Recap:

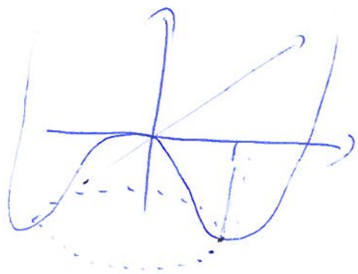
Gauge symmetries forbid mass terms for gauge bosons: $m_W^2 W_\mu W^\mu$

Observation ("Higgs-Mechanism"):

add scalar particle: $(D_\mu \phi)(D^\mu \phi)^* - \mu^2 \phi^* \phi - \lambda(\phi\phi^*)^2$

$D_\mu = \partial_\mu - ie A_\mu$

$(A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha)$



$v^2 = -\mu^2 / 2\lambda$

do physics around minimum: $\phi = \sqrt{\frac{1}{2}}\left(v + \eta(x) + i\xi(x)\right)$

$$\Rightarrow \mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2}_{\text{kinetic terms}} \underbrace{- v^2 \lambda \eta^2}_{m_\eta} + \underbrace{\frac{1}{2} e^2 v^2 A_\mu A^\mu}_{m_A} \underbrace{- e v A_\mu \partial^\mu \xi}_{\text{WTF?}}$$

Trick: $\phi \simeq \sqrt{\frac{1}{2}}(v + \eta(x))\, e^{i\xi/v}$ $\Rightarrow$ perform gauge trafo with $-\xi/v = \alpha$

$\Rightarrow$ Goldstone-boson ξ disappears from theory and becomes 3rd d.o.f. of A_μ; + 1 scalar particle

C1

This scalar particle is called the

Higgs-boson

We further need today the Goldstone theorem:

if the vacuum is ~~[illegible]~~ invariant under M generators of the gauge theory (that is generated by $N > M$ generators) then there are $N - M$ Goldstone bosons.

$\Rightarrow$ the gauge boson associated with an unbroken generator remains massless

(if the gauge is local: eaten by gauge bosons)

Also: generalization of $U(1)_{em}$ with charge operator $\hat{Q}$

e.g. $\hat{Q}\, u = 2/3$ $\qquad$ $\hat{Q}\, e = -1$

and $U(1)_{em}$ generated by $\hat{Q}$: $e^{i\alpha(x)\hat{Q}}$

~~this scalar particle is called a Higgs boson~~

$\hookrightarrow$ on towards the SM : $SU(2)_L \times U(1)_Y$

$$\psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim 2_L \,,\ Y_1 \qquad \left(\text{or } \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim 2_L ,\ \tilde{Y}_1 \right)$$

$$\psi_2 = u_R \quad \sim 1_L ,\ Y_2 \qquad (\text{or } \nu_R \sim 1_L ,\ \tilde{Y}_2)$$

$$\psi_3 = d_R \quad \sim 1_L ,\ Y_3 \qquad (\text{or } e_R \sim 1_L ,\ \tilde{Y}_3)$$

note: • maximal parity violation

• different charge of u, d (and ν, e)

• doublet has same Y

• u_R, d_R to have mass terms

• property of $\frac{\sigma_3}{2}$ is called „weak isospin"

$$I_3^{u_L} = \frac{1}{2} \qquad I_3^{d_L} = -\frac{1}{2} \qquad I_3^{u_R} = 0 \text{ etc.}$$

„singlet"

„generated by $\exp\{ i \frac{\sigma_3}{2} \alpha(x) \}$" $\equiv \exp\{ i \hat{I}_3 \alpha(x) \}$

+ $U(1)_Y$ generated by $\exp\{ i/2\, \hat{Y} \beta(x) \}$

$\odot$ ☺

The transformation rules

$$\psi_1 \rightarrow \psi_1' = \exp\left\{\tfrac{1}{2} y_1 \beta(x)\right\} \mathcal{U}_L \psi_1$$

$$\psi_2 \rightarrow \psi_2' = \exp\left\{\tfrac{1}{2} y_2 \beta(x)\right\} \psi_2$$

$$\psi_3 \rightarrow \psi_3' = \exp\left\{\tfrac{1}{2} y_3 \beta(x)\right\} \psi_3$$

with $\mathcal{U}_L = \exp\left\{ i \, \sigma_i/2 \; \alpha_i(x) \right\}$ $\quad \sigma_i$: Pauli matrices

$\Rightarrow$ expect 4 gauge bosons

The scalar particle to allow for massive weak bosons:

„Higgs-Doublet" $\quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \sim 2_L \,,\; y_\Phi$

has the usual potential: $V = \mu \, \Phi\Phi^\dagger + \lambda (\Phi^\dagger \Phi)^2$

$$\Rightarrow \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} \equiv \frac{1}{2} v^2$$

and choose $\phi_3^2 = v^2$.

