Recap:

$$\langle \Phi \rangle = \sqrt{\tfrac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$ with $m_h^2 = 2 \lambda v^2$

only unknown parameter of the Standard Model (till Wednesday 9 am ?)

responsible for
1) SSB
2) Mass of fermions ($\leftrightarrow$ irresponsible for mass in the Universe!")
arxiv: 1206.1114 Wilczek

Collider limits: $(\underbrace{114.4}_{LEP} \ldots \underbrace{129}_{LHC})$ GeV

[125±2 GeV hints ?]

indirect limits: electroweak precision observables
e.g. S-parameter

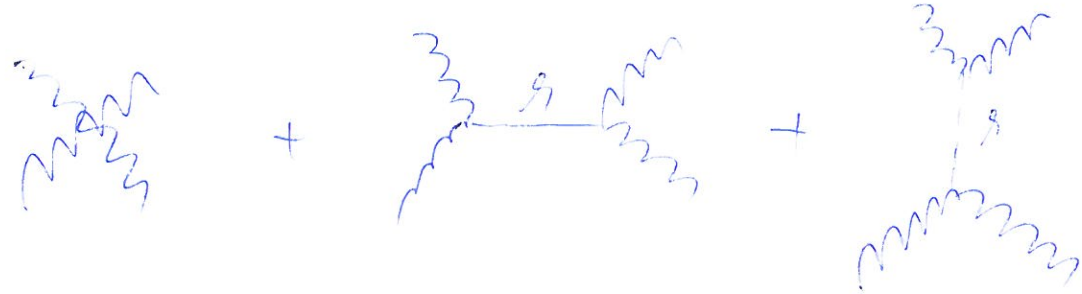
logarithmic dependence on m_h

$$m_h = 94^{+29}_{-24} \text{ GeV}$$

$$m_h \leq 125 \text{ GeV} \quad (\text{with LEP: } 171\text{ GeV})$$

another indirect limit: Unitarity

$W^+W^- \to W^+W^-$



can be messy... useful property:

equivalence theorem!

Consider polarization vectors (sheet 4)

$$\varepsilon^{\mu}_{\lambda=0} = \frac{1}{m_W}\left(|\vec{k}|, E\sin\Theta, 0, E\cos\Theta\right)$$

$$\varepsilon^{\mu}_{\lambda=\pm 1} = \frac{1}{\sqrt{2}}\left(0, \mp\cos\Theta, -i, \pm\sin\Theta\right)$$

@ high energy dominated by $\varepsilon_{\lambda=0}$

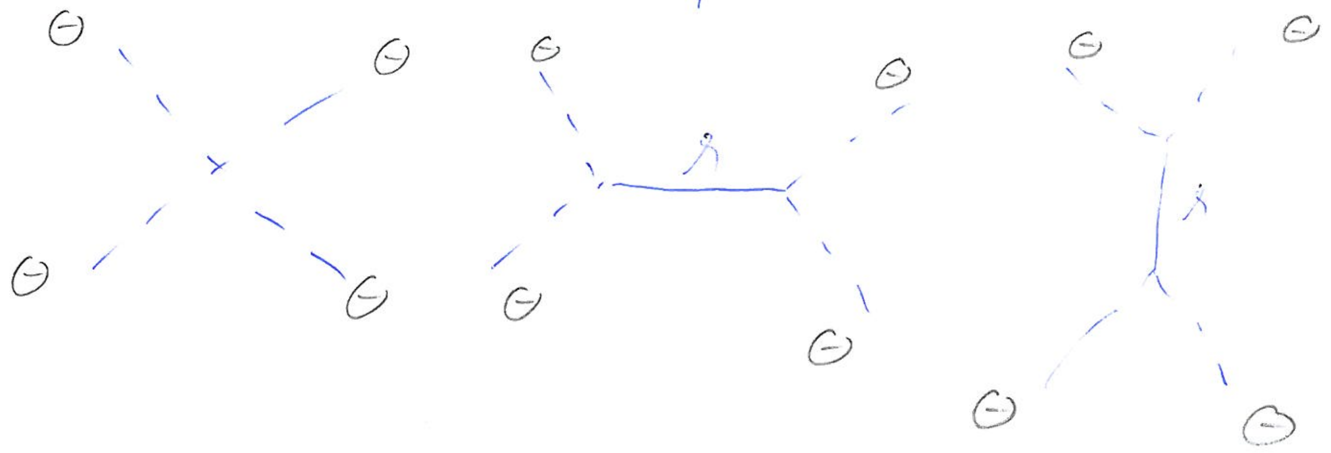
$\Rightarrow$ longitudinal polarizations

$\Rightarrow$ enough to consider Goldstone bosons

(recall: $$\Phi = \underbrace{\exp\left\{i\frac{\vec{\sigma}\vec{\Theta}}{v}\right\}}\begin{pmatrix}0\\ \frac{v+h(x)}{\sqrt{2}}\end{pmatrix}$$

$\Theta_{1,2,3}$ are GB)

Therefore consider more simple processes



need Feynman rules:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ v + h - i\Theta_3 \end{pmatrix}$$

insert in potential

$$V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

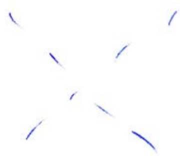
$$\Rightarrow V = \frac{m_H^2}{8v^2} \left(\sum \Theta_i^2 \right)^2 + \frac{m_H^2}{2v} h \sum \Theta_i^2 + \ldots$$

we need charged GB: $\Theta_\pm = \sqrt{\frac{1}{2}} (\Theta_1 \pm i\Theta_2)$.

Useful relation: $\Theta_1^2 + \Theta_2^2 = 2\,\Theta_+ \Theta_-$

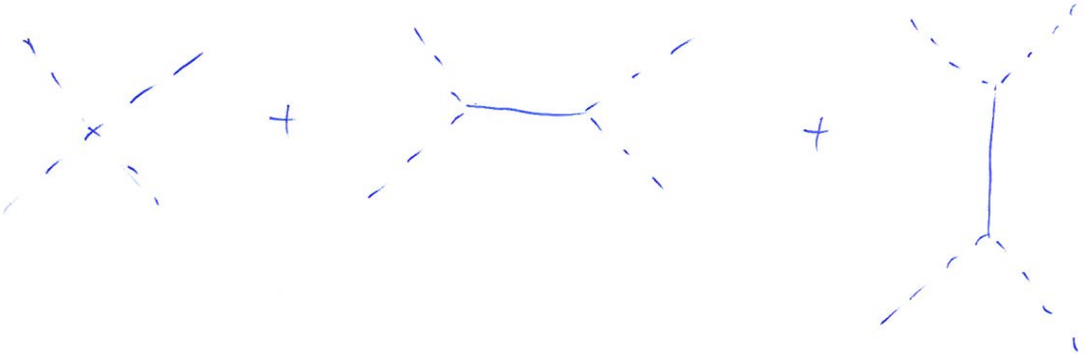
$$\Rightarrow V = \frac{m_h^2}{2v^2} \Theta_+ \Theta_- \Theta_+ \Theta_- + \frac{m_h^2}{v} h\, \Theta_+ \Theta_- + \ldots$$

$\Rightarrow$ Feynman rules



$-2i \dfrac{m_h^2}{v^2}$ (factor 4 is combinatorics)

$-i\, m_h^2 / v$



$$\Rightarrow i\mathcal{M} = \frac{-2i\, m_h^2}{v^2} + \left(\frac{-i\, m_h^2}{v}\right)^2 \left[\frac{i}{s-m_h^2} + \frac{i}{t-m_h^2}\right]$$

$$\boxed{\mathcal{M} = -\frac{m_h^2}{v^2}\left(2 + \frac{m_h^2}{s-m_h^2} + \frac{m_h^2}{t-m_h^2}\right)}$$

Matrix element for longitudinal $W_L W_L \to W_L W_L$

Now we recall lecture from May, 14:

<u>partial wave decomposition</u>

$$\mathcal{M} = 16\pi \sum_l (2l+1)\, P_l(\cos\theta)\, a_l \quad \text{with} \quad |\mathrm{Re}\{a_l\}| < \frac{1}{2}$$

$\Rightarrow$ the high energy matrix element

$$|\mathcal{M}| \approx 2\, m_h^2/v^2 < 16\pi \cdot \frac{1}{2}$$

(take the leading a_0 - term)

$$\Rightarrow \boxed{m_h < \sqrt{4\pi v^2} = 870\ \text{GeV}} \quad \text{or: } \lambda < 2\pi$$

(perturbative) unitarity limit

we have assumed that
a_0 is leading amplitude...
cancellation?

In a similar manner: for fixed m_h get $\sqrt{s}_{max}$

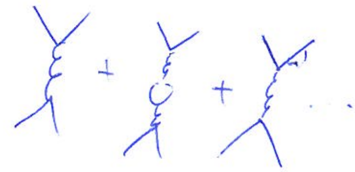
$$\Rightarrow \sqrt{s} \lesssim \text{TeV}$$

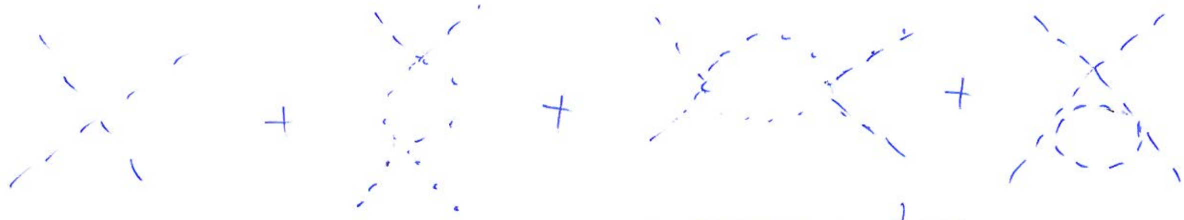
Main message: something should regularize longitudinal gauge boson scattering, and it better should do it before $\simeq$ TeV
(if something else than Higgs is there, it should show up @ TeV scale)

$\Rightarrow$ LHC

3) theoretical limits on Higgs

recall: $\alpha(Q^2) = \dfrac{\alpha_0(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log Q^2/\mu^2}$

running coupling from 

here: 

+ gauge + top + ...

$\Rightarrow \dot{\lambda} = \dot{\lambda}(Q^2) = \tilde{f}(\lambda, y_t, g, g')$

Note: y_t largest among all Yukawas and gauge-couplings

or: $\dfrac{d\lambda}{dQ^2} = f(\lambda, y_t, g, g')$..
or $\log Q^2$

2 interesting limits: a) large λ
b) small λ

a) large λ:

$$\frac{d\lambda}{d\log Q^2} \simeq \frac{3}{4\pi^2}\lambda^2 \Rightarrow \lambda \nearrow \text{ for } Q^2 \nearrow$$

"Landau pole"

<u>BUT</u>: λ should be less than, say, 4π !

$\Rightarrow \lambda$ at our energies $< \lambda_{max} \Rightarrow m_H < \ldots$

thus: maximal Higgs mass $m_h^{max}(Q^2)$

„Triviality bound" (only theory which has no Landau pole: $\lambda = 0$ „trivial theory")

b) small λ

$$\frac{d\lambda}{d\log Q^2} \simeq -\frac{3}{16\pi^2}\, y_t^4 \Rightarrow \lambda \downarrow \text{ for } Q^2 \nearrow$$

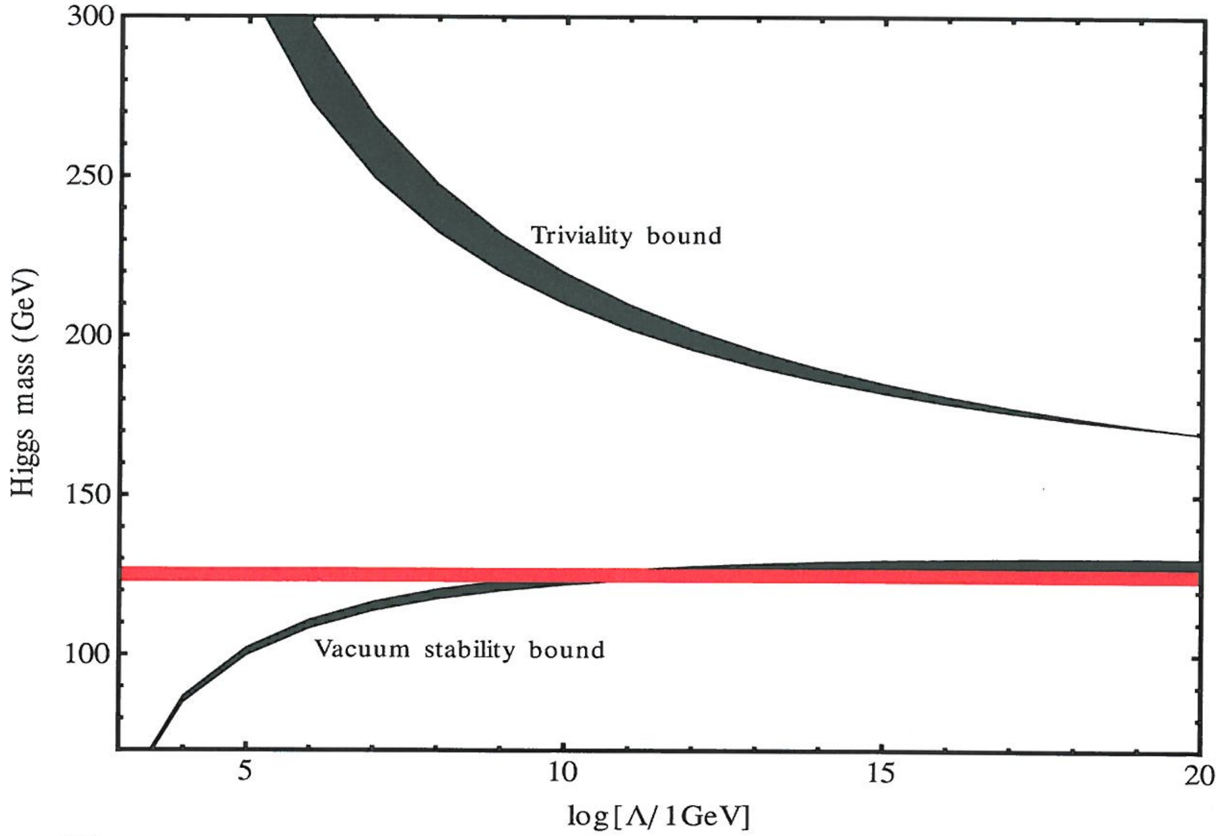
<u>BUT</u>: λ should not become negative !

$\Rightarrow$ λ at our energies $> \lambda_{min}$ $\Rightarrow$ $m_h > \ldots$

thus: minimal Higgs mass $m_h^{min}(Q^2)$

„stability bound"

$\Leftrightarrow$ scale at which new physics should have shown up !



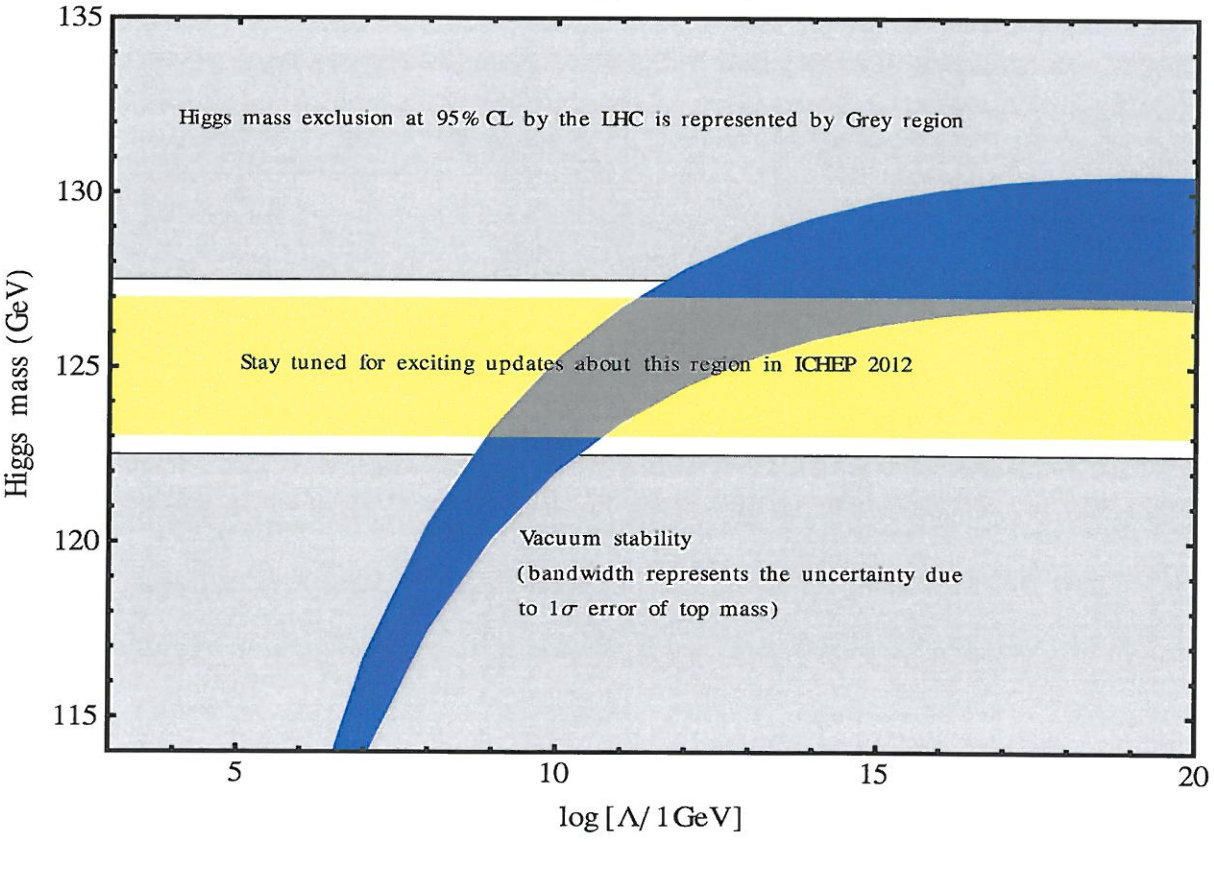


Figure 1: Plots calculated and made by Kher Sham Lim (MPIK).

$\Rightarrow$ the value $m_h \simeq 125 \pm 2$ GeV seems to be special: $\lambda(M_{Pl}) = 0$ seems possible

$\rightarrow$ quartic coupling generated radiatively

Also: $X = \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 6\lambda - 6 y_t^2 = 0$ at M_{Pl}

seems possible.

„Veltmann throat"

conection to Higgs-mass:

$$\delta m_h^2 = \frac{\Lambda^2}{32\pi^2 v^2} X$$

„hierarchy problem"
quadratic conection
to scalar masses