)\,u_d$$

with $\gamma_0^{\dagger} = \gamma_0$ and $\gamma^{i\dagger} = -\gamma^i$

$$\mathcal{L}_{CP} = V_{ca} V_{db}^{*} \; \bar{u}_a \gamma_\mu (1-\gamma_5) u_c \; \bar{u}_b \gamma^\mu (1-\gamma_5) u_d$$

$\Rightarrow$ if V_{ij} are real: $\mathcal{L}_{CP} = \mathcal{L}^{\dagger}$ and theory is CP invariant

For $N > 3$: V contains phase and CP violation

Parametrization of V

$$V = R_{23}(\theta_{23}) \, R_{13}(\theta_{13}, \delta) \, R_{12}(\theta_{12})$$

with $R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$; $R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13} e^{i\delta} & 0 & 1 \end{pmatrix}$ $\qquad c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

a (phenomenological) Wolfenstein - parametrization:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - {}^1\!/_2\, \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$+ \mathcal{O}(\lambda^4)$$

comes from: $s_{12} \equiv \lambda$

$$s_{23} \equiv A\lambda^2$$

$$s_{13}\, e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)$$

$\lambda = 0.2253$ $\quad A = 0.808$

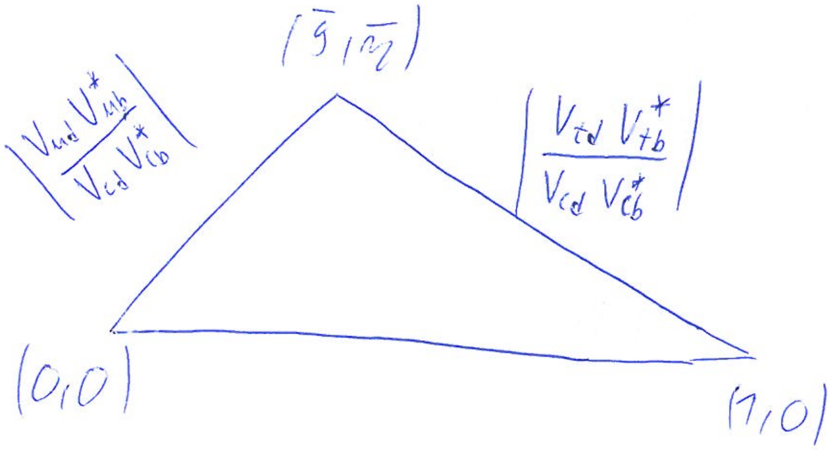
$\bar\rho = 0.132$ $\quad \bar\eta = 0.341$

$[\rho(1-\lambda^2/2)]$

Unitarity of V gives rise to <u>unitarity triangles</u>

e.g. $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \qquad (*)$

$$V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0$$



$(*)$ normalized

<u>Final remarks</u>

1) GIM-mechanism (Glashow-Iliopoulos-Maiani)

absence (= strong suppression) of flavor-changing neutral currents (FCNC)

e.g. $b \to s\gamma$

$$\mathcal{A} \propto V_{qb} V_{qs}^* f(m_q)$$

$$\simeq V_{qb} V_{qs}^* (1 + \varepsilon_q) \simeq \mathcal{O}(\varepsilon_q) \ll 1$$