we choose hypercharge such that

$$\boxed{\hat{Q} = \hat{I}_3 + \frac{Y}{2}}$$

is fulfilled !

from $\bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1$ and $\bar{\psi}_i \gamma^\mu \frac{Y_i}{2} \psi_i$ "motivated"

electric charge — weak isospin — hypercharge

Therefore: if the Higgs-vev conserves this operator Q, $U(1)_{em}$ will be conserved (even though $SU(2)_L$ and $U(1)_Y$ are broken) and the photon will be massless!

i.e. the gauge boson associated with Q

$$\Rightarrow \quad Y_\phi = 1 \quad \leftrightarrow \quad Q \begin{pmatrix} 0 \\ v \end{pmatrix} = \left[\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

vacuum has to have $Q = 0$

$$= 0$$

or $\begin{pmatrix} 0 \\ v \end{pmatrix} \to e^{i\alpha(x)Q} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$

vacuum invariant under $U(1)_{em}$ trafo

$\Rightarrow$ write $\Phi(x) = \exp\left\{ i \frac{\vec{\sigma}\vec{\Theta}}{v} \right\} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} / \sqrt{2}$

to get rid of $\vec{\Theta}$ by an appropriate gauge $\Rightarrow$ 3 gauge bosons massive!

proof that I can do this:

$$\Phi = (1 + i\sigma_1 \Theta_1/v + i\sigma_2 \Theta_2/v + i\sigma_3\Theta_3/v) \begin{pmatrix} 0 \\ v+h \end{pmatrix} / \sqrt{2}$$

$$= \ldots = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \Theta_2 + i\Theta_1 \\ v + h - i\Theta_3 \end{pmatrix}$$

- The covariant derivative for Φ:

$$\partial_\mu \to D_\mu = \partial_\mu + \frac{1}{2}(\underbrace{ig W_\mu^i \sigma_i}_{\text{fields of } SU(2)_L} + i g' \underbrace{Y_\Phi}_{\text{Hypercharge}} \underbrace{B_\mu}_{\text{field of } U(1)_Y})$$

note:
- different charges g, g'
- Y_Φ in trafo + cov. der.

~~• VEV is neutral~~

consider the kinetic term:

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) \quad \text{contains } W_\mu^i ,\ B_\mu$$

- only interested in terms with v
- define $W_\mu^\pm = \sqrt{\tfrac{1}{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$; $(W_\mu^-)^\dagger = W_\mu^+$

$$\Rightarrow \mathcal{L} = \frac{1}{2} v^2 g^2 W_\mu^+ W^{\mu -}$$

$$+ \frac{v^2}{8} (W_\mu^3 , B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

first term: $\boxed{m_W^2 = \frac{1}{4} v^2 g^2}$

2nd term: „non-diagonal mass matrix"

↳ diagonalize!

Note: rank of mass matrix = 1

$\Rightarrow$ 1 massless state !

Diagonalization gives

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \qquad M_A = 0$$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \qquad \boxed{M_Z^2 = \frac{v^2}{4}\left(g^2 + g'^2\right)}$$

$$\tan \Theta_W \equiv g'/g$$ Weinberg angle

$$\Rightarrow \boxed{\begin{aligned} A_\mu &= c_W B_\mu + s_W W_\mu^3 \\ Z_\mu &= c_W W_\mu^3 - s_W B_\mu \end{aligned}}$$

"physical fields" with diagonal mass terms

Note: $$\frac{M_W}{M_Z} = \cos^2 \Theta_W$$

ρ-parameter: $$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \Theta_W} = \frac{(80.399\,\text{GeV})^2}{(91.1876\,\text{GeV})^2 \cdot 0.7688}$$

very important to test SM with respect to extensions!

we need to rewrite the covariant derivatives in terms of the physical fields

$\partial_\mu \to D_\mu$ for Ψ_1 has terms $Z_\mu(\ldots)$

$$+ A_\mu \left[g\, I_3^{u_L} s_W \, \bar{u}_L \gamma^\mu u_L + g' \frac{Y_1}{2} c_W \, \bar{u}_L \gamma^\mu u_L \right]$$

$$\Longrightarrow \boxed{g\, s_W = g' c_W = e}$$

Examples: $Y_{u_L} = Y_{d_L} = 1/3$

$Y_{d_R} = -2/3 \qquad Y_{u_R} = 4/3$

$Y_{\nu_L} = -1$

($Y_{\nu_R} = 0$: „total singlet") leave out! ($m_\nu = 0$)

Neutral current terms: $Z_\mu(\ldots] \subset \frac{e}{2 c_W s_W} Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f$

	u	d	ν	e
$2v_f$	$1 - 8/3\, s_W^2$	$-1 + 4/3\, s_W^2$	1	$-1 + 4 s_W^2$
$2a_f$	1	-1	1	-1