Recap.:

$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$

$\downarrow$ $W^\mu_{1,2,3}$ $\downarrow$ B^μ

$W^{\mu\pm} = \sqrt{\tfrac{1}{2}}\,(W_1^\mu \mp i W_2^\mu)$

$Z^\mu = c_W W_\mu^3 - s_W B_\mu \perp A^\mu$

$\tan\theta_W = g'/g = \frac{U(1)_Y \text{ coupl.}}{SU(2)_L \text{ coupl.}}$

vector coupling of A_μ

$V-A$ " of $W_\mu^\pm$

$\cancel{V}, V - a_f A$ " of Z_μ ($\rightarrow$ neutral currents)

SSB by Higgs-Doublet $\Phi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} / \sqrt{2}$

v = vacuum expectation value $= 2 m_W / g$

$h(x)$: Higgs-particle

Interpretation of Fermi-theory

$\Psi_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \sim 2_L , Y = -1$

$$\mathcal{L}_{fund} = -\frac{g}{\underbrace{2}_{L = \frac{1}{2}(1-\gamma_5)}\underbrace{\sqrt{2}}_{\text{Def. of } W_\mu^+}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right] W^{\mu +}$$

$$+ \frac{1}{2} m_W^2 W_\mu^+ W^{\mu -} + \mathcal{L}_{kin}$$

(this term comes from $\bar{\Psi}_1 W_\mu^i \frac{\sigma_i}{2} \Psi_1$)

@ low energy: $\partial_\mu W^{\mu +} = 0$ (kinetic term is irrelevant)

$\Rightarrow$ Euler Lagrange is simply $\frac{\partial \mathcal{L}_{fund}}{\partial W_\mu^+} = 0$

$$\Rightarrow W^{\mu -} = \frac{g}{2\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma^\mu (1-\gamma_5) e \right] \frac{1}{m_W^2}$$

insert back in $\mathcal{L}_{fund}$

$$\Rightarrow \mathcal{L}_{eff} = -\frac{g^2}{8 m_W^2}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu\right]\left[\bar{e}\gamma^\mu(1-\gamma_5)\nu_e\right]$$

$$\Rightarrow \boxed{\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}}} \Rightarrow \frac{g}{m_W} \simeq \frac{1}{123\ \text{GeV}}$$

from μ-decay

$$\Rightarrow \boxed{v = 246\ \text{GeV}}$$

with $m_W \simeq 80$ GeV : $g \simeq 0.65$ (Note: $e = \sqrt{4\pi\alpha} \simeq 0.3$)

↳ Fermi theory is „effective theory", i.e. „ultraviolet completion" is not observable.

⟷ fundamental energy scale of theory $\gg$ energy scale of observation,

i.e. $m_W \gg m_\mu$

Running coupling, β-function, QCD

For QED: $$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log Q^2/\mu^2}$$

μ^2: reference scale

$Q^2 \nearrow \quad \alpha \nearrow$

The β-function of QED is $\beta_\alpha = \frac{\alpha^2}{3\pi}$ ($\times N_\ell$ for N_ℓ charged leptons)

Definition: $$\boxed{\frac{d\alpha}{d\log Q^2} = \beta_\alpha}$$

(same definition for other couplings, masses, ...)

formal solution of this diff. eq. would give α^3

$\Rightarrow$ trick: $g \equiv \alpha^{-1}$

$$\frac{d\alpha}{d\log Q^2} = \frac{d}{d\log Q^2}\frac{1}{g} = -\frac{1}{g^2}\frac{dg}{d\log Q^2} = \frac{1}{3\pi}\frac{1}{g^2}$$

$$\Rightarrow \frac{dg}{d\log Q^2} = -\frac{1}{3\pi} \Rightarrow g(Q^2) = -\frac{1}{3\pi}\log Q^2 + C$$

C from boundary condition = reference measurement

$$g(\mu^2) = -\frac{1}{3\pi}\log\mu^2 + C$$

$$\Rightarrow \quad g(Q^2) = -\frac{1}{3\pi}\log Q^2 + g(\mu^2) + \frac{1}{3\pi}\log \mu^2$$

$$= g(\mu^2) - \frac{1}{3\pi}\log Q^2/\mu^2$$

$$\Rightarrow \quad \alpha(Q^2) = \frac{1}{\frac{1}{\alpha(\mu^2)} - \frac{1}{3\pi}\log Q^2/\mu^2} = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log Q^2/\mu^2}$$

q.e.d.

Note that in QCD:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{4\pi}\left(\frac{11}{3}N - \frac{2}{3}N_f\right)\log Q^2/\mu^2}$$

where $N = 3 \quad (SU(N) = SU(3))$

$N_f = 6 \quad (u, c, t, d, s, b)$

$\Rightarrow$ 1) $\alpha_s \searrow$ for $Q^2 \nearrow$ as long as $N_f \leq 16$

<u>asymptotic freedom</u>

2) $\alpha_s \nearrow$ for $Q^2 \searrow$ <u>confinement</u>

confinement happens when $\alpha_s = \infty$, or

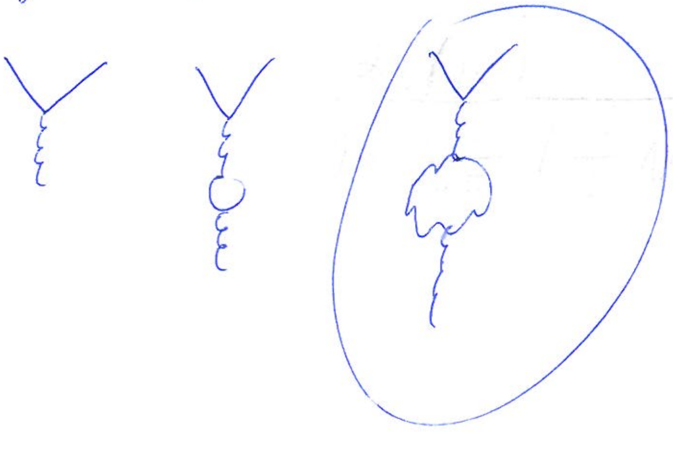
$$Q^2 = \mu^2 \exp\left\{\frac{-12\pi}{(33-2N_f)\,\alpha_s(\mu^2)}\right\} \equiv \Lambda^2$$

use $\alpha_s(M_Z^2) = 0.12 \;\Rightarrow\; \Lambda \simeq 51$ MeV

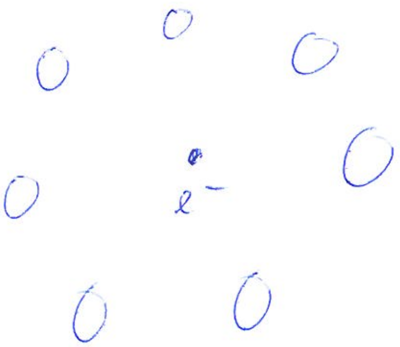
usually: $\Lambda \simeq (200 \ldots 300)$ MeV

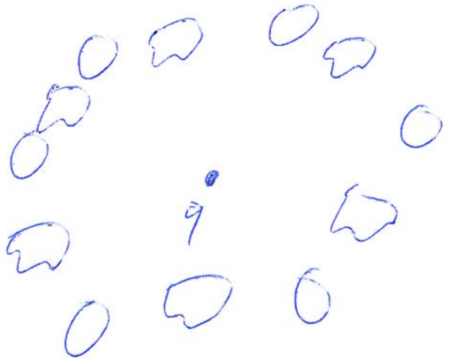
Hadronization scale of QCD

Origin of different behavior with respect to QCD



Self-interacting gauge fields





$e^\pm$-Pairs

$Q^2 \nearrow$ see more charge

$Q^2 \searrow$: <u>screening</u>

screening of $q\bar{q}$ pairs

<u>anti-screening</u> of gluons

wins unless $N_f \geq 16$

Fermion masses

nice feature: fermion masses generated by Higgs-doublet as well!

$$\mathcal{L} = -g_d \overline{\Psi}_1 \, \phi \, d_R \quad - g_u \overline{\Psi}_1 \, \tilde{\phi} \, u_R$$ "Yukawa coupling"

Y: $\quad -\frac{1}{3} \quad +1 \quad -2/3 \qquad\qquad -\frac{1}{3} \quad -1 \quad +4/3 \qquad \Psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L$

with $\tilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} \phi_2^* \\ -\phi_1^* \end{pmatrix} \longrightarrow \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} / \sqrt{2}$

terms are invariant:

1) $\Psi_1 \to U \Psi_1$; $\phi \to U \phi \Rightarrow \overline{\Psi}_1 \phi \to \overline{\Psi}_1 \phi$

and singlet d_R added to generate dim 4 term and settle hypercharge balance

2) $\overline{\Psi}_1 \tilde{\phi} \to \overline{\Psi}_1 U^+ i \sigma_2 U^* \phi^* = \overline{\Psi}_1 i\sigma_2 \phi^*$

because: $-i\sigma_{\substack{1\\2\\3}} \, i\sigma_2 (-i\sigma_{\substack{1\\2\\3}}^*) = i\sigma_2$

recall: $U = \exp\left\{ i \frac{\sigma_i}{2} \alpha_i \right\} \qquad \sigma_i = \sigma_i^+$

$$\Rightarrow \mathcal{L} = \underbrace{-g_d \frac{v}{\sqrt{2}}}_{m_d} \bar{d}_L d_R \underbrace{- g_u \frac{v}{\sqrt{2}}}_{m_u} \bar{u}_L u_R + \underbrace{g_d \bar{d}_L d_R\, h(x)/\sqrt{2}}_{\text{Higgs-fermion-interaction}}$$

$\sim g_d = \frac{m_d}{v}$

- Note that mass term $m \bar{d}_L d_R$ is forbidden by gauge invariance! $\Rightarrow$ only via Higgs

- Note that one Higgs-doublet is enough to create mass terms for up- <u>and</u> down quarks

- recall: no ν_R $\Rightarrow$ no mass for neutrino

- <u>same</u> thing breaks electroweak symmetry and creates fermion masses.

- coupling of Higgs to fermions (and gauge bosons) is proportional to <u>mass</u>

Theoretical constraints on the Higgs

(winter term: „Higgs physics at the LHC" by Plehn
+ „SM II: theory" by Lindner)

various approaches: direct (→ André Schöning)
indirect (today + 2.7.)
theoretical (2.7.)

we know everything about the Higgs, except for: its mass $\boxed{m_H^2 = 2 v^2 \lambda}$

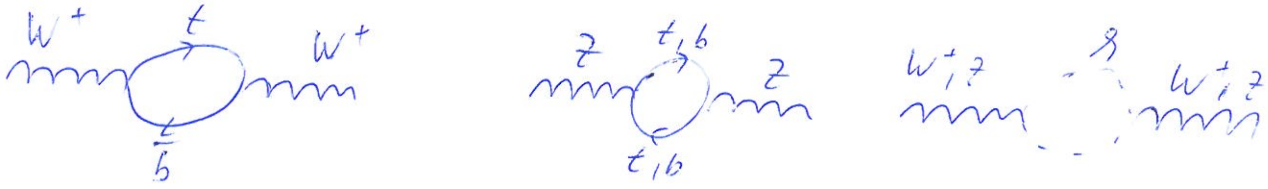
1) direct limits: collider

current status 95% CL:
- LEP: 114.4 GeV lower limit
- D∅+CDF: exclude $147 < m_H < 179$ GeV
- ATLAS: excludes (110.0 ... 117.5) GeV
  (118.5 ... 122.5) GeV
  (129 ... 539) GeV
- CMS: excludes (127 ... 600) GeV

$\Rightarrow$ $\boxed{\text{probably between } (114.4 \dots 129)\text{ GeV}}$

2) indirect limits

m_H shows up in loop-diagrams, e.g.



shifts pole of propagator, i.e. mass

$$\Rightarrow \Delta\rho = \frac{3 G_F}{8\pi^2\sqrt{2}}\left[m_t^2 + m_b^2 - 2\,\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\log\frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 s_W^2 \log\frac{m_H^2}{m_Z^2}\right]$$

- $m_{t,b}$ part $\to 0$ for $m_t = m_b$
- $\Delta\rho \to 0$ for Yukawa couplings $= 0$
  for gauge couplings $= 0$
  "custodial symmetry"

- scalar contribution logarithmically ...
- observed to be very close to $\Delta\rho \simeq 0$

Fit to 9 and many many many more observables...

$$m_H = 94^{+29}_{-24}\ \text{GeV} \quad (68\%\ \text{C.L.})$$

$$\Rightarrow\ m_H \leq 125\ \text{GeV} \quad (95\%)$$

include LEP: $m_H \leq 171\ \text{GeV}$ (95%